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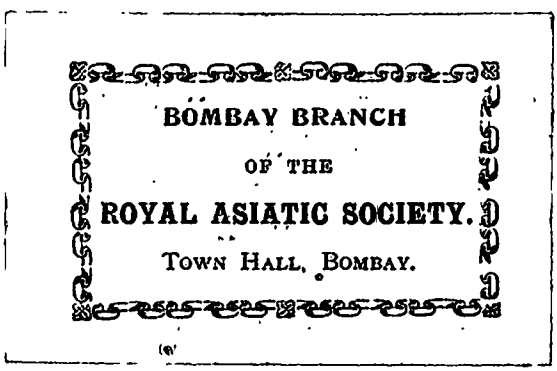

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OBSERVATIONS

ON

REVERSIONARY PAYMENTS;

ON

SCHEMES for providing ANNUITIES for WIDOWS, and for Perſons in OLD AGE;

ON

THE METHOD OF CALCULATING THE VALUES OF ASSURANCES ON LIVES;

AND ON

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THE NATIONAL DEBT.

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ALSO,

ESSAYS on different Subjects in the Doctrine of LIFE-ANNUITIES and POLITICAL ARITHMETIC;

A Collection of NEW TABLES, and a POSTSCRIPT on the POPULATION of the KINGDOM.

BY RICHARD PRICE, D.D. F.R.S.

TO WHICH ARE ADDED,

ALGEBRAICAL NOTES, the SOLUTION of ſeveral NEW PROBLEMS in the DOCTRINE of ANNUITIES,

And a GENERAL INTRODUCTION.

BY WILLIAM MORGAN, F.R.S.

FIFTH EDITION.

VOL. I.

LONDON:

PRINTED FOR T. CADELL, IN THE STRAND.

M.DCC.XCII.

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TO

THE RIGHT HONOURABLE

THE

EARL OF SHELBURNE,

THIS WORK

Is, with all GRATITUDE and RESPECT,

INSCRIBED,

BY

His LORDSHIP'S

Moſt obliged, and

Moſt obedient humble Servant,

March 29th,
1783.

RICHARD PRICE.

GENERAL INTRODUCTION.

THE following invaluable work is one of the fruits of a life which was uniformly devoted to ſtudious enquiry, and to the promotion of the beſt intereſts of mankind. The motives that firſt led the author to engage in theſe labours were the moſt humane and honourable, and the ſucceſs that attended them, he conſidered as the higheſt reward and gratification he could enjoy. Even in preparing his notes for the preſent edition, which unfortunately were never completed, he mentions his life as then drawing near its cloſe, but that he had reaſon to reflect with ſatisfaction on the time which he had employed in thoſe purſuits.

The different Prefaces which have been affixed to the former editions render it un-

neceſſary to enter minutely into the origin and progreſs of this work. But it may not be improper to obſerve, that at the time of its firſt publication in 1769, the advice and inſtruction which it contained could not poſſibly have been communicated at a more ſeaſonable opportunity.—The various Societies for the benefit of age and widows which, like the preſent Tontines, were then continually riſing up to allure and to defeat the hopes of the ignorant and diſtreſſed, were become an object of ſerious alarm, and if the evil had not been effectually oppoſed, it is difficult to imagine the extent to which it might have proceeded. On the firſt appearance of this work the rage for eſtabliſhing *new* Societies immediately ſubſided, a *partial* reformation took place in ſome of thoſe which had been already formed, and in a ſhort time the greater part of them, convinced of their miſtakes, diſſolved themſelves. A few, indeed, perſevered in an obſtinate adherence to their original plans, but they have lately exhibited a melancholy proof of their own folly, and of the truth and juſtice of the admonitions which had been waſted upon them.

It

It was Dr. *Price*'s intention to have written a new Preface to the preſent edition, in which he meant to have continued his account of the ſtate and progreſs of the few Societies which now remain, and alſo to have inſerted whatever additions and remarks he might have thought neceſſary to the improvement of the following volumes.—But he died ſoon after the *firſt* of them was printed, and has left only a few detached hints and obſervations in regard to the plan which he was to have adopted.—Being anxious to exert every effort in my power towards fulfilling his intentions I ſhall endeavour, though in a much inferior manner, to purſue the method which he had pointed out for himſelf; but not without lamenting the event which has obliged me to engage in ſo arduous an undertaking.

Beſides the peculiar ſatisfaction which he derived from this work as having been inſtrumental in doing much good, Dr. *Price* was alſo accuſtomed to reflect on it with pleaſure as it contained, in his opinion, many improvements in the doctrine of Annuities and in Political Arithmetic.—It was his wiſh therefore to have rendered it as com-

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plete as poſſible; and the great number of tables and valuable obſervations with which he has enriched each edition, and particularly the *laſt*, are a proof of his zeal and ſucceſs in accompliſhing this purpoſe.—To the preſent edition a new table has been added of the values of two joint lives, computed under his direction from the probabilities of life at *Northampton*, reckoning intereſt of money at 6 *per cent.* and alſo three other tables of the values of a ſingle life, communicated and computed by myſelf, from the ſame table of obſervations at the ſeveral rates of 6, 7, and 8 *per cent.* Theſe I believe were all the *new* tables, excepting thoſe in the firſt Appendix, which Dr. *Price* meant to have added to this edition:—nor did he ſeem to entertain the moſt remote idea of making any material alterations either in the arrangement or the matter of the ſecond volume, which he did not live to correct.—When this edition was put to the preſs it was done in ſome haſte, and his other engagements prevented him from attending to the correction of it before the two firſt chapters were printed off. Had not this been the caſe, he meant inſtead of the preſent

ſent anſwers to the 11th and 12th queſtions in the firſt Chapter, to have ſubſtituted others from a paper communicated by myſelf to the *Royal Society* in the year 1788, which was honoured with their approbation, and publiſhed in the 78th Volume of their Tranſactions. But being diſappointed in this intention, he reſerved what he had to ſay on the ſubject for a Note in the ſecond Appendix.—As far as relates to theſe queſtions I have endeavoured to ſupply the deficiency. I have alſo explained ſome of the notes in this Appendix which appeared to me to be too conciſe, and have deduced from the real probabilities of life other ſolutions of ſuch Problems as had been derived from Mr. *De Moivre*'s hypotheſis,—intending by this means to give a further proof that it can ſeldom or ever be neceſſary to have recourſe to ſuch an expedient in the doctrine of Annuities.—In conſequence of a reference made by Dr. *Price* in a note at the end of his third Eſſay (Vol. I.), I have inſerted all the different rules reſpecting the values of reverſions depending on ſurvivorſhips between three lives, which I had communicated to the *Royal Society* in the year

1789

1789 and 1791, and which were publiſhed in the 79th and 81ſt Volumes of the Philoſophical Tranſactions.—Theſe, together with a few notes in the ſecond Volume (ſome of which had become indiſpenſably neceſſary ſince the publication of the laſt edition), comprehend all that I have ventured to add to this work.—The text has been preſerved invariably the ſame, and the additions have conſequently been kept ſeparate; ſo that, if any errors have been committed, they may be eaſily aſcribed to their real author.

As all the ſolutions in the firſt Chapter, except thoſe of the 11th and 12th queſtions, are ſtrictly true (*) according to any table of the probabilities of life, I do not know that this part of the work admitted of any other improvement than the addition in thoſe two queſtions of correct rules in lieu of the approximations which had been taken from Mr. *Simpſon's* Select Exerciſes. But in regard to the ſecond Chapter, which contains an account of the different Societies for the benefit of age and widows, the circumſtances

(*) The *Solutions* of the 15th and 16th queſtions are true, and the *rules* deduced from them are ſufficiently correct for any uſeful purpoſe.—See Note (H) 2d vol.

and

and even plans of ſome of theſe have undergone ſuch conſiderable alterations ſince the year 1783, when the laſt edition was publiſhed, that it becomes neceſſary, agreeable to the author's own intention, to give a more accurate account of their preſent ſtate.

The LONDON ANNUITY SOCIETY for the benefit of widows had been eſtabliſhed about five years before the firſt publication of the Treatiſe on Reverſionary Payments, and, like all the other inſtitutions of the ſame kind, had been founded on ſuch principles as muſt have inevitably produced its ruin. The premiums it required were almoſt *one half* too low for the annuities it promiſed, and nothing could have ſaved it but an immediate and entire alteration of its plan.—Happily, the advice and inſtruction which were offered to the members by Dr. *Price* were not waſted upon them. The premiums in a ſhort time were increaſed, and the annuities reduced ſo far as to ſecure the permanence of the Society; and ſubſequent experience has proved the wiſdom and ſucceſs of this reformation.—In the year 1780 the annuities of thoſe widows whoſe huſbands had continued their payments 15 years were raiſed from £20 to £24; and

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ſo favourable had events appeared to prove during the courſe of the ten following years, that it was reſolved in 1790 to lay the accounts of the Society before Dr. *Price* and myſelf, in order that we might determine whether they were not capable of making ſtill further additions to their annuities.

In January 1789 the Society conſiſted of 328 members, holding 377 policies, the number of widows was 85, and their annuities amounted to £1967; their income, including intereſt on £70,500 ſtock in the 3 *per cents*, general premiums, annual payments, &c. exceeded their claims and expenditure above £2000 a year, and their real *ſurplus* Stock, after diſcharging every probable demand from all the preſent and future widows, appeared by a minute inveſtigation to be ſo conſiderable as to juſtify an addition to the annuities, provided a few alterations were made in their plan.—In conſequence therefore of our joint opinion and advice thoſe alterations have in a great meaſure been adopted, the Society, in addition to the annuity of £20 if a ſubſcriber lives one year and £24 if he lives 15 years, engages to pay a further annuity of £1 for every year that a ſubſcriber lives beyond

beyond 24 years, and there is every reaſon to believe that this addition may be made without in the leaſt endangering its ſtability. —The friends and founders of this inſtitution muſt derive much pleaſure from beholding it in a ſtate ſo proſperous, and alſo from reflecting that, by their candour and prudence in immediately receding from errors when pointed out to them, they have not only preſerved the Society from ruin, but are likely to render it a permanent benefit to the public.

THE LAUDABLE SOCIETY *for the benefit of widows*, when firſt eſtabliſhed in the year 1761, proceeded upon principles as irrational as thoſe of the *London Annuity Society*, and therefore the ruinous conſequence of its plan, if not ſpeedily reformed, was equally neceſſary and inevitable. The ſame friendly admonitions were offered by *Dr. Price* to each of thoſe inſtitutions, and happy had it been for the one if in this inſtance it had imitated the prudence and diſcretion of the other. But the *Laudable Society*, though warned of its danger in every new publication of this work, and admoniſhed to conſent to ſuch a reformation as might have pre-

prevented the impending miſchief, determined to perſevere in error, and to reject all inſtruction however frequently repeated or ſtrongly enforced. The more reſpectable part of its members were alſo continually urging the neceſſity of reforming their plan, but no arguments could prevail upon the majority, excepting reluctantly to conſent to ſome feeble and trifling alterations which could have no other effect than by poſtponing to increaſe their calamity (*).—About four years ago, after every argument had proved ineffectual, the declining ſtate of its affairs became ſo manifeſt as to awaken this infatuated Society to a ſlight ſenſe of its danger, and it then raiſed its annual payments from 5 to 6 guineas, and lowered its Annuities to £10 if a member lived 3 years, £20 if he lived 7 years and £30 if he lived 13 years. By this meaſure all the former defects of the inſtitution were ſaid to be remedied and "the Scheme rendered of that "public utility for which it was eſtabliſh-"ed (†)." But in the deplorable ſtate of

(*) See Vol. I. p. 114.

(†) See the Deed of Settlement in 1788.

the

the Society at that time it was eaſy to foreſee, that ſuch alterations could not poſſibly produce any ſubſtantial benefit.—All therefore that has been ſaid of its folly and danger in the firſt Volume of this work (*) has been ſince verified. Its number of members from 700 in 1772 and 550 in 1782 had ſunk to 330 in the laſt year; at the ſame time the number of its annuitants had increaſed to 217, and it's income by falling £1740 ſhort of its expenditure, had rendered it neceſſary to ſink ſo much of its capital in order to make up the deficiency. By perſevering in this plan its ſtock, amounting to £42,200 in the 4 *per cent.* Bank Annuities, muſt in a few years have been exhauſted, the payment of the annuities ſtopped, and the whole muſt have terminated in a melancholy ſcene of diſappointment and diſtreſs. —The ruin which, though always certain, had in former years appeared too diſtant or perhaps imaginary to be regarded, ſeemed now to be haſtening ſo faſt as to excite a general alarm. It was therefore at length determined to examine the real ſtate of the Society's

(*) See page 107 to page 115.

affairs, and to adopt ſuch meaſures as might effectually ſave it from bankruptcy.—— On this occaſion they were pleaſed to have recourſe to my aſſiſtance, and I am ſorry to obſerve that after a minute inveſtigation, the finances of the Society were proved to be in a very unfavourable ſtate. ——From the computations which were laid before the Directors it appeared that their whole ſtock could not even pay *four-fifths* of the claims of the preſent annuitants, and that there was a real deficiency in their accounts of near £30,000. Under theſe circumſtances it was propoſed and recommended that the annuities of the preſent widows be immediately reduced *one half*, and that thoſe of all future claimants ſhould be reduced *one fifth* (*). Theſe meaſures

(*) This diſtinction between the two claſſes of annuitants may perhaps at firſt ſight appear to be unreaſonable. But the objection will be removed, if it be conſidered that many of the preſent widows, by having received more than their due, have been the chief cauſe of bringing the Society into its difficulties, and that the future claimants are certainly entitled to ſome allowance not only on this account, but becauſe their huſbands are alſo now paying *higher premiums* than thoſe of the preſent annuitants had done.

have

have ſince been adopted, and it is indeed to be lamented that the diſtreſſed ſtate of the Society ſhould have rendered them ſo indiſpenſibly neceſſary. Had the reformation commenced with the admonitions of Dr. *Price*, the reduction in the annuities would have been much more inconſiderable, and poſſibly, inſtead of affording a ſtriking example to the public of the fatal conſequences of an obſtinate perſeverance in error, they might by this time, like the *London Annuity Society*, have been enabled to increaſe their buſineſs and reputation by increaſing their annuities. But the Society, by a late reſolution, is for ever cloſed againſt the further admiſſion of *new* members, and it is probable therefore that it may never again be neceſſary to make its concerns the object of public notice.

THE SCOTCH ESTABLISHMENT *for providing for the widows and orphans of the miniſters and profeſſors*, continues to promiſe all that can be expected from it. Since the former edition of this work an attempt has been made by Sir *Francis Hutchinſon* and others to eſtabliſh a ſimilar inſtitution for providing for the widows and orphans of

the clergy in *Ireland*; but it has met with an oppoſition that has defeated it, and thus a very great public benefit has been prevented.

THE SCHEME deſcribed in page 119, Vol. I. *for providing for the nominees of Eaſt India commanders*, is likely to anſwer all the purpoſes for which it was eſtabliſhed. The ſums payable to nominees for the contributions there mentioned, have been raiſed to £600, the number of members who have died has been leſs than the calculations ſuppoſe on which the ſcheme has been founded, and there is every reaſon to expect that this Society will ſoon find itſelf capable either of increaſing its claims ſtill further, or of lowering the contributions required from its members.

Having been able to obtain no information, either from Dr. *Price's* papers or by any other means, reſpecting the progreſs of any of the *foreign ſchemes* mentioned in the firſt Volume from page 121 to page 127, I can add nothing to what has been there ſaid concerning them.

THE LAUDABLE SOCIETY *for the benefit of age* is now almoſt the only one remaining of the

the numerous inſtitutions of this kind which had been eſtabliſhed before the firſt publication of this treatiſe. Excepting the *Amicable Society for the benefit of age*, which had reduced its annuities from £26 to £8, when the laſt edition was publiſhed in 1782, and which is probably now ſecure, I believe they have all either diſſolved themſelves or terminated in ruin. Unfortunately for the *Laudable Society*, it had perſiſted too long in error, like the other *Laudable Society for the benefit of widows*, before it had recourſe to any effectual reformation, and in conſequence of this obſtinacy it has involved itſelf in ſimilar difficulties. The number of its members from 1300 was reduced in 1785 to 204, of whom 152 were between 40 and 50 years of age, who of courſe muſt very ſoon become claimants,—the number of its annuitants amounted to 564, its ſtock which was originally £96,000 had diminiſhed more than *one half*, and though the annuities, which at firſt were £44, had been gradually reduced till they were now only £8, its annual outgoings ſtill exceeded its income above £2300, and this exceſs was continuing to increaſe ſo faſt that in a few years its whole

ſtock would have been exhauſted. Had the firſt admonitions of Dr. *Price* been immediately and properly regarded, the annuities need not have been reduced ſo low as £8. But by paying the early claimants ſo much more than their due, and obſtinately perſiſting in this plan for ſo many years, the Society, when it adopted the meaſure of paying this reduced annuity, had become unable to perform it, and was obliged in the year 1786 even to lower its ſcanty allowance ſtill further, by reſtraining the annuity to only *ſix guineas*. This was done in conſequence of my having been employed at that time, by the deſire of the Society, in examining the real ſtate of its affairs, and recommending ſuch a reduction. By going on to pay the annuity of £8 it appeared that in leſs than 18 years its ruin muſt certainly have been completed, and that it could not afford, with any proſpect of ſecurity, to allow its preſent and future claimants a higher annuity than ſix guineas. An exception however was made in regard to the 52 members under 40 years of age, whoſe payments entitled them to an annuity of £8 even if they were then juſt entering into the Society.

Society. No alteration therefore has been made in thoſe annuities, nor do I think that this exception will be attended with any danger.—In its preſent reduced ſtate there is every reaſon to believe that the Society will be permanent; but it is not probable that the acceſſion of *new* members will ever render it of ſufficient importance to publiſh any account of its circumſtances in future.

THE AMICABLE SOCIETY *for a perpetual Aſſurance Office* has been eſtabliſhed ever ſince the year 1706, and ſeems to have withſtood the trials of time and experience ſo long, that it is no wonder if its ſafety and permanence ſhould be conſidered inviolable. From the nature of the inſtitution this muſt be the caſe to a certain degree; but it is by no means perfectly ſecure.—There are evils to which it is expoſed from imprudent meaſures which, by deſtroying its credit, may eventually deſtroy the inſtitution itſelf.—In the ſixth Section of the ſecond Chapter of this work it was ſhewn that the Society was in danger of being too much encouraged by its apparently proſperous ſtate in 1780, in conſequence of an increaſe which in a few years had nearly tripled the number

of its ſhares; and it was alſo demonſtrated there, that if no alteration in its plan took place, the dividend among nominees, though then at near £200 for each nominee, would in time fall below £150, the guarantied dividend, and produce the neceſſity of either entering into its capital or lowering this dividend.—I do not know that any attention has been paid to theſe obſervations, and if we may judge from the conduct of other Societies on ſimilar occaſions, it is not probable that they will be much regarded in the preſent circumſtances of this Society.—When it was determined in the year 1770 that the claims on each number ſhould not be leſs than £150, the members did not exceed 1120.—From that period the Society increaſed very rapidly, and in ten years had nearly attained its full complement of 2000. Hence it is evident that near *two thirds* of the preſent members have been admitted within theſe 20 years, and that the claims muſt now bear a much ſmaller proportion to the number of ſhares than they will do at the end of 20 or 30 years from this time.—If the whole annual contributions therefore be divided among the nominees of thoſe that

now

now die in the year, the dividend muſt be much more conſiderable than it will be in future; for thoſe nominees not only receive the ſums to which they are entitled, but alſo the *ſurplus*, which ſhould be yearly laid by and improved in order to form a fund which ſhall be ſufficient to pay the claims when their number ſhall have arrived at its *maximum*. In the year 1786 the dividend upon each ſhare amounted to £303, and in ſeveral of the preceding years to more than £200. But ſince the year 1787 it has fallen far ſhort of this ſum; nay, in one of thoſe years the dividend from the annual contributions was ſo near the guarantied dividend that, with the view I ſuppoſe of preſerving the credit of the Society, it was thought proper to ſell out a part of its Stock in order to make an addition of £15 to each claim.—The miſchievous tendency of this meaſure, if repeated always under ſimilar circumſtances, may be eaſily perceived; but it is poſſible, eſpecially as the Society hath lately increaſed the number of its ſhares from 2000 to 4000, that in this and ſome following years, it may not be neceſſary to have recourſe to it, and that the dividend

may

may again exceed £200. The time however, though perhaps diſtant, muſt inevitably arrive when this will not be the caſe. —The average number of annual deaths muſt go on increaſing with the average age of the members for many years to come. The average of the dividends therefore muſt go on diminiſhing, till at laſt it gets below £100, which it appears to have often done before the year 1757.—Had the Society adopted the more rational plan which has been propoſed in the firſt Volume of this Work (*) it might have always, for the ſame contributions, provided for claimants nearly double the ſum of £100; but it does not ſeem at preſent diſpoſed to profit by inſtruction.—On the contrary, it has lately rendered its plan more abſurd and inequitable than ever. For when the number of its ſhares was extended to 4000, it was determined that every ſubſcriber, when the exiſting number exceeded 2000, ſhould be required to pay a contribution of £6. 4s. inſtead of £5 *per annum*. It muſt be needleſs to point out the folly and injuſtice of

(*) See page 150, &c.

ſuch

ſuch a diſtinction; and it is to be lamented that a Society which, with ſome improvements in its plan, is capable of doing much good, ſhould chuſe to adopt meaſures that tend to diſgrace and injure it, rather than regard the advice and inſtruction which have been repeatedly offered to it in this treatiſe with no other view than that of promoting its credit and uſefulneſs.

THE SOCIETY *for Equitable Aſſurances on Lives and Survivorſhips* is certainly the moſt extenſive and important of all the inſtitutions of the ſame kind in this Kingdom, and is the only one that has been uniformly guided in its practice by calculation. In the different editions of this work an account of the ſtate and progreſs of the Society has been regularly given to the time in which thoſe ſeveral editions were publiſhed, and this makes it neceſſary that the account ſhould be continued to the preſent time.—In the year 1782 new Tables of premiums were computed agreeable to the probabilities of life in *Northampton* in lieu of the tables then in uſe which had been computed from the probabilities of life in *London*. But in order to prevent too great a reduction at one time

time, an addition of 15 *per cent.* was then charged upon thoſe premiums; and with the view of making an adequate compenſation to the exiſting members for their former payments, which had been ſo much higher than thoſe to be required in future, an addition was made to their claims of £1. 10*s* *per cent.* for every premium they had paid previous to this reduction.—The conſequence of theſe meaſures proved highly favourable to the Society; for its buſineſs increaſed ſo faſt that in the year 1785 it was nearly doubled.—At this period it was determined to enter minutely into the ſtate of the Society, by computing the value of each ſeparate aſſurance, and aſcertaining from thence the real ſurplus it poſſeſſed after diſcharging every poſſible demand upon it. The reſult of this laborious enquiry was a reſolution to take off the 15 *per cent.* which had been charged upon the premiums in the year 1782, and to make a further addition to the claims of £1. *per cent.* for every payment that had been made prior to the firſt of January 1786.—By thus lowering its premiums and compenſating its members without diminiſhing its capital, the Society has

has not only preſerved the credit which it derived from that capital, but has enlarged its uſefulneſs to a greater degree than ever.—The aſſurances which in 1782 were limited to £2000 on one and the ſame life have been extended, firſt to £3000 in 1784, and then in 1790 to £5000. The buſineſs has conſequently been more than doubled ſince the year 1785, and events have proved ſo favourable to the Society that it has lately been determined to make another addition to the claims of £1. *per cent.* for every payment which has been made prior to the firſt of January 1792.—In conſequence of theſe different additions many of the older members, when claimants, will be entitled to 50 and ſome of them even to 70 and 80 *per cent.* more than the ſums which they had originally aſſured.—It is hardly poſſible to deviſe a more ſafe and equitable method than this of making the members to participate of the profits of the Society. Its operation is gradual, every perſon is benefited in proportion to the money which he has paid, the grand object of making the aſſurance is promoted in the moſt deſirable manner by increaſing that proviſion which it was deſigned

to ſecure for a ſurviving family, and if the Society perſeveres in the ſame plan, it ſucceſs will continue, and probably in a few years ſtill further additions may be expected. Its preſent capital (which it is to be hoped will always be preſerved ſacred and inviolate) guards the Society againſt the effects of any extraordinary mortality. The number of its members is alſo now become ſo conſiderable that the decrements of life in each year are nearly the ſame, and the annual increaſe of the Society is continually rendering this uniformity more certain.

From the year 1768 to the year 1791 (a term of 23 years) the decrements of life in the Society to thoſe in the *Northampton* table of obſervations have been

from 10 to 20 as 1 to 4
20 to 30 as 4 to 7
30 to 40 as 3 to 5
40 to 50 as 3 to 4
50 to 60 as 3 to 4
60 to 80 as 9 to 10.

Hence it appears from this comparative view that the Society was perfectly juſtified in taking off every additional charge on its premiums. The probabilities of life among its

its members being ſo much higher than they are in the table of obſervations from which thoſe premiums are computed, will generally produce ſuch a balance in its favour, that together with the annual intereſt of its *ſurplus* Stock will be more than ſufficient to diſcharge every reaſonable expence of management, or even to ſecure it from the danger of any *temporary evil.* Provided therefore the ſame caution be continued in admitting none but good lives to be aſſured, and the ſame wiſe reſolution be maintained of preſerving the capital undivided, there can be no doubt but that the credit, the proſperity, and the uſefulneſs of this Society will go on to improve in an accelerated progreſſion.

In the firſt Volume of this work (*) an account is given of a plan for enabling the labouring poor to provide ſupport for themſelves in ſickneſs and old age by ſmall weekly ſavings from their wages. Since the firſt publication of this plan Dr. *Price* had be-

(*) Note page 140.

ſtowed

ſtowed great attention on the ſubject; and being furniſhed with more information than he then poſſeſſed, he was enabled to compute the valuable Tables which are inſerted in the firſt Appendix.—Theſe were intended to have ſerved as the foundation of a plan to be eſtabliſhed by the legiſlature, which ſhould hold forth ſuch particular encouragement to the poor as ſhould induce them to give it the preference to thoſe inſufficient and erroneous plans which are now forming all over the Kingdom. Nothing could be more important than this deſign. It was founded on the beſt principles, and had the happieſt tendency to aſſiſt the poor by encouraging induſtry and ſaving, and by raiſing them above the wretched neceſſity of depending upon the pariſh for ſubſiſtence when rendered incapable of providing for themſelves. The bill for eſtabliſhing this plan was brought into parliament about two years ago, and after having paſſed the houſe of Commons was rejected by the Lords.—But it is to be hoped that this want of ſucceſs will not prevent the ſubject from being ſoon reſumed: For its uſefulneſs and importance, when properly underſtood, can hardly fail to engage

gage every branch of the legiſlature in its ſupport. It is obſerved by Dr. Price (*) that the poor's rate in the year 1777 amounted to £1,556,804. Since that time it has increaſed near half a million; and the truth ſeems to be, that there is a growing diſtreſs among the poor occaſioned moſt probably by the price of proviſions having increaſed more in proportion than the price of labour.—From an impartial compariſon of the one with the other, it will appear to be hardly poſſible now for a poor man, if he have a wife and family, to ſubſiſt on his wages.—This melancholy truth, were I diſpoſed to enter further into the ſubject, might be proved in the moſt convincing manner from the valuable information communicated to Dr. *Price* by Mr. *Pitt*, the member for Dorſetſhire, whoſe humanity has led him to make particular enquiries into the ſtate of the poor of this Kingdom.—But it will be beſt to refer the detail of thoſe proofs to the author himſelf; by whoſe benevolent example it is to be wiſhed that others may be ſtimulated to the purſuit of ſimilar en-

* Note page 141, Vol. I.

quiries.

quiries. Happy would it be for the ſtate were thoſe perſons who are called of higher rank to intereſt themſelves more in adopting proper meaſures for relieving the diſtreſs of the labouring poor. By the formation of rational plans for this purpoſe the many wretched ſchemes which are now continually riſing up to delude their hopes would diſappear, the indigent labourer and his family would look forward without anxiety to ſeaſons of ſickneſs and old age, and the more wealthy would enjoy the advantage of a reduction in the enormous expence with which the pariſhes are now loaded for the maintenance of their poor.—It is much to be lamented that the different parochial clubs which are eſtabliſhed in this kingdom ſhould in general be founded on ſuch erroneous principles, and that a diſpoſition ſo laudable in itſelf ſhould not be properly directed and encouraged by the legiſlature. But of all the phantoms which are held up to entice and to deceive the multitude, none are more miſchievous or deſerve more ſevere reprehenſion than the TONTINES which have lately become ſo prevalent in every part of this country. By theſe, while the adven-

turer

turer is lured with the extravagant hope of making his fortune in a ſhort period and at a ſmall expence, the worſt ſpirit of gambling and idle ſpeculation is called forth, and thoſe baneful effects which are produced by a State Lottery in *London* are extended to the remoteſt corners of the Kingdom. It is not only the perſon who can afford to ſubſcribe ſixpence or a ſhilling a week from his income that becomes the dupe of thoſe bubbles; they are crowded in the poorer parts of the country with domeſtic ſervants whoſe wages do not exceed £3 or £4—a ſum which even properly applied is hardly ſufficient for their maintenance. This claſs of ſubſcribers muſt neceſſarily either involve themſelves in poverty and diſtreſs to complete their payments, or, which is more probable, they muſt find themſelves unable to go on with them, and by this means loſe all the money they have already advanced.—It is indeed no wonder, conſidering the ignorance which prevails on the ſubject, that ſo many ſhould be captivated with the advantages which are promiſed them in theſe tontines, and it may not perhaps be improper to ſtate a few facts which, if they produce no other effect,

will however tend to moderate the expectations and conſequently to leſſen the diſappointment of the ſubſcribers at the final diviſion of their ſtock. The well known increaſe of money when improved at compound intereſt and the continual mortality of the human race (which is proved by the melancholy experience of every day) are the two principles upon which all tontines are founded, and from which they derive all their encouragement. But it is impoſſible to apply thoſe principles more improperly than to the preſent purpoſe. In the ſhort term of ſeven years the accumulation of money at ſimple and compound intereſt is much the ſame, and the decrements of life are ſo inconſiderable as to produce little or no effect in increaſing this accumulation.—A weekly payment of ſixpence improved at 4 *per cent. compound* intereſt for ſeven years will amount to 10*l.* 5*s.* 3*d.* but at *ſimple* intereſt it will amount to 10*l.* 3*s* 10*d.* and at *no intereſt at all* to 9*l.* 2*s.*—The addition therefore to the principal from the mere operation of compound intereſt is ſo inconſiderable that were all the ſubſcribers to live to the end of the term, each ſhare would be increaſed by this means only 1*l.* 3*s.* 7*d.*—Again; let

the

the number of ſubſcribers to the tontine be 10,000, and let them conſiſt of perſons of all ages under 60 years.—According to the probabilities of life at *Northampton* (which however are much too low in the preſent caſe) 8647 of thoſe perſons will ſurvive a term of ſeven years; ſo that if the whole 10,000 lived to make their laſt payment, and none of them died till juſt before the final diſtribution of the ſtock, the ſhare of each ſurvivor would be no more than 11*l.* 17*s.* 6*d.*—But it is to be obſerved that theſe lives will be continually dying from the time of the firſt ſubſcription to the concluſion of the tontine, and that theſe deaths by leſſening the weekly contributions will reduce the ſhare of each ſurvivor to *eleven guineas* nearly. When the expences of management are alſo deducted, and allowance is made for the loſs which is likely to be ſuſtained by inveſting the ſubſcriptions in the Public Funds, where the improvement of of money is always uncertain, but particularly ſo at the very high price which they now bear, it is more than probable that the ſhares will fall greatly below the ſums which I have ſtated, and that the ſurviving members

will have the ſatisfaction to find at the end of ſeven years that they have barely received their principal, after having endangered the loſs of the greateſt part of it by dying in the mean time. The only ſource from which thoſe tontines can derive any additional increaſe muſt be from the inability of ſome of the members to go on with their ſubſcriptions. But this is an evil of the worſt kind, and defeats the very end for which thoſe plans are ſaid to be principally intended. Inſtead therefore of relieving they will add to the miſeries of the poor, and the only perſons that will be benefited by them (excepting indeed the *treaſurers* and *ſecretaries*) will be the more wealthy ſubſcribers, whoſe ſhares will have acquired their chief increaſe from the ſpoils of the diſtreſſed.—I am ſorry to ſee thoſe ſchemes adopted and encouraged by ſo many reſpectable perſons, who have contributed very much by their character and ſituation in life to ſpread the contagion; and by ſubmitting to become *truſtees* of the different ſocieties, they have unfortunately given them a weight and credit which they would not otherwiſe have enjoyed.—In one of the moſt numerous of thoſe tontines, which

which conſiſts of 43,000 members, it has been urged in its defence that it would have a tendency *to improve the morals of the people* by leading them to habits of ſaving. Although no doubt can be entertained of the good intentions of thoſe who *patronize* this ſcheme, it muſt, I think, be acknowledged that the method they have choſen of reforming the poor is rather equivocal.—It is not likely that an indigent man, when allured to ſave by the ſame motives which ſtimulate a gambler to his ruin, ſhould be *improved in his morals* by this means, or that he ſhould be much prejudiced in favour of this diſpoſition when his hopes and expectations are kept alive by a deluſion. After looking forward during the term of ſeven years to the accumulation of an immenſe fortune (*), and finding himſelf at laſt barely in poſſeſſion of the miſerable pittance he has paid, it will be no wonder if the mortifying diſappointment he feels ſhould lead him to ſquander away a ſum, too ſcanty for eſtab-

(*) One of the ſubſcribers to the *Briſtol* Tontine wrote to inform me that he expected to receive *One thouſand pounds* at the final diviſion of the Stock.

liſhing him in trade or for any other valuable purpoſe, and that, inſtead of convincing him of the good effects of ſaving, it ſhould confirm him in the habits of diſſipation.

Conſidered in every point of view, a tontine for a ſhort term of years is abſurd in the higheſt degree. If it be extended to the whole duration of life, and include perſons of all ages, a term of 80 or 90 years muſt elapſe before all the lives become extinct, and the ſubſcriptions will neceſſarily accumulate during this time to a large amount. But the probability is ſo remote that any particular member will be the ſurvivor of thouſands to enjoy this accumulation, and the value of money is ſo diminiſhed when the payment of it is poſtponed for ſuch a long ſucceſſion of years, that, were the ſubject properly underſtood, this hopeleſs and unprofitable method of ſpeculation which is now encouraged by the ignorance and credulity of the multitude, would be univerſally reprobated. If, inſtead of a periodical ſubſcription during life with the view of appropriating the accumulation to the laſt ſurvivor, the whole money be paid at once, and the intereſt be annually divided among the ſubſcribers till they

they are reduced to one perſon who is to receive the principal, the proſpect in this caſe is not more encouraging.—Suppoſe 1000 perſons between 10 and 20 years of age to ſubſcribe £100 each, and that the intereſt at 4 *per cent.*, or £4000, be annually divided among them. In the courſe of 20 years the dividend of each member will be raiſed from £4 to £5 *per annum*—in 37 years it will be doubled, or increaſed to £8 *per annum*—in 54 years it will be quadrupled or increaſed to £16 *per annum*—in 64 years it will be decupled or increaſed to £40 *per annum*—and in about 80 years the laſt ſurvivor will come into poſſeſſion of the whole principal.—The preſent value of £100,000 to be received at the end of 80 years is £4330, and as it is nearly 1000 to one againſt any individuals coming into poſſeſſion of it, the value of each ſubſcriber's expectation in this principal is only 4*l.* 6*s.* 7*d.*—It has lately been the cuſtom to borrow money by the aſſiſtance of *tontines*, where the ſubſcriber ſinks his principal upon condition of having his annual intereſt increaſed. But I do not know that a worſe method can be adopted; for it has the peculiar property of being

being injurious both to the borrower and the lender.—If the annual intereſt be much increaſed it is highly injurious to the *borrower*; ſince it evidently appears from what has been already proved, that the addition to be allowed yearly on each ſhare as an equivalent for the principal is altogether inconſiderable. But it is neceſſary to allure the ſubſcribers by an immediate benefit, as well as by the proſpect of future advantages, and therefore this addition is generally much greater than it ought to be; ſo that were the difference between the common intereſt and that which is allowed to the ſubſcribers to be properly improved during the continuance of the tontine, it would in moſt caſes amount to a ſum which would be double or even triple of that principal for which it is given as an equivalent.—The ſubſcriber on the other hand is no gainer. His property is depoſited where he cannot diſpoſe of it without conſiderable loſs, or continue it without being flattered with hopes which can never be realiſed; for if he comes into poſſeſſion of the whole dividend, it will be at the end of 80 years, when it is more than an equal chance that he does not live ſix months to enjoy

enjoy it.—I am much concerned to obſerve the legiſlature of this kingdom giving a ſanction and encouragement to the very evil which I have been endeavouring to expoſe. Wretched indeed muſt be the condition of that country where the morals of the people are ſacrificed to the intereſts of the revenue, and the public exigencies are provided for by the pernicious aſſiſtance of lotteries and tontines. When the ſpirit of idle ſpeculation and gambling is thus cheriſhed and maintained by thoſe who ought to exert their power in ſuppreſſing it, what miſchievous conſequences may not be apprehended? The abſurd ſchemes of every deſigning projector, encouraged by ſuch flagrant examples, will ſucceed in impoſing upon the credulity of the ignorant and diſtreſſed, who, being deceived in their firſt hopes, will be allured to go on by other ſchemes ſtill more flattering and deluſive, till their rage for adventure, increaſing with diſappointment, will extinguiſh every habit of honeſt induſtry, and render them at laſt totally corrupted and depraved.—Being convinced of the certainty of theſe and of worſe conſequences, I ſincerely wiſh that the facts which I have ſtated may engage the attention

tion of thoſe from whom the evil at preſent derives its chief ſtrength and ſupport; for unleſs the reformation begin with them, it will be of little uſe to attempt it by reaſon and argument.

P. S. It was Dr. *Price*'s intention, as appears from ſome references in the firſt Volume, to have continued in this edition his account of the public revenue and expenditure to the beginning of the year 1791. —He had alſo propoſed to have publiſhed the three plans for paying off the national debt which were communicated by him to Mr. *Pitt* in conſequence of an application having been made to him for his opinion and advice on this ſubject.—One of theſe plans was adopted by the miniſter without the ſlighteſt acknowledgment, and now forms the foundation of an Act for reducing the public debt, which was eſtabliſhed by the legiſlature in 1786, and has contributed more than any other meaſure to raiſe the credit of the preſent Adminiſtration.—Being apprehenſive of rendering this introduction too long by attempting at this time to fulfil the intentions

tions of Dr. *Price*, I ſhall reſerve the further proſecution of the ſubject for a ſeparate publication, in which I may be led by the duty and affection I owe to his memory, to ſtate ſome facts which his own modeſty perhaps might have never ſuffered him to publiſh, but which ought to have been mentioned by the Miniſter with particular reſpect and gratitude.

CHATHAM-PLACE,
29th February 1792.

W. M.

PREFACE

TO THE

FIRST EDITION.

BEFORE the Reader enters upon this Work, it will not be improper to give him the following information concerning it.

A few years ago, many gentlemen, of the firſt eminence in the law, formed themſelves into a *Society*, for providing annuities for the widows of all ſuch perſons in judicial offices, barriſters, civilians, and ſolicitors, as ſhould chuſe to become members. A plan was agreed upon and printed; but, ſome doubts happening to ariſe with reſpect to it, the directors reſolved to aſk the opinion and advice of three gentlemen, well known for their ſkill in calculation. This occaſioned a further reference to me; and the iſſue was, that the plan being found to be inſufficient, the whole deſign was laid aſide.

About

About the ſame time, ſeveral other ſocieties were formed with the ſame views; but all on plans alike improper and inſufficient. Finding, therefore, that the public wanted information on this ſubject, I was led to undertake this work; imagining, that it might be ſoon finiſhed, and that all I could ſay might be brought into a very narrow compaſs. But in this I have been much miſtaken. A deſign, which I at firſt thought would give little trouble, has carried me far into a very wide field of enquiry: and engaged me in many calculations that have taken up much time and labour. I ſhall, however, be ſufficiently rewarded for my labour, ſhould it prove the means of preventing any part of that diſtreſs, which is likely to be hereafter produced by the ſocieties now ſubſiſting for the benefit of widows.——I have proved the inadequateneſs of their plans, by undeniable facts and mathematical demonſtration.—I have, further, given an account of ſome of the beſt plans, which are conſiſtent with a ſufficient probability of permanency and ſucceſs.——Should, therefore, any of theſe ſocieties determine to reform themſelves; or ſhould any inſtitutions of the ſame kind be hereafter

hereafter eſtabliſhed, they will here find direction and aſſiſtance.

In Queſtion VI. Chap. I. a general method is deſcribed of finding the values, in *ſingle* and *annual* payments, of all life-annuities which are to begin after a given term of years; and, in the 4th Section of the 2d Chapter, the plans of the ſocieties for granting ſuch annuities are particularly conſidered, and proved to be extremely deficient.—— Indeed, the general diſpoſition which has lately ſhewn itſelf to encourage theſe ſocieties, is a matter of the moſt ſerious concern; and ought, I think, to be taken under the notice of the Legiſlature. The leading perſons among the *preſent* members, will be the *firſt* annuitants; and they are ſure of being gainers: And the more inſufficient the ſcheme is, on which a ſociety is formed, the greater will be the gains of the firſt annuitants. The ſame principle, therefore, that has produced and kept up other *bubbles*, has a tendency to preſerve and promote theſe; and, for this reaſon, it is to be feared, that, in the preſent caſe, no arguments will be attended with any effect. The conſideration, that "the gain made by ſome in theſe ſocieties,

" will be ſo much plunder taken from " others," ought immediately to engage all to withdraw from them, who have any regard to juſtice and humanity; but experience proves, that this argument, when oppoſed to private intereſt, is apt to be too feeble in its influence.

It cannot be ſaid with preciſion, how long theſe ſocieties may continue their payments to annuitants, after beginning them. A continued increaſe, and a great proportion of young members, may ſupport them for a longer time than I can foreſee. But the longer they are ſupported by ſuch means, the more miſchief they muſt occaſion.—So, a tradeſman, who ſells cheaper than he buys, may be kept up many years by increaſing buſineſs and credit; but he will be all the while *accumulating* diſtreſs; and the longer he goes on, the more extenſive ruin he will produce at laſt.

In the latter end of the firſt Chapter, I have ſtated very particularly, the method of computing the values of *aſſurances* on lives and ſurvivorſhips, in all caſes where no more than two lives are concerned: And, in the third Eſſay, I have pointed out a conſiderable error,

error, into which there is danger of falling in computing ſome of theſe values. The ſocieties and offices for tranſacting buſineſs in this way, are very uſeful; and it is neceſſary that they ſhould go upon the beſt principles, and poſſeſs all the information that can be given them.

But there is no part of this work in which the public is ſo much concerned, as the third Chapter. It will be there proved, that had the ſums raiſed for public ſervices ſince the REVOLUTION, been much greater than they have been, the increaſe of the public debts to their preſent ſtate might have been prevented in the eaſieſt manner, and at a trifling expence. A method, likewiſe, of reducing within due bounds theſe debts, heavy as they now are, will be propoſed.—All competent judges will, I believe, ſee, that this method, being founded on the moſt perfect improvement that can be made of money, is the moſt expeditious and effectual that the natures of things admit of. Nor, in my opinion, if the nation is not yet too near the *limit* of its reſources, can there be any good reaſon againſt carrying it into execution.——It is well known, to what prodigious ſums, money,

improved for ſome time at *compound intereſt*, will increaſe (*a*). A ſtate, if there is no miſapplication of money, muſt neceſſarily make this improvement of any ſavings, which can be applied to the payment of its debts. It need never, therefore, be under any difficulties; for, with the *ſmalleſt* ſavings,

(*a*) A *penny*, ſo improved from our Saviour's birth, as to double itſelf every 14 years, or which is nearly the ſame, put out to 5 *per cent.* compound intereſt at our Saviour's birth, would, by this time, (that is, in 1773 years) have increaſed to more money than would be contained in 150 millions of globes, each equal to the earth in magnitude, and all ſolid gold (*).

(*) This computation is made on the ſuppoſition that a cubic inch of gold is worth about £ 35.—But ſince a cubic inch of water is known to weigh 254 grains nearly, and the ſpecifick gravities of *ſtandard* gold and water are to each other as 18,888 to one, it will follow that a cubic inch of gold, according to the *Mint* price of 3*l.* 17*s.* 10½*d.* per oz. will amount to 38*l.* 1*cs.* 1¼*d.* and conſequently that a penny improved at compound intereſt ſo as to double itſelf once in 14 years, from the time of our Saviour's birth to the year 1773, would have accumulated only to 107 millions of ſuch globes as are mentioned by Dr. *Price*,—but this is abundantly ſufficient to prove the ſtrength of his argument.—In the preſent year, it would have accumulated on the above ſuppoſition to 274 millions of thoſe globes.

If inſtead of ſuppoſing money to double itſelf every 14 years it be improved at 5 *per cent.* compound intereſt, the accumulation in 1791 years will amount only to 144 millions, and in 1773 years to no more than 60 millions of globes. The difference therefore between the ſuppoſition of money's doubling itſelf every 14 years and of its being improved at 5 *per cent.* compound intereſt, is far from being inconſiderable, eſpecially in a long term of years. ED.

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it may, in as little time as its intereſt can require, pay off the *largeſt* debts.

In the *firſt* Eſſay I have made many obſervations on the expectations of lives, the pernicious influence of great towns on health, and manners, and population; the increaſe of mankind; and other ſubjects in the doctrine of Annuities and Political Arithmetick.—— In the Laſt Eſſay I have ſtated carefully the proper method of forming Tables of the probabilities of human life, from given obſervations: And, at the cloſe of this Eſſay, beſides ſeveral new Tables, I have thought it neceſſary to give Mr. *Simpſon*'s Tables of the values and expectations of LONDON lives. ——I have alſo, in the Notes at the end of this work, given the Demonſtrations of the Anſwers to the *Queſtions* in Chap. I. Theſe Demonſtrations I have choſen to keep out of ſight in the body of the work, in order to avoid diſcouraging ſuch readers as may be unacquainted with mathematics.

Upon the whole. A great part of the work now offered to the Public is, I believe,

new; and I am in hopes alſo, that it will be found to contain ſome improvements in thoſe branches of philoſophical enquiry, which are the ſubjects of it.

PREFACE

PREFACE

TO THE

THIRD EDITION.

THAT favourable reception of this Work, which has occaſioned the preſent Edition of it, ſo ſoon after two former editions, is ſuch a proof that it has been of ſome uſe to the public, as amply rewards me for the attention and labour which I have beſtowed upon it. In reviſing it on the preſent occaſion, I have been anxious about improving it as far as poſſible. Several additional facts and obſervations have been inſerted in different places, particularly in the Firſt Eſſay and the Poſtſcript to it.—That

part of the Second Section, Chap. II. which treats of the *Scotch* eſtabliſhment, has been new compoſed, and carefully accommodated to the more accurate information concerning it, with which I have been favoured.

The SUPPLEMENT is an addition which was made to the *ſecond* edition.—The obſervations in it on the preſent ſtate of our population have been a good deal enlarged. —This is a very ſerious and important ſubject. If, indeed, there has been that diminution of our people which the evidence I have produced ſeems to ſhew, it muſt alarm every one who wiſhes well to his country, and it ought to engage the immediate and vigorous attention of government.——Many differ from me on this point; and I wiſh I could find ſufficient reaſon to believe as they do. Several great manufacturing towns have, I know, increaſed; but theſe are nothing to the whole kingdom; and even by their increaſe, our population may, on the whole, have loſt more than it has gained.—In truth; it would have been ſtrange if our numbers had not been declining; for I can ſcarcely think of any great cauſe of depopulation,

lation, which has not for the laſt 80 years been operating among us.

The prodigious traffic now carried on in Life-annuities, and the rage for forming and encouraging Annuity Schemes, which has for ſome time been ſpreading through the kingdom, has rendered the information which I have meant to convey in the following work particularly neceſſary. And I have had the pleaſure to obſerve that it has been attended to. Several of the Annuity Societies in LONDON have been diſſolved; and there is reaſon to hope, that thoſe which ſtill remain will not be able much longer to ſupport themſelves on their preſent plans, in oppoſition to the evidence of demonſtration, and the calls of juſtice and humanity.—— Theſe *Bubbles*, however, are of little conſequence, compared with that GRAND NATIONAL EVIL, which is the ſubject of the ſecond chapter of this treatiſe. This is an evil on which I could not imagine, that any ſuch efforts as mine would make any great impreſſion. Perhaps, indeed, the united efforts of all the independent part of the kingdom would now be too weak

weak to ſave us from the diſtreſs with which it threatens us.

Much has been ſaid for ſome time of a plan mentioned in PARLIAMENT, at the end of the laſt ſeſſion, for paying off the NATIONAL DEBT. This raiſed ſome expectations; and I will beg leave here to give a brief account of it.

After providing for all the current ſervices, there remains this year (1773) a *ſaving* or *overplus* of 1,200,000*l.* With this ſum, and a profit of 150,000*l.* from a Lottery conſiſting of 60,000 tickets, a MILLION AND A HALF of the 3 *per cent.* annuities, purchaſed at 90, will be paid off.—When this was propoſed to the Houſe of Commons, it was at the ſame time announced, that it would be the COMMENCEMENT OF A PLAN FOR PAYING OFF THE NATIONAL DEBT; for, if no extraordinary ſervices ſhould call for any other application of the public ſurpluſſes, the ſame payment increaſed by the intereſt of former payments, is intended to be made every year while the peace laſts: And thus, reckoning compound intereſt

intereſt at 3 *per cent.* SEVENTEEN MILLIONS will be paid off during a peace of ten years.

On this plan I will take the liberty, with all the deference which becomes me to the ſtation and abilities of the propoſer of it, to offer the following remarks.

1ſt. It implies, that there is to be a *Lottery* every year during the whole continuance of peace.—Formerly, lotteries were expedients for procuring money on more advantageous terms, to which government had recourſe, when preſſed by the neceſſities of war. They are now, it ſeems, to be eſtabliſhed as *permanent* reſources never to be given up or ſuſpended.—This muſt ſhock every perſon who is duly acquainted with the miſchief occaſioned by lotteries, particularly among the lower claſſes of people. The rage for gaming threatens the ruin of all that is virtuous and manly among us. It is increaſing faſt, and wants not to be foſtered by government.

2dly. The *ſurplus* of the preſent year is in part the effect of ſome *extraordinary* ſavings in the laſt year, (1772) which cannot be

be expected another year: And, I believe, that thoſe who are beſt acquainted with this ſubject, muſt be ſenſible that there is no ſufficient reaſon to expect, while the augmentation of the navy is continued, a conſtant *ſurplus* of ſo much as a MILLION *per ann.* I mean this on the ſuppoſition, that the produce of the *Sinking Fund* will continue what it is taken for this year, and what is has been the laſt three years, or 2,600,000*l.* But this is certainly more than can be depended on. The difficulties of the *Eaſt India Company*; that ſtagnation of credit which has lately diſtreſſed the public; and many other cauſes, may poſſibly occaſion *Deficiences.* Should there, however, be an *increaſe*, it will be owing, I am afraid, to a very bad cauſe: I mean, to an increaſe of our importations proceeding from luxury, and turning the balance of trade againſt us; and, conſequently, draining the kingdom of its *ſpecie*, and leaving it more and more to the precarious and dangerous ſupport of paper-money. But,

3dly, Let the *ſurplus* of the public revenue prove what it will, there is too much probability that, even during the continuance of peace, ſome emergencies or other will be often

often furniſhing reaſons or pretences for employing it in other ways than the payment of the public debts. This has been the caſe hitherto; and from the year 1730 to the preſent time, it has never happened, that we have gone on above three or four years together employing *ſurpluſſes* in diſcharging debts. Though in profound peace, there have been calls for a different application of them; nor can I imagine what reaſon there is for believing that our circumſtances are ſo much changed for the better, that there will ariſe no ſuch calls for ten years to come, ſhould the peace laſt ſo long. But,

4thly, The moſt capital defect in this plan is, that its operation is to ceaſe as ſoon as a war begins. That is; it is to ceaſe at the very time when it would operate to moſt advantage, and make the quickeſt progreſs in redeeming the public debts. This has been demonſtrated in the chapter on public credit in this Treatiſe, and in my *Appeal to the Public on the Subject of the National Debt.*

Is it then any wonder, that ſuch a plan has had no effect on public credit?—Does it mean any more than that the ſurpluſſes of the revenue

revenue ſhall be applied to the diſcharge of our debts, when there are no other uſes for them?—And was there ever a time when this was not done? Is not this the very plan we have been purſuing theſe forty years, and to which we owe our preſent incumbrances? Certain it is, that nothing but a plan that ſhall go on operating uniformly in *war* as well as in *peace*, or the eſtabliſhment of a permanent fund that ſhall never be diverted; that is, in other words, a return to the ſcheme adopted by the legiſlature in 1716; and which even now ſtands eſtabliſhed by law, but which, through the unpardonable miſconduct of men in power, has been defeated of its good effects: Nothing, I ſay, but this can do us any eſſential ſervice; or, in our preſent circumſtances, be much more than trifling with the difficulties and dangers of the public.—Eſtabliſh ſuch a fund—Conſign it to a particular commiſſion, acting under penalties, in ſuch a manner as ſhall take it out of the hands of the *Treaſury*, and form a check even on the *Houſe of Commons* itſelf.—Supply from time to time all deficiencies juſt as if no ſuch fund exiſted; and, by theſe and other meaſures, convince the kingdom that

ſomething

ſomething effectual is meant, and that the public debts are indeed in the way to be extinguiſhed.—LET THIS BE DONE; and we may ſoon ſee a new ſtate of things; public credit may revive; and the kingdom enjoy at leaſt a chance for being preſerved.—By the confidence which ſuch a meaſure would give in government ſecurity; but more eſpecially, by the increaſing ſums which would be thrown annually into the public markets, and returned to the public creditors, the 3 *per cents.* would be ſoon raiſed to *par*, and in ſome time probably far above *par*. It is well known, what an effect *borrowing* every year has in ſinking funds. Paying every year would certainly have an equal contrary effect. In a time of war, particularly, it would give ſuch a demonſtration to the public, that an *irrevocable* plan of redemption was at laſt eſtabliſhed, as could not but produce the happieſt effects. It would indeed in theſe circumſtances be neceſſary to borrow an extraordinary ſum annually equal to the appropriation. That is; ſuppoſing the fund to ſet out with a million *per ann.* it would be neceſſary to borrow ſo much more annually than would have been wanted

wanted had the fund been capable of being diverted. But this being done to convey a conviction with which the very power of borrowing was connected; and to preſerve a *fund* on which the very being of the ſtate depended; no bad conſequences could follow. The annual charge on the public, occaſioned by the war, would be even *leſs* than it muſt have otherwiſe been. For, let us ſuppoſe *ten millions* neceſſary to be borrowed every year to defray the expences of war, *nine millions* only of which would have been wanted, had not the million *ſurplus* been locked up.——Suppoſe farther, that the ſcheme, by keeping up public credit, and throwing money every year into the hands of lenders, enables government to borrow at 1 *l. per cent.* leſs intereſt than would be otherwiſe required; that is, at 4 inſtead of 5 *per cent.*—In theſe circumſtances, there would ariſe a preſent ſaving to the kingdom of 50,000 *l. per ann.*; for the intereſt of *ten millions* at 4 *per cent*, is 50,000 *l.* leſs than the intereſt of *nine millions* at 5 *per cent.* And ſuch a ſaving, repeated every year of a war, would be an object of ſome importance to the kingdom.—Indeed, there may be no poſſibility

poſſibility of conceiving what important effects in this way, the eſtabliſhment of ſuch a ſcheme might produce. During its progreſs in diſcharging our debts, and before it could give any relief by the annihilation of taxes, it might *ſave* the kingdom, by preſerving it from difficulties which would have ſunk it. And every one muſt be ſenſible of this, who has conſidered what danger there is that a war, ſhould it become unavoidable before our debts are put into any certain courſe of redemption, will either entirely overwhelm public credit, or ſo much weaken it, as to produce an impoſſibility of borrowing except on very exorbitant intereſt, and, conſequently, of finding taxes ſufficiently productive to pay ſuch intereſt. The general apprehenſion now is, that the nation is overloaded; and that its debts will never be paid. This keeps the funds near 18 *per cent.* lower than they were in the laſt *peace.* In the next war ſuch apprehenſions will increaſe, and produce great danger. But ſhould it be then ſeen, that a plan for redeeming our debts, the moſt efficacious poſſible, was going on; and, in conſequence of being guarded in ſome ſuch manner as I

have hinted, *would* not (or *could* not *eaſily*) be revoked; in theſe circumſtances, all danger would be ſo far leſſened, that it might be practicable to find new taxes which would ſupport the expences of war during the operations of the ſcheme.

But I am got far beyond the limits I preſcribed myſelf when I begun this Preface.—As the national debt is a ſubject unſpeakably intereſting to this nation, I could not allow myſelf to omit any thing that appeared to me of conſequence upon it; and the Reader of this Treatiſe will on this account, I hope, excuſe me, if I have detained him here too long and too unprofitably. Much has been before ſaid on this ſubject by writers of more conſequence to no purpoſe; and we ſhall purſue the path we are in, till the edge of the precipice towards which we are advancing awakens us, and ruin becomes unavoidable.—The diſtreſs occaſioned by the ſhock lately given to the bubble of paper-credit, is, I am afraid, a prelude to greater calamities, and a warning to prepare for them.

PRE-

PREFACE

TO THE

FOURTH EDITION.

THIS work having been for ſome time out of print, I reſolved about four years ago to prepare for the preſs a new edition of it, expecting that I ſhould have only a few corrections and additions to make of no particular conſequence. But in this expectation I have found myſelf greatly miſtaken. Such a variety of new matter came in my way, and ſuch means of improving this work were communicated to me, as have led me to beſtow upon it more attention and labour than can be eaſily imagined, and to increaſe it from one to two volumes.

It is probable that nothing could have engaged me to undertake ſo much labour

had I foreſeen it; but having begun, I could not avoid going on; and I was encouraged by the reflection on the favourable manner in which the former editions of this work had been received, and alſo by the hope that on *one* ſubject of human enquiry I ſhould be able to produce a work more complete than any that the public has been yet furniſhed with.

The additions of moſt conſequence in the preſent edition are the following.

There has been added to the *Second* Chapter an account of ſeveral *foreign* Societies; and a continuation of the hiſtory of ſuch annuity ſocieties as are ſtill ſubſiſting in *London*, to the time when that Chapter was printed off; that is, to the beginning of the year 1782.——The largeſt of theſe additions reſpect the *Amicable Corporation for Perpetual Aſſurances* at *Serjeant's Inn*; and the Society in *Chatham Place* for *Equitable Aſſurances on Lives and Survivorſhips*.—— The former of theſe Societies, ſhould they think proper to look into theſe obſervations, will, I am perſuaded, find that they require their attention.——But it is the Society

Society laſt mentioned, which, through the whole of this Treatiſe, I have had chiefly in view. Having for many years been concerned in adviſing this Society (the firſt of the kind in the world, and increaſing faſt), I have been anxious about giving it all the information and aſſiſtance poſſible.——The additional obſervations addreſſed to it in this edition will be found chiefly in this Volume from page 170 to 178; and in the Second Volume from p. 101 to p. 112, and from p. 189 to p. 199. And, from theſe obſervations, it may be learnt in particular, that though this Society, in conſequence of having happily begun too high, has already found itſelf capable of making great abatements in its demands, it is ſtill capable of making farther abatements.

In the former editions of this work, I had intimated that a publication of the tables by which its buſineſs is tranſacted would be proper, together with an account of the principles aſſumed, and the method taken in compoſing them. This is an information to which the public has a right, and which is now given it in different parts

of this work, and particularly in the Second Volume from p. 101 to p. 112.

Many corrections have been made and ſeveral additional notes inſerted in the Firſt Eſſay. Some of theſe have been occaſioned by two late publications; one by Mr. *Wales*, maſter of the mathematical ſchool at *Chriſt-Church*; and the other by Mr. *Howlet*. See, particularly, the notes in p. 243, 251, 254, and 255.——The Table in p. 298 is alſo now firſt inſerted.

To the Second Eſſay a *Poſtſcript* has been added, the principal intention of which is to point out the reaſons for diſcarding the valuation of ſingle and joint lives derived from Mr. *De Moivre*'s hypotheſis; and alſo to deſcribe a method of computing theſe values from any given tables of mortality, which, while it leaves no poſſibility of any miſtakes, renders ſuch computations as eaſy and expeditious as their nature will allow.

The Third Eſſay is the ſame that it has been in all the former editions. But to the *Fourth* ſome additions have been made; particularly, the notes in p. 343, 345, and 346.

346. The two firſt of theſe notes have in view a remark of Mr. *Wales*'s; and the deſign of the laſt is, to retract an aſſertion in the former editions concerning the duration of life in old age in great towns, and to ſhew the reaſon why a greater proportion of the inhabitants of *London* died formerly in old age than have died lately.

But the additions of moſt conſequence are the Tables in the Second Volume, with the remarks explaining the conſtruction, uſe, and application of them.——In the general introduction to theſe Tables, and in the remarks on Table XVI. p. 48. it is ſhewn (and, I think, undeniably) that the Tables of the values of lives deduced from the *London* bills of mortality err only by giving them too high; and that, with reſpect to the main body of the inhabitants, the unfavourableneſs of *London* to the duration of life continues much the ſame that it uſed to be.

Tables I. II. III. and IV. and Tables LVIII. LIX. LX. and LXI. are an abridgement of Mr. *Smart*'s tables of compound intereſt, and contain all that is important in them.

The VIth Table, ſhewing the mean pro-

babilities of the duration of life according to a regiſter of mortality at NORTHAMPTON, has been inſerted in all the former editions; but it is now given more correctly; and tables deduced from it have been added, of the *expectations* of life and the values of *ſingle* lives and of any *two joint* lives at all ages, and for three rates of intereſt. The labour of computing theſe tables was undertaken in order to ſet aſide all occaſion for uſing the defective valuation of lives founded on Mr. *De Moivre*'s hypotheſis; but not having being able to finiſh theſe computations till a great part of this Treatiſe had been printed off, I have been obliged to continue the uſe of the old tables ſo far as to take from them many of the examples of the ſolutions of queſtions in the firſt and following chapters.

When tables of the values of *two joint* lives are given, the values of *three joint* lives may be deduced from them, with perfect eaſe, by Mr. *Simpſon*'s rule inſerted in the Second Volume of this Treatiſe, p. 118.

This rule ſaves ſo much trouble, that I have thought it worth while to procure calculations of the XXXVIIth and XXXVIIIth tables,

Tables, on purpoſe to determine how far it may be depended on. The reſult is given in the remarks and compariſons from p. 117 to p. 120. And it ſeems to appear that it finds the values of *three joint* lives ſo nearly as to leave little room for wiſhing in this inſtance any greater degree of correctneſs.

The rules from p. 170 to p. 182 deſcribe a method of deducing, with ſufficient accuracy, the value of any life or number of lives at *all* rates of intereſt, from the correct value given at *one* rate of intereſt.——In computing, therefore, tables of the values of lives, according to any given obſervations, no more will hereafter be neceſſary than to compute them for *any one* rate of intereſt.

All improvements, however, of this kind would be of little conſequence, were there no tables which ſtate correctly the laws that govern human mortality in different ſituations. One principal part of my buſineſs in this work has been to frame ſuch tables; and any one who will look over the collection of tables in the Second Volume, and particularly from p. 121 to p. 161, will ſee that I have

have been furniſhed with the beſt means of doing this.

With reſpect to the Tables, in particular, deduced from the *Swediſh* Obſervations, I cannot heſitate to pronounce that they exceed in correctneſs every thing of this kind which has been hitherto offered to the public; and that nothing is wanting to make our knowledge in this inſtance complete, but ſimilar obſervations in other kingdoms.—By theſe Tables I have been enabled to ſtate minutely the different rates of mortality at all ages among males and females; and to form tables of the values of ſingle and joint lives for *each* ſex, as well as for both ſexes collectively; in conſequence of which, I have been farther enabled to determine the increaſe of the values of annuities payable during ſurvivorſhip, occaſioned by the longer duration of lives among females; and thus to furniſh a direction of ſome importance to the various ſocieties in this kingdom and abroad for providing annuities for widows.

I muſt not in this place neglect to acknowledge the great obligation I am under to Mr. WARGENTIN of the Royal Academy of Sciences at STOCKHOLM, for the communications

munications which have enabled me to make the additions to this Treatiſe laſt-mentioned. It will be found alſo, that I have been much indebted to Mr. OEDER of *Oldenburgh* for ſeveral uſeful communications.

In the obſervations and rules which follow the LVIth Table, p. 226, I have given a general account of the method of computing the values of *preſentations* to livings, and of the renewals of leaſes held either for terms certain or for lives.

The rules from p. 244 to p. 252, ſhew how to deduce from the values given of any annuities payable *yearly* their values when payable *half-yearly* or *quarterly*, or when ſecured by land and payable half-yearly. And the laſt Table in p. 252, exhibits the particular differences between theſe values for two rates of intereſt.

The collection of Tables is followed by a SUPPLEMENT which formed a part of the Third Edition of this work. In the preſent Edition ſeveral notes have been added to this Supplement; but the addition of moſt conſequence is the POSTSCRIPT on the ſubject of the population of the kingdom.——

In

In the former Editions, and alſo in the publication entitled an *Eſſay on the Population of England from the Revolution*, &c. I gave an account of ſeveral facts which ſeemed to me to ſhew, that our population has declined. Great pains have been taken to prove this to be a miſtake (*a*) In the *Poſtſcript* juſt mentioned, I have entered a little farther into this controverſy; and it will appear that though I ſtill retain my former opinion, yet I wiſh to be conſidered as far from being decided in it, and therefore as

(*a*) Much has been ſaid alſo about a miſtake into which it is ſuppoſed I have fallen in eſtimating the quantity of gold coin in the kingdom. The truth, in this inſtance, is briefly this.———The third proclamation for calling in the gold coin brought in near *double* the ſum that was expected. In conſequence of this, an eſtimate which I had publiſhed in the Firſt Tract on Civil Liberty, proved ſhort of the truth about *three millions and a half*; and it appears now, that, excluſive of two millions purchaſed by the bank and melted into bars, the gold coin of the kingdom was (in 1773) about *ſixteen millions*, inſtead of *twelve millions and a half*, as I had reckoned it. And this is the account I have given in the *laſt* edition, p. 74, of the Tract juſt mentioned, except that being then not informed of the coin purchaſed by the bank, I have not mentioned it.

open

open to receive any evidence which can be produced to overthrow it.

Being willing to comprize in this Edition all that I have written on the duration of human life, and the values of life-annuities, I have inſerted at the end of the Second Volume Three Eſſays on theſe ſubjects, which have been publiſhed in the *Philoſophical Tranſactions of the Royal Society*.

A pretty copious Index cloſes the whole.

The additions I ſhall laſt mention are thoſe which relate to public credit and the national debt; and I have choſen to mention them laſt, on account of their particular nature and importance.

In the Preface to the Third Edition, (ſee p. lviii) I took notice of a plan announced in 1773 by Lord *North* to the Houſe of Commons, for paying in the ten following years 17 millions of the public debt. It is neceſſary I ſhould here juſt mention, that this plan was never afterwards heard of. ——The remarks I have made upon it, were followed with a propoſal for expediting a plan of redemption in ſuch a manner,

as

as to cauſe an appropriation of a million *per ann.* to diſcharge, in forty years, a *hundred millions* of the public debts then bearing 3 *per cent.* intereſt. This propoſal has not been continued in this Edition, becauſe I intend ſoon to lay before the public a plan more efficient, and better adapted to the preſent ſtate of our funds. I muſt, however, obſerve that, having now no hope that an efficient plan of redemption will ever be eſtabliſhed, I think with regret of the time and attention I have beſtowed on this ſubject. Nothing relieves me, but the reflection that the object about which I have loſt my time, has been the removal of an evil which, if no ſuch meaſures as I have propoſed are adopted, muſt bring on a *cataſtrophe* which will make this country a warning and a terror to the world.

At the end of the chapter on public credit I have, in this Edition, inſerted a brief hiſtory of the Sinking Fund; and alſo a particular account of the increaſe of the public debts from 1776 to 1783, and of the ſtate of our finances at the time of ſigning the Preliminaries of peace in *January* laſt. This account is, I believe, as correct as it is poſſible

poſſible at preſent to make it; and I have choſen for many reaſons that it ſhould form a part of this work. Hereafter, probably, it will be read with amazement. Our folly, in this inſtance, is without example. Lord NORTH enjoys the ſingular diſtinction of having contributed more to it than any former miniſter. By a war which has degraded the kingdom, and a diſſipation of treaſure which was never equalled, he has, in the ſhort compaſs of ſeven years, doubled a debt before too heavy to be endured. And let future generations riſe up; and, if poſſible, let them call him—*Bleſſed.*

Newington-Green,
March 29th, 1783.

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ERRATUM.

In Page 205, Vol. I. Line 6, for "*Enquiries,*" read "*Enemies.*"

CHAP.

CHAP. I.

Queſtions relating to Schemes for granting Reverſionary *Annuities, and the Value of Aſſurances on Lives.*

QUESTION I.

"A SET of married men enter into a "ſociety for ſecuring annuities to "their widows. 'What ſum of money, "in a ſingle preſent payment, ought every "member to contribute, in order to entitle "his widow to an annuity of 30 *l. per* "*ann.* for her life, eſtimating intereſt at "4 *per cent?*"

ANSWER.

It is evident, that the value of ſuch an expectation is different, according to the different ages of the purchaſers, and the proportion of the age of the wife to that of the huſband. Let us then ſuppoſe, that every perſon in ſuch a ſociety is of the ſame age with his wife, and that one with another all the members when they enter may be reck-

oned 40 years of age, as many entering above this age as below it. It has been demonſtrated by Mr. *De Moivre* and Mr. *Simpſon*, that "the value of an annuity on the *joint continuance* of any two lives, ſubtracted from "the value of an annuity on the life in expectation," gives the true preſent value of annuity on what may happen to remain of the latter of the two lives after the other.

In the preſent caſe, the value of an annuity to be enjoyed during the *joint continuance* of two lives, each (*a*) 40, (*b*) is 9.826, according

(*a*) See Table II. at the end of this work.

(*b*) The values of *joint* lives and reverſions, as deduced from the *Breſlaw* obſervations, are not given in any part of this work from Mr. *De Moivre*'s rules in his treatiſe on annuities on lives. For theſe rules are approximations, which give reſults ſo far from the truth, as to be, not only uſeleſs, but dangerous. In the ſecond eſſay in the Appendix, a particular account of this will be given, and alſo of the method in which theſe values have been calculated.

Mr. *De Moivre* has calculated the values of *ſingle* lives, on the ſuppoſition of an *equal decrement of life* thro' all its ſtages till the age of 86, which he conſidered as the utmoſt probable extent of life. Thus, let there be 56 perſons alive at 30 years of age. It is ſuppoſed that one will die every year till, in 56 years, they will be all dead. The ſame will happen to 46 at 40, in 46 years. To 36 at 50, in 36 years, and ſo on for all other ages. The number of years which a given life wants of 86, he calls the *complement* of that life. Fifty-ſix, therefore, is the *complement* of 30; 46 of 40, and 36 of 50.

This hypotheſis eaſes very much the labour of calculating the values of lives; and at moſt ages between 30 and 70 or 75, it is ſo conformable to Dr. *Halley*'s Table of Obſervations, that I ſhall not, in theſe queſtions, think it neceſſary

ing to the probabilities of life in the Tables of Obſervations formed by Dr. *Halley*, from the bills of mortality of *Breſlaw* in *Sileſia*. The value of a ſingle life 40 years of age, as given by Mr. *De Moivre*, agreeably to the ſame Table, is 13.20 *(a)*; and the former ſubtracted from the latter, leaves 3.37, or the true number of years purchaſe, which ought to be paid for any given annuity, to be enjoyed by a

neceſſary to diſtinguiſh between the values of ſingle lives as deduced from this Table, and the ſame values deduced from the hypotheſis.

In order to avoid putting the reader to trouble, I have given this table among other tables in the Appendix. And I have alſo given two tables which I have formed from the bills of mortality at *Northampton* and *Norwich*. Theſe laſt anſwer more nearly to Mr. *De Moivre*'s hypotheſis than Dr. *Halley*'s table; and the difference between the values of *ſingle* and *joint* lives by the *hypotheſis*, and the ſame values computed ſtrictly from the tables, is generally leſs in theſe tables than in Dr. *Halley*'s, as will be ſhewn in the laſt Eſſay. When, therefore, in the courſe of this work the values of *ſingle* and *joint* lives are mentioned, as given agreeably to Dr. *Halley*'s table, it muſt be underſtood, that they are taken from the two tables in the laſt leaves of this work, and given in ſtrict agreement only to the *hypotheſis*; and that for this reaſon, they are in reality ſtill more conformable to the *Northampton* and *Norwich* tables.

The inhabitants of *London*, as is well known, not living ſo long as the reſt of mankind, the values of *ſingle* and *joint* lives there, are conſiderably leſs than thoſe juſt mentioned. And, therefore, whenever I have had *London* lives in view, I have given particular notice of it, and taken their values from Mr. *Simpſon*, who has calculated them from the *London* tables of obſervation. See the Tables in the Appendix.

(a) See Table I. at the end of this work.

perſon 40 years of age, *provided* he ſurvives another perſon of the ſame age, intereſt being reckoned at 4 *per cent. per annum.* The annuity, therefore, propoſed in this Queſtion being 30 *l.* the preſent value of it is 30 multiplied by 3.37, or 101*l.* 2*s.*

By calculating from Mr. *Simpſon*'s Tables *(a)*, formed from the bills of mortality of *London*, this value comes out 102*l.*

The difference in the value of the reverſion will be inconſiderable, whether the common age is taken a few years more or leſs than 40. Thus married men of 30 ought not, according to Dr. *Halley*'s Table, to give two-fifths of a year's purchaſe more, for any given reverſionary annuity for their wives, than married men of 50, provided they are of the ſame ages with their wives; and one quarter more, according to Mr. *Simpſon*'s Table. If the wives are younger (as is generally the caſe) there will indeed be a conſiderable difference; for the value now determined would be 120*l.* according to the *Breſlaw* Obſervations, ſuppoſing the two lives to be 40 and 33, or that wives are one with another ſeven years younger than their huſbands; and 118*l.* 10*s.* according to the *London* Obſervations.

(*a*) See Tables X. and XI. Appendix.

QUESTION II.

" Suppoſing ſuch a ſociety as that deſcrib-
" ed in the preceding Queſtion, to be limit-
" ed to a certain number of members, and
" conſtantly kept up to that number, by the
" admiſſion of new members as old ones are
" loſt, in conſequence of their own deaths,
" and the deaths of their wives: What is the
" number of annuitants which, in ſome time
" after its eſtabliſhment, will come to be
" conſtantly upon it?"

ANSWER.

Since every marriage produces either a widow or widower; and ſince all marriages taken together would produce as many widows as widowers, were every man and his wife of the ſame age, and the chance equal which ſhall die firſt; it is evident, that the number of widows that have ever exiſted in the world, would, in this caſe, be equal to *half* the number of marriages. And what would take place in the world, muſt alſo, on the ſame ſuppoſitions, take place in this ſociety.——In other words; every *other* perſon in ſuch a ſociety leaving a widow, there muſt ariſe from it a number of widows equal to half its own number.—But this does not determine what number, all living at one and the ſame time, the ſociety may expect will

come to be conſtantly upon it. For if every widow lived no more than a year, the ſociety would never have more annuitants upon it, than came on in a year. And on the contrary, if none ever died, the number of annuitants would go on increaſing for ever.—'Tis, therefore, neceſſary, in order to anſwer the preſent enquiry, to determine how long the *duration* of *ſurvivorſhip* between perſons of equal ages will be, compared with the *duration* of *marriage*. And the truth is, that, ſuppoſing the probabilities of life to decreaſe uniformly (*a*), the former is equal to the latter; and conſequently, that the number of *ſurvivors*, or (which is the ſame ſuppoſing no ſecond marriages) of *widows* and *widowers* alive together, which will ariſe from any given ſet of ſuch marriages conſtantly kept up, will be equal to the whole number of marriages; or *half* of them (the number of widows in particular) equal to *half*

(*a*) That is, ſuppoſing that out of any given number alive at any age, the ſame number will die every year 'till all are dead. See the preceding note. That on this hypotheſis, the duration of ſurvivorſhip is equal to the duration of marriage, when the ages are equal; or, in other words, that the *expectation* of two joint lives, the ages being equal, is the ſame with the *expectation* of ſurvivorſhip, may be learnt from the 18th and 20th problems of Mr. *De Moivre*'s treatiſe on annuities; and a demonſtration of it, together with a particular explanation of this ſubject, may be found at the beginning of the firſt Eſſay to which I muſt beg the reader to turn, if he is at any loſs about the full meaning of what is here ſaid.

the

the number of marriages.—Now, it appears that in moſt towns the decreaſe in the probabilities of life, is in fact nearly uniform. According to the *Breſlaw*, the *Northampton* and *Norwich* Tables of Obſervation, almoſt the ſame numbers die every year from 20 years of age to 77 (*a*). After this, indeed, fewer die, and the rate of decreaſe in the probabilities of life is retarded. But this deviation from the hypotheſis is inconſiderable; and its effect, in the preſent caſe, is to render the duration of ſurvivorſhip *longer* than it would otherwiſe be. According to the *London* Table of Obſervations, the numbers dying every year begin to grow leſs at 50 years of age; and from hence to extreme old age, there is a conſtant retardation in the decreaſe of the probabilities of life (*b*). Upon the whole, therefore, it appears in anſwer to the preſent Queſtion, "that according to the *three* "*former Tables* of Obſervations, and ſuppo- "ſing no widows to marry, the number "enquired after is *ſomewhat greater* than "half the number of the ſociety; but, ac- "cording to the *London Table*, a *good deal* "*greater*."

It muſt be carefully remembered, that this has been determined on the ſuppoſition, that

(*a*) See Tables V. VI. and VII. Appendix.

(*b*) The reaſon of this difference between the *London* and other Tables, will be given at the end of the fourth Eſſay.

huſbands and their wives are of equal ages, and that in this caſe it becomes an equal chance which ſhall die firſt. In reality neither of theſe ſuppoſitions is juſt. Huſbands in general are older than their wives; and, in equal ages, the mortality of males has been found to be greater than the mortality of females. For both theſe reaſons, it is much more than an equal chance that the huſband will die before his wife, or that the woman ſhall be the ſurvivor of a marriage, and not the man. This will increaſe conſiderably the duration of ſurvivorſhip on the part of the woman, and conſequently the number enquired after in this Queſtion. The marriage of widows will alſo diminiſh this number, and the operation of theſe cauſes will be different in different ſituations. But it is by no means to be expected (in the ſituation of the ſocieties I have in view) that the diminution from the latter cauſe will be conſiderable enough to overbalance the operation of all the other cauſes which have been mentioned, and reduce the number under conſideration ſo low, as half the number of marriages (*a*).

SCHOLIUM.

In *London* it appears, that there is a retardation of the decreaſe in the probabilities

(*a*) It will be obſerved hereafter, that this obſervation has been found to be true in fact.

of

of life, which renders the duration of ſurvivorſhip between two lives of equal ages, conſiderably longer than their joint continuance. It ſeems worth obſerving, that this is the reaſon why, though the probabilities of life, and therefore the values of ſingle and joint lives, are leſs in *London* than in other places, yet the values of reverſions depending on ſurvivorſhips, are in ſome caſes greater there. It is proper to add, that this likewiſe is the reaſon why, in calculating the values of joint lives and reverſions, the preſent value of an annuity payable yearly to the ſurvivor of two equal lives, may come out equal to, or even greater than, the preſent value of a like annuity for the joint lives. As an annuity, during ſuch ſurvivorſhip, will probably not become payable for ſome years, and therefore the money given for it will have time to accumulate, it is manifeſt, that the value of it could never be equal to the value of an annuity on the joint lives, the payment of which begins immediately, were not the obſervation now made true.

QUESTION III.

" Such a ſociety as that deſcribed in the " preceding Queſtions being ſuppoſed; in " what time will the number of annuitants " upon it come to a *maximum*?"

ANSWER.

ANSWER.

In order to be more clear in anſwering this Queſtion, I will firſt ſuppoſe the ſociety to comprehend in it from its firſt eſtabliſhment, *all* the married perſons of *all* ages in any town or country, where the number of people continue conſtantly the ſame. In this caſe, the whole collective body of members will be, at their greateſt age, at the time of the eſtabliſhment of the ſociety; and the number of members, together with the number of widows left every year, will, taking one year with another, admit of no increaſe or diminution. The number of widows in life together, derived from any given number coming on a ſociety every year, will increaſe continually, 'till as many die off as are added every year; that is 'till they come to die off as faſt as poſſible. But they cannot die off as faſt as poſſible, 'till the whole collective body of widows are at their greateſt age: or, 'till there is among them the greateſt number poſſible of the oldeſt widows; and, therefore, not 'till there has been time for an acceſſion to the oldeſt widows, from the youngeſt part of the widows that come on annually.

Let us, for the ſake of greater preciſion, divide the whole medium of widows that come on every year, into different claſſes according to their different ages, and ſuppoſe ſome to be left at 56 years of age, ſome at 46, ſome

ſome at 36, and ſome at 26. The widows, conſtantly in life together, derived from the firſt claſs, will come to their greateſt age, and to a *maximum*, in 30 years, ſuppoſing, with Mr. *De Moivre*, 86 to be the utmoſt extent of life. The ſame will happen to the ſecond claſs in 40 years, and to the third in 50 years (*a*). But the whole body, compoſed of theſe claſſes, will not come to a *maximum*, 'till the ſame happens to the fourth or youngeſt claſs; that is, not 'till the end of 60 years. After this, the affairs of the ſociety will become *ſtationary*, and the number of annuitants upon it of all ages will keep always nearly the ſame.

Such is the anſwer to this Queſtion, ſuppoſing a ſociety to begin with its complete number of members, conſiſting of married perſons of all ages, in the ſame proportion to one another, with the proportions in which they exiſt in the world.——If it begins with its complete number of members, but at the ſame time admits none above a particular age: If, for inſtance, it begins with 200 members all under 50, and afterwards limits itſelf to this number, and keeps it up by admitting every year, at all ages between 26 and 50, new members as old ones drop off;

(*a*) In note (A), at the end of this treatiſe, a rule is given by which the numbers alive at the end of any particular number of years may be very eaſily determined.

in

in this caſe, the period neceſſary to bring on the *maximum* of annuitants will be juſt doubled. For, in the firſt place, the whole collective body of members will be 60 years in getting to their greateſt age, as may eaſily appear from what has been juſt ſaid. The annual medium of widows therefore, that will come on the ſociety will increaſe continually for 60 years; it being evident, that the older any ſet of married men are, taken one with another, the faſter they will leave widows. And after this annual medium is increaſed to a *maximum*, 60 years more will be neceſſary to bring to a *maximum* the number in life together, derived from ſuch a *fixed* annual medium conſtantly coming on.——If ſuch a ſociety is any number of years in gaining its *maximum* of members, the time neceſſary to bring on the *maximum* of annuitants will be ſtill further prolonged, and will be equal to twice 60 years with that number of years added.—Moſt of the ſocieties for granting annuities to widows are of this kind; and, therefore, ſuppoſing them to gain their complete number of members in ten years, and for ever afterwards to preſerve it, the number of annuitants upon them will go on increaſing for 130 years—It is proper, however, to be remembered, that the increaſe will be quicker at firſt, and afterwards ſlower; and that, within 20 or 30 years of the end

of

of this term, it will be ſo ſlow as ſcarcely to be ſenſible, though ſtill real.

All who will beſtow due attention on this ſubject muſt ſee theſe deciſions to be juſt; and a demonſtration of them might be given, in a form more ſtrictly mathematical, were it neceſſary.

QUESTION IV.

"Suppoſe the members of ſuch a ſociety "as that deſcribed in the preceding Queſti- "ons, to chuſe making *annual payments du- "ring the continuance of marriage*, in lieu of "the ſum which the reverſionary annuity for "their widows is worth in *preſent money*: "What ought theſe *annual payments* to be, "eſtimating intereſt at 4 *per cent*?"

ANSWER.

This will be eaſily determined, by finding what annual payments, during two joint lives of given ages, are equivalent to the value of the reverſionary annuity in *preſent money*.— Suppoſe, as in Queſtion I. the two joint lives to be each 40, and the reverſionary annuity 30 *l. per annum*. An annual payment during the continuance of two ſuch lives is worth, according to Dr. *Halley*'s Table of Obſervations, 9.82 (*a*) years purchaſe. The annual

(*a*) See Table II. in the laſt leaves of this work.

payment

payment then ought to be ſuch as being multiplied by 9.82, will produce (*a*) *l.* 101.1, the preſent value of the annuity in one payment by Queſtion I. Divide then *l.* 101.1 by 9.82, and the *quotient*, or *l.* 10.3 will be the anſwer.——This is very nearly the annual payment of all the members at an average, ſuppoſing equal numbers to offer themſelves for admiſſion of every age between 30 and 50. As much as ſome give leſs, others ought to give more, according to their exceſs of age. Thus, the annual payment of a married perſon, 30 years of age, ought to be *l.* 9.39; and of a perſon 50 years of age *l.* 11.33.——If the values of joint lives and of the reverſionary annuity are taken agreeably to the *London* Table of Obſervations, theſe annual payments will be, for 30 years of age (*b*), *l.* 10.9—for 40, *l.* 12.5,—for 50, *l.* 14.5. If

(*a*) Particular notice ſhould be taken of the method of notation here uſed, becauſe it will be carried through the whole of this work.—— The figures on the right hand of the full-point ſignify the decimal parts of 1*l.* Thus; *l.* 101.1, is 101 and the 10th of 1*l.* or *l.* 101 and 2*s.*——*l.* 9.39, is *l.* 9, and 39th hundredths of 1*l.* or *l.* 9: 7*s.*: 10*d.*——*l.* 11.33, is *l.* 11, and 33 hundredths of 1*l.* or *l.* 11 : 6*s.*: 7*d.*———In general; it ſhould be remembered, that 2 ſhillings allowed for every unit in the firſt place of decimals, and two-pence half-penny for every unit in the ſecond place of decimals, will give, nearly enough, the value of the decimal part of every ſuch expreſſion.

(*b*) The value of two joint lives of 30, taken from Table XI. Appendix, is 9.6. This ſubtracted from the value of the life in expectation, or from 13.1, by Table X. gives 3.5, the

If either the rate or intereſt is ſuppoſed lower, or wives are ſuppoſed younger than their huſbands, the annual payments will be increaſed. But their is no occaſion for pointing out particularly the difference. It may be eaſily found in any caſes by the directions now given. There is, however, one obſervation which ought to be here carefully attended to.—This method of calculation ſuppoſes, that the firſt annual payment is not to be made 'till the end of a year. If it is to be made *immediately*, the value of the joint lives will be increaſed one year's purchaſe; and, therefore, in order to find in this caſe the annual payments required, the value in preſent money found by Queſt. I. muſt be divided by the value of the joint lives increaſed by unity, and, in this way, the preceding values at 4 *per cent.* according to the *Breſlaw* Obſervations, will be found to be *l.* 8.62—*l.*9.35.—*l.*10.07—According to the *London* Obſervations, *l.*10,—*l.*11.2,—*l.*12.7,

the number of years purchaſe which an annuity for a life of 30 years of age, *after* another life of the ſame age, is worth. This remainder, multiplied by 30, gives 105*l.* the value in a ſingle payment, ſuppoſing the reverſionary annuity to be 30*l.* And 105*l.* divided by 9.6, gives *l.*10.9, the value of the ſame annuity in annual payments, during the joint continuance of the two lives, according to the *London* obſervations.—By ſimilar operations all the other values above given have been found.

QUESTION V.

" A ſociety may chuſe to make abate-
" ments in theſe annual payments, and to re-
" quire the remainder of the value of the
" reverſionary annuity to be given, in fines
" or premiums, at the time of admiſſion. It
" may, for inſtance, chuſe to fix the annual
" payments of all the members to 5 guineas.
" What, in this caſe, would be the premium
" due at admiſſion, the annuity being ſup-
" poſed 30 *l. per annum*, and intereſt being
" at 4 *per cent?*"

ANSWER.

From the whole preſent value of the annuity in one payment, ſubtract the value of 5 guineas *per annum*, during the joint lives; and the remainder will be the anſwer.

Suppoſing the joint lives, both 40, the whole preſent value of the annuity in one payment is, according to the *Breſlaw* Obſervations, *l.*101.1, by Queſt. I.—The value of 5 guineas *per annum*, or of *l.*5.25 *per annum*, during two ſuch joint lives, is *l.*5.25, multiplied by the value of the joint lives; that is, 4.25, multiplied by 9.82, or *l.*51.55; and this ſubtracted from *l.*101.1, gives *l.*49.5, the anſwer required for two lives at the age of 40. The anſwer found in the ſame way for two lives whoſe common age is 30, is *l.*46.5,—and for two lives at 50, 50*l.*

Accord-

According to the *London* Obſervations, theſe values are, for two lives, at 30, *l.*54.6.—At 40, *l.*59. 4.—At 50, *l.*63.3.

If the firſt of the annual payments is to be made immediately, the true anſwer will, in every inſtance, be the values found in the manner now directed, diminiſhed by the annual payment; or, in the preſent caſe, 5 guineas leſs than the values ſpecified.

The values, in *premiums* and *annual payments*, of any other reverſionary annuity, will be as much greater or leſs than theſe, as the annuity itſelf is greater or leſs.

QUESTION VI.

"A perſon 35 years of age wants to buy "an annuity, for what may happen to re- "main of his life after 50 years of age. "What is the value of ſuch an annuity in "*ready money*, and alſo in *annual payments*, "till he attains to the ſaid age; that is, in "annual payments for 15 years, ſubject in "the mean time to failure, ſhould his life "fail?"

ANSWER.

The preſent value of ſuch an annuity is the *preſent* value of a life at 50, in money to be received 15 years hence, and the payment of which depends on the contingency of the continuance of the given life 15 years. That is; it is equal to the value of a life at 50,

multiplied by the preſent value of 1*l.* to be received at the end of 15 years, and alſo by the probability that the given life will continue ſo long.—A life at 50, according to Mr. *De Moivre*'s valuation of lives, and reckoning intereſt at 4 *per cent.* is worth 11.34 years purchaſe. The preſent value of 1*l.* to be received at the end of 15 years, is, by Table I, (ſee the laſt leaves of Vol. II.) 0.5553. And the probability that a life at 35, will continue 15 years, is, according to the *Breſlaw* Obſervations $\frac{1}{4}\frac{4}{9}\frac{6}{0}$(*a*). And theſe three values, multiplied by one another, give *l.*4.44, or the number of years purchaſe that ought to be given for the annuity.—The annuity then being ſuppoſed 50*l.* its value in preſent money is 222*l.*

(*a*) The probability that a given life ſhall continue any number of years, or reach a *given age*, is (as is well known) the fraction, whoſe *numerator* is the number of the living in any Table of Obſervations oppoſite to the *given age* and *denominator*, the number oppoſite to the preſent age of the given life.—Thus, in the preſent inſtance; 346 is the number in Dr. *Halley*'s Table oppoſite to 50, and 490 the number oppoſite to 35.—$\frac{3}{4}\frac{4}{9}\frac{6}{0}$. (or the odds of 17 to 7) is, therefore, the probability that a perſon whoſe age is 35 ſhall attain to 50, or live 15 years. In the ſame manner it will appear, that, according to the ſame Table, the probability that a perſon at this age ſhall live 25 years, is $\frac{2}{4}\frac{4}{9}\frac{2}{0}$; or nearly an even chance.

At *Northampton* and *Norwich* a perſon at the ſame age, has an even chance of living 26 years; but in *London*, ſcarcely 20 years. See Tables V. VI. VII. and VIII. at the beginning of Vol. II. I will add, though foreign to my preſent purpoſe, that a perſon at the ſame age has in theſe towns a better chance of living one year, than in *London*, in the proportion of 3 to 2.

In

In order to find this value in *annual payments*, while the given life is attaining 50, it is neceſſary to find the value of an annuity for 15 years, ſubject to failure on the extinction of the given life. And the value of ſuch annuity is, evidently, the laſt value ſubtracted from the value of the given life; or, in the preſent inſtance, *l.*4.44, ſubtracted from *l.*13.97. (See Table I. at the end of Vol. II.) that is, *l.*9.53.—222*l.* then, being the preſent value of an annuity of 50*l.* for the remainder of a life now 35, after attaining 50; and 9.53 being the number of years purchaſe, which ought to be given for an annual payment to laſt 15 years, if a life now 35 laſts ſo long, it follows that the value of the ſame annuity in annual payments, till this life attains 50, is 222*l.* divided by 9.53; or *l.*23.3.

This calculation ſuppoſes, that the firſt of the annual payments is not to be made till the end of a year. If the firſt payment is made immediately, the value will be, the *ſingle payment* divided by the value of the life for the given term increaſed by unity; that is, in the preſent caſe, 222*l.* divided by 10.53; or *l.*21.08.

If the value of the annuity is required in a ſingle payment, over and above any given annual payment; deduct the value of the annual payment from the whole value in a ſingle preſent payment, and the remainder will

be the anſwer.—Thus, let 5 guineas, in the preſent inſtance, be the given annual payment for the aſſigned term; and let the enquiry be, how much more in preſent money the ſuppoſed annuity is worth. By what has been juſt ſaid, 9.53, multiplied by 5 guineas, that is, 50*l.* is the value of the annual payment; and this ſum deducted from 222*l.* leaves 172*l.* the anſwer.

If the annual payment begins immediately, its value is 10.53, multiplied by 5 guineas, and the anſwer comes out *l.*166.75.

In this way may be found the value, in ſingle and annual payments, of any other annuity, payable to an aſſigned life, after a given term of years, taking any valuation of lives or intereſt of money. But care muſt be taken to remember, that it is the title to the annuity that will commence at the end of the given term, and that the firſt payment is not to be made till a year afterwards; that is, in the caſe here ſpecified, not till the end of 16 years.

SCHOLIUM.

The value of the *remainder* of two joint lives, after a given term of years, is likewiſe the value of 1*l.* due at the end of the given term multiplied by the value of two joint lives, each older by the given term than the given lives; and this product, multiplied by the probability, that the given joint lives ſhall not

not fail in the given term; or (which is the ſame) by the product of the two probabilities, that the ſingle lives ſhall each continue the given term. And the value of an annuity, on any given joint lives for a term of years beginning now, is this laſt value ſubtracted from the whole preſent value of the joint lives. Thus; the value of two joint lives, one 40 years of age, and the other 50, (ſee Table II. at the end of Vol. II.) is 8.91; which, multiplied by 0.6755, the value of 1*l.* due 10 years hence, and by $\frac{443}{511}$ (the probability that a life at 30 ſhall continue 10 years) and alſo by $\frac{346}{443}$, (the probability that a life at 40 ſhall continue 10 years) give 3.92, the preſent value of the remainder of two joint lives, aged 30 and 40, after 10 years; and this value, ſubtracted from 10.43, (the value in Table II. ibid. of two joint lives, aged 30 and 40) leaves 6.51, their value for 10 years.

As the value of the longeſt of two lives is always the value of the *joint* lives, ſubtracted from the ſum of the values of the two *ſingle* lives; their value alſo for any *given term*, is the value of the *joint* lives for the given term, ſubtracted from the ſum of the values of the *ſingle* lives for the given term.

The truth of theſe rules may eaſily appear without particular proof. I have, however, pointed out the method of demonſtrating them in a note (*a*) at the end of this work.

a See note (B) at the end of Vol. II.

By ſimilar operations, may be found the values of 3 or more *joint* lives, or the longeſt of *three* or more lives, for a given term of years, or of what ſhall remain of them after a given term of years.

QUESTION VII.

" The preſent value is required of an annuity to be enjoyed by one life, for what " may happen to remain of it beyond another life, after a given term: that is, provided *both* lives continue, from the preſent time, to the end of a given term of " years?"

ANSWER.

Find the value of the annuity for two lives greater, by the given term of years, than the given lives. Diſcount this value for the given term; and then, multiply by the probability, that the two given lives ſhall *both* continue the given term; and the product will be the anſwer.

EXAMPLE.

Let the two lives be each 30. The term ſeven years. The annuity 10*l*. Intereſt, 4 *per cent*.——The given lives, increaſed by 7 years, become each 37. The value of two joint lives each 37, is (by Table II. in the laſt leaves of the next Volume) 10.25.

The

The value of a ſingle life at 37, is (by Table I. ibid.) 13.67. The former, ſubtracted from the latter, is 3.42, or the value of an annuity for the life of a perſon 37 years of age, after another of the ſame age, by Queſt. I.—3.42 diſcounted for 7 years, (that is, multiplied by 0.76, the value of 1*l.* due at the end of ſeven years by Table I. at the beginning of Vol. II.) is 2.6. —The probability that a ſingle life at 30 ſhall continue 7 years, is (by the hypotheſis explained page 2.) $\frac{49}{56}$ (*a*). The probability, therefore, that two ſuch

(*a*). In this caſe, it is on ſome accounts beſt, as well as eaſieſt, to take the probabilities of life from the hypotheſis, rather than immediately from the Tables.—Fifty-ſix perſons being ſuppoſed alive at 30, one will die every year, according to the hypotheſis. At the end of ſeven years then, the number of the living will be 49, and $\frac{49}{56}$, or the odds of 7 to 1, is, by note, p. 18, the probability, that a life, aged 30, will continue 7 years; and this fraction, multiplied by itſelf, is the probability, that two lives of this age, ſhall *both* continue 7 years. In general, it muſt be remembered, that the probability, that any two or more events ſhall *all* happen, is the product ariſing from multiplying by one another, the probabilities of all the events taken ſeparately. The probability, therefore, that any number of perſons will *all* live any given time, is rightly found by multiplying into one another the probabilities that each of them will live that time.—It may further be of uſe to ſome, that I ſhould obſerve here, that the difference between unity and the fraction expreſſing the probability that an event will happen, gives the probability that it will *not* happen. Thus; the probability, that a perſon 40 years of age will live 11 years, is, by the *Breſlaw* Table (or Table V. beginning of Vol. II.) $\frac{335}{445}$. The probability, therefore, that he will *not* live 11 years, is $\frac{335}{445}$ ſubtracted from

ſuch lives ſhall both continue 7 years, is $\frac{2401}{3136}$, or, in decimals 0.765. And 2.6, multiplied by 0.765, is 1.989, the number of years purchaſe which ought to be given for an annuity, to be enjoyed by a life now 30 years of age, after a life of the ſame age, provided both continue 7 years. The annuity then being 10*l.* its preſent value is *l.*19.89.

By ſimilar operations, it may be found, that ſuppoſing the term one year, and the ages and the rate of intereſt the ſame, the preſent value of the ſame reverſionary annuity is *l.*32.4; and that if the term is 15 years, the value is *l.*9.7.

For two lives each 40, theſe values are *l.*30.33.—*l.*17.44.—*l.*7.3. the term being 1, 7, or 15 years.

For two lives each 50, the ſame values for the ſame terms, are *l.*28.2,—*l.*13.86,—*l.*4.34 (*a*).

Theſe values, according to the *London* Obſervations and Mr. *Simpſon*'s Tables of the values of ſingle and joint lives, are,

from unity or $\frac{110}{445}$. In like manner: The probability that two perſons aged 30 ſhall *both* live 7 years, being 0.765, the probability that they will *not* both live ſo long, or that *one or other* of them will die in 7 years, is 0.765 ſubtracted from unity, or .235.

If any reader is unwilling to take theſe aſſertions for granted, he ſhould conſult the beginning of Mr. *De Moivre*'s, or Mr. *Simpſon*'s Treatiſes on the Doctrine of Chances, where he will find them demonſtrated.

(*a*) See Note (C) at the end of Vol. II.

For

For 2 lives at 30—*l.*32.05—*l.*18.62—*l.*7.66.
at 40—*l.*30.7— *l.*15.6— *l.*5.45.
at 50—*l.*29.36—*l.*12.33—*l.*3.24.

QUESTION VIII.

" Let the ſcheme of a ſociety for granting " annuities to widows, be, that if a member " lives *a year* after admiſſion, his widow ſhall " be entitled to a life annuity of 20*l.* If " *ſeven* years, to 10*l.* more, or 30*l.* in the " whole. If *fifteen* years, to another addi- " tional 10*l.* or 40*l.* in the whole. What " ought to be the annual payments of the " members for the ages of 30, 40, and 50, " ſuppoſing them of the ſame ages with their " wives, and allowing compound intereſt at " 4 *per cent* ?"

ANSWER.

According to the *hypotheſis*, explained p. 2; and, therefore, very nearly, according to the Tables of Obſervation for *Breſlaw*, *Norwich* and *Northampton*, or Tables V. VI. VII. at the beginning of Vol. II:

*l.*8.44—*l.*8.69—*l.*9.05.

According to the *London* Obſervations.

*l.*9.41—*l.*10.17—*l.*10.92.

Theſe

Theſe values are eaſily deduced from the values in the laſt Queſtion. For example. The value of 10 *l. per annum* for life to 40 after 40, provided the joint lives do not fail in *one* year, is, according to the *hypotheſis*, *l.*30.33. The value of 20 *l. per annum*, in the ſame circumſtances, is therefore *l.*60.66.—In like manner, the value of 10 *l.* after *ſeven* years, is, *l.*17.44. And of 10 *l.* after 15 years *l.*7.3.—Theſe values together make *l.*85.4, or the value of the expectation, deſcribed in this Queſtion, in a *ſingle preſent payment*; which, divided by 9.82, the value (by Table II. at the end of Vol. II.) of two joint lives at 40, gives *l.*8.69, the value of the ſame expectation in *annual payments*, during the joint lives.—In the ſame manner may be found the anſwer in all caſes to any Queſtions of this kind.

Theſe calculations ſuppoſe, that the annual payments do not begin till the end of a year. If they are to begin *immediately*, the true *annual payments* will be, as was before obſerved, the *ſingle* payments, divided by the value of the joint lives increaſed by unity; and in the preſent caſe they will be, by the *hypotheſis*,

*l.*7.75—*l.*7.9—*l.*8.07.

By the *London* Obſervations,

*l.*8.52—*l.*9.06—*l.*9.51.

By

By the method of calculation now explained may be eaſily found in all caſes, ſuppoſing the annual payments previouſly ſettled, what the reverſionary annuities are correſponding to them in value.—Thus, the annuities being the ſame with thoſe mentioned in this Queſtion, the *mean* annual payments for all ages between 30 and 50, are nearly 8*l.* according to the *higheſt* probabilities of life; 9*l.* according to the *loweſt*; and 8 guineas the *medium* (*a*); intereſt being at 4 *per cent.* and the firſt payment to be made immediately.

If the mean annual payments, beginning immediately, are fixed to five guineas, the correſponding life annuities will be nearly (by the *hypotheſis*) 12*l.* if the contributor lives a year, and 24*l.* if he lives ſeven years; or (by the *London* Obſervations) 12*l.* if he lives a year, and 20*l.* if he lives ſeven years (*b*).

It

(*a*) The value of this expectation, ſuppoſing married men 40 years of age, and their wives 30, is, in a *ſingle* payment, 113*l.* In annual payments beginning immediately, *l.* 9.88, by the *hypotheſis*. And 10*l.*—and *l.* 10.93, by the *London* Obſervations.

(*b*) If the annuities in expectation are 14*l.* provided a member lives a year, and 20*l.* provided he lives ſeven years, the proper *mean ſingle* payments for all ages, taken one with another, under 50 or 52, is 50 guineas nearly, according to all the Tables of Obſervation, ſuppoſing equality of age between men and their wives. And the addition which ought to be made, on account of exceſs of age on the man's ſide, is, taking the neareſt and the eaſieſt

It is obſervable, that the difference in the values of the annuities, ariſing from difference of ages and the difference in the probabilities of life, is leſs in this Queſtion than in Queſtion 4th; and that, conſequently, the plan propoſed in it, is the ſafeſt, as well as the moſt equitable and encouraging, that a ſociety can adopt.

It is neceſſary to remark here further, that *yearly* payments which begin immediately, are more advantageous than *half-yearly* payments which begin immediately. In an Eſſay publiſhed in the *Philoſophical Tranſactions*, vol. 66. p. 109, and inſerted in the 2d volume of this work, I have ſhewn that in the caſe of life annuities, *half-yearly* payments which begin at the end of half a year, are nearly a *fifth* of a year's purchaſe better than *yearly* payments which begin at the end of a year. And it is manifeſt, that *half-yearly* payments, which begin immediately, are no

eaſieſt round ſums, about a guinea and $\frac{1}{2}$ for every year as far as 17 years; or, in the annual payments, (ſuppoſed 5 guineas) $\frac{1}{2}$ a guinea *per annum* for five years exceſs, and $\frac{1}{2}$ a guinea more for every four years exceſs beyond five years, till the exceſs comes to be 17 years. And, I believe, that 60 guineas in *ſingle payments*, and ſix guineas in *annual payments* beginning immediately, may very well be ſtated as the *loweſt common* payments proper to be required, ſuppoſing all married men under 52, taken into a ſociety, without enquiring into the difference of age between them and their wives, the annuities being all along ſuppoſed to be *life* annuities, and intereſt reckoned at 4 *per cent.*

more

more than half a year's purchaſe better than thoſe which begin at the end of half a year. But *yearly* payments, which begin immediately, are a *whole year*'s purchaſe better than the ſame payments to begin at the end of a year. The difference of value, therefore, between *yearly* and *half yearly* payments, ſuppoſing both to begin immediately, is three tenths of a year's purchaſe in favour of the former.——The whole of this ſubject may be ſeen accurately ſtated in the eſſay juſt referred to.

QUESTION IX.

"The value is required of an annuity to "be enjoyed for what may happen to re- "main of one life after another, provided the "life in expectation continues a given time?"

ANSWER.

Find by Queſtion VI. the preſent value of the annuity for the remainder of the life in expectation, after the given time, and multiply this value by the probability, that the other life ſhall fail within that time. Find alſo by Queſtion VII. the value of the reverſion, provided *both* lives continue the given time. Add theſe values to one another, and the *ſum* will be the anſwer in a ſingle preſent payment.

EXAMPLE.

EXAMPLE.

An annuity of 10*l.* for the life of a perſon now 30, is to commence at the end of 11 years (*a*), if another perſon now 40, ſhould be then dead; or, if this ſhould not happen, at the end of any year beyond 11 years in which the former ſhall happen to ſurvive the latter: What is the preſent value of ſuch an annuity, reckoning intereſt at 4 *per cent.* and taking the probabilities of life as they are in Dr. *Halley*'s Table, or Table V. at the beginning of Vol. II.?

The value of 10*l.* *per annum*, for the remainder of the life of a perſon now 30, after 11 years, found by Queſt. VI. is *l.*69.43.—The probability that a perſon 40 years of age ſhall live 11 years, is, by Dr. *Halley*'s Table, $\frac{335}{445}$. The probability, therefore, that he will die in 11 years, is $\frac{335}{445}$ ſubtracted from unity (*b*), or $\frac{110}{445}$; which multiplied by *l.*69.43, gives *l.*17.16.—The value of the reverſion, provided *both* live 11 years, found by Queſt. VII. is 17*l.* And this value added to the

(*a*) That is, the title to the annuity is to commence at the end of 11 years, and the firſt payment to be made a year afterwards, in caſe the life in expectation ſhould continue ſo long, and the other fail. But if *both* lives ſhould continue the given term, the firſt payment is always to be made at the end of the year, in which the former life ſhall happen to ſurvive the latter. See Queſt. VI.

(*b*) See the Note, p. 23.

former,

former, makes *l.* 34.16. the value required in a *ſingle preſent payment*; which payment divided by *l.* 11.43, (the value by Table II. at the end of Vol. II. of two joint lives, aged 30 and 40, with unity added) gives 3*l.* (*a*); or the value required in annual payments during the joint lives, the firſt payment to be made immediately.—If, every thing elſe being the ſame, the aſſigned term is 15 years, the value required will be 29*l.* in a *ſingle payment*, and *l.* 2.55. in *annual payments*.

QUESTION X.

"What money in hand, and alſo in annual payments during life, ought a perſon of an aſſigned age to give for a ſum of money, payable at his death to his heirs (*b*)?—In other words, what money in hand, and in annual payments during life, ought a perſon of a given age to pay for an *aſſurance* of any given ſum on his life?"

ANSWER.

Subtract the value of the life from the *perpetuity*. Multiply the remainder by the

(*a*) See the demonſtration of this rule in Note (D) Vol. II.

(*b*) This Queſtion is the ſame with Problem 16th, in Mr. *De Moivre*'s Treatiſe on Annuities, and Problem 26th, in Mr. *Simpſon*'s Select Exerciſes; but the anſwers there given are right only when applied to reverſionary *eſtates*, and therefore muſt be materially wrong when applied to reverſionary *ſums*, as will appear from the *Scholium* to this Queſtion, and from Note (E) Vol. II.

product

product of the given ſum into the intereſt of 100 *l.* for a year: and this laſt product, divided by 100 *l.* increaſed by its intereſt for a year, will give the anſwer in a *ſingle preſent* payment. And this payment, divided by the value of the life, will give the anſwer in *annual* payments, during the continuance of the life.

Example. Let the life be 30. The ſum 100 *l.* The rate of intereſt 4 *per cent.* And the valuation of lives, that in Table I. at the end of Vol. II. The perpetuity, therefore (*a*), is 25. The intereſt of 100 *l.* for a year is 4 *l.* 100 *l.* increaſed by its intereſt for a year, is 104 *l.* And the value of the life 14.68—The value of the life, ſubtracted from the perpetuity, gives 10.32, which, multiplied by the product of 100 *l.* into 4, or by 400, gives 4128. And this, divided by 104, gives *l.* 39.7, the value of 100 *l.* payable at the death of a perſon aged 30, in a ſingle preſent payment.—And this payment, divided by 14.68, is *l.* 2.7, the ſame value in annual payments during the continuance of the life.

Theſe values found in the ſame way agreeably to the valuation of lives for *London*, in Table X. at the beginning of Vol. II. are *l.* 45.76, and *l.* 3.49.—If the life is 36, and intereſt 4 *per cent.* theſe values are 43 *l.* and *l.* 3.1, by Table I. at the end

(*a*) That is; the value of the *fee-ſimple* of an eſtate found by dividing 100 *l.* by the rate of intereſt.

of

of Vol. II. and *l.* 4.1, by Mr. *Simpſon*'s valuation of lives for *London* in Table X.—If intereſt is reckoned at 3 *per cent.* the ſame values are, by *De Moivre*'s valuation of lives, for 30 years of age, *l.*48.14, and 2.86—For 36 years of age, *l.*51.43, and *l.*3.28.

It appears here, that difference of intereſt makes no conſiderable difference in the anſwers to Queſtions of this kind, except when the values are required in a ſingle payment.

If the firſt of the annual payments is to be made immediately, the ſingle payment is to be divided by the value of the life, with unity added to it, agreeably to what has been already obſerved; and the annual payments in this caſe (intereſt ſuppoſed at 4 *per cent.*) will be by Mr. *De Moivre*'s valuation of lives (or Table I. at the end of the next Vol.) for a life at 30, *l.*2.53—At 36, *l.*2.9.

If the payments are half-yearly payments beginning immediately, the ſingle payment muſt be divided by the value of the life increaſed by ſeven tenths, (ſee Queſt. VIII.) And the half-yearly payments, for the age of 36, will be half 2.96, or 1.48. And half 1.48, or 74, is likewiſe nearly the proper quarterly payments.

Again; if an annual payment, beginning immediately, of *l.*2.9, ought (reckoning intereſt at 4 *per cent.*) to purchaſe 100*l.* payable at the failure of a life now 36; 5*l.* by the rule of proportion, ought to purchaſe 172*l.* And in like manner, it may be found,

that the ſame annual contribution, in half-yearly or quarterly payments, beginning immediately, ought to purchaſe 170 *l.*—Theſe ſums, according to the *London* Obſervations, are 132 *l.* and 130 *l.* nearly.

The reaſon of mentioning theſe particulars will be ſeen in the next chapter.

SCHOLIUM.

If the reverſion is not a *ſum*, but an annuity for ever, or an *eſtate* in *fee-ſimple*, to be entered upon after a given life, its preſent value, *in a ſingle payment*, will be "the value "of the life ſubtracted from the perpetuity, "and the remainder multiplied by the annuity, or the annual rent of the eſtate."—And the value, in *annual payments*, will be, as before, the ſingle payment divided by the value of the life.—Univerſally. It ought to be remembered, that a reverſionary *eſtate*, after any given life or lives, is worth as much more than a correſponding reverſionary *ſum*, as 100 *l.* increaſed by its intereſt for a year, is greater than 100 *l.*—Thus, the preſent values, in ſingle and annual payments, of 4 *l.* *per annum* for ever, and of 100 *l.* in money after any aſſigned life, are to one another, (intereſt being at 4 *per cent.*) as 104 to 100, or 1.04 to 1.—The reaſon of this difference is, that the calculations ſuppoſe, that the reverſionary *ſum*, and the firſt yearly rent of the *eſtate*, or firſt payment of the annuity, are

are to be received at the ſame time, after the extinction of the lives in poſſeſſion. It is eaſy to ſee, that this is a circumſtance which muſt make the latter of moſt value. But to prevent any doubts about it, I ſhall explain it more particularly in a note in the Appendix (*a*)

QUESTION XI.

" A perſon of a given age, having a yearly income which will fail with his life, " wants to make proviſion for another perſon of a given age, in caſe the latter ſhould happen to ſurvive. What ought the former to give in a ſingle payment, and alſo " in annual payments during their joint lives " for a given ſum, payable at his death to " the latter?"

It is manifeſt, that the value of the given ſum in this caſe, muſt be leſs than in the caſe ſtated in the laſt Queſtion; becauſe, here the payment of it is ſuſpended on the contingency, that one life ſhall ſurvive another; whereas, in the other caſe, it is *certainly* to be paid at the failure of a given life.

ANSWER.

Find, by the ſolution of problem 32d, p. 297, Mr. *Simpſon*'s Select Exerciſes, the

(*a*) Vid. Appendix, note (E).

value of an eſtate, correſponding to the given ſum, and depending on the given ſurvivorſhip. Divide this value by 1*l.* increaſed by its intereſt for a year, and the quotient will be the value of the given ſum in a ſingle preſent payment. And the ſingle payment, divided by the value of the given joint lives, will be the anſwer in annual payments during the joint lives.

The ſolution I have referred to is as follows:

" Find the value of an annuity on two " equal joint lives, whereof the common age " is equal to the age of the older of the two " propoſed lives; which value ſubtract from " the perpetuity, and take half the remain- " der. Then ſay, as the *expectation* of the " duration of the younger of the two lives is " to that of the elder, ſo is the ſaid half re- " mainder to a 4th proportional, which will " be the number of years purchaſe to be gi- " ven for the eſtate when the life in expec- " tation is the oldeſt of the two. But if this " life is the youngeſt, then add the number " of years purchaſe juſt found to the value " of the joint lives, and let the ſum be ſub- " tracted from the perpetuity, and you will " alſo have the anſwer in this caſe." (*a*)

Let

(*a*) Mr. *Simpſon* has given the following examples of this ſolution, adapted to *London* lives:—— Example I. " Suppoſe the age of the expectant to be 40, of the *poſ-* " *ſeſſor* 30. The rate of intereſt 4 *per cent.* and the

" given

Let the life in expectation be 30; and the other life 40: The ſum, 100*l.* Intereſt, 4 *per cent.* The valuation of lives, Mr. *De Moivre*'s, or that in Table I. at the end of the next Vol.

The *expectation* of the firſt life, is 28; of the ſecond life 23, by Mr. *De Moivre's hy-*

" given legacy 5000*l.* or 200*l. per annum.* Then the " value of two equal joint lives of 40, being 8.1, (ſee " Table XI.) and the perpetuity 25, the remainder or " difference will be here 16.9; whereof the half is 8.45. " Therefore, it will be as 23.6 to 19.6, ſo 8.45 to 7.02 " years purchaſe, or *l.*1404, the required value."

Example II. " Let the age of the *expectant* be 30, of " the *poſſeſſor* 40, and the reſt as in the preceding exam- " ple. Here the value of the joint lives 30 and 40, will " be 8.8; which added to 7.02, (found above) the ſum " will be 15.82; whence the anſwer, in this caſe, is " 9.18 years purchaſe, or 1836."

I have ſhewn, that the values of reverſionary *eſtates*, and reverſionary *ſums*, are not the ſame as is here ſuppoſed.—The rule gives the true value when applied to the former; but, when applied to the latter, the values given by it muſt be divided by 1*l.* increaſed by its intereſt for a year, as above directed.—The ſame obſervation is to be applied to Mr. *Simpſon*'s next Problem, or the 33d.

In theſe Examples 23.6 and 19.6, are the expectations, in Table IX, of 30 and 40, according to the *London* Tables of Obſervation; and the method of finding them for any age, and from any Tables of Obſervation, is explained at the beginning of the firſt Eſſay.

In Mr. *De Moivre's hypotheſis*, the expectation of a life is always *half* the complement. See note, p. 2.—Sometimes the *complement* of a life is mentioned without any view to Mr. *De Moivre*'s hypotheſis, and it then means double the *expectation* of the life, whatever that may be, according to any Table of Obſervations.

potheſis. The value of the joint lives is 10.43. by Table II. at the end of the next volume. The value of two joint lives, both 40, is 9.82, by the ſame Table. The eſtate correſponding to 100 *l.* is 4 *l. per Ann.* and the preſent value of ſuch an eſtate to be entered upon by a perſon 30 years of age, provided he ſurvives a perſon 40 years of age, is, by the rule juſt quoted, *l.*33.32. And this value, divided by 1 *l.* increaſed by its intereſt for a year, or by 1.04, is *l.*32.03. the value in a *ſingle preſent payment* of the ſum of 100 *l.* dependent on the given ſurvivorſhip. And this ſingle payment divided by 10.43, is *l.*3.07, the required value in *annual payments*, during the joint lives, if the firſt payment is not to be made till the end of a year. But if the firſt payment is to be made immediately, the required value in *annual payments* will be *l.*32.03, divided by 11.43, or *l.*2.8.—Theſe values, according to the *London* Obſervations, or Mr. *Simpſon*'s Tables founded upon them, are *l.*35.30, in a *ſingle payment*, and *l.*3.6, in *annual payments*, beginning immediately.

Mr. *Simpſon*, in the Problems following that here quoted, has given ſolutions of moſt other Queſtions, concerning the values of reverſions depending on ſurvivorſhips, where the whole duration of two or three lives is concerned. And I am acquainted with no other ſolutions of theſe Queſtions, which are

appicable to all Tables of Obſervations, and which at the ſame time (proper regard being paid to the correction explained in the laſt Queſtion) may be conſidered as ſufficiently correct. (*a*)

QUESTION XII.

"Suppoſe an inſtitution for the relief of "widows to extend its aſſiſtance likewiſe "to the families of married men, provided "they leave no widows. Suppoſe, for in- "ſtance, that in this caſe children are to be "entitled to 100*l.* What is ſuch an expec- "tation worth, in preſent payment, intereſt "being at 4 *per cent.* ?"

ANSWER.

If 40 is the mean age at which members are admitted on ſuch an inſtitution, and 32 the mean age of their wives, the anſwer (ſuppoſing no ſubſequent marriages) is, by the 33d Problem in Mr. *Simpſon*'s Select Exerciſes, p. 298, and the correction already explained, *l.* 13.80, (*b*) taking the expectations and values agreeably to Mr. *De Moivre*'s hypotheſis.

But

(*a*) See the third Eſſay.

(*b*) This Problem and its ſolution are given by Mr. *Simpſon* in the following words: "A. and his heirs are "entitled to an eſtate of a given value, upon the deceaſe

But there is a reduction neceſſary, on account of the chance there is, that a widower may marry again. Suppoſe, therefore, only half of all widowers to marry a ſecond and third time, and that two fifths of ſuch widowers ſurvive theſe ſubſequent marriages. In this caſe, $\frac{1}{2}$ added to $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{7}{10}$ of all who become widowers, will die without leaving widows, and therefore $\frac{7}{10}$ of *l.*13.8, or *l.*9.66, will be the anſwer. If only *one fourth* of

" of B, provided B ſurvives A; to find the value of " their expectation in *preſent* money."—Solution. " Find " the value of an annuity on the longeſt of two equal " lives, whereof the common age is that of the older of " the lives A and B; which value ſubtract from the " perpetuity, and take half the remainder; then it will " be, as the expectation of duration of the younger of " the lives A and B, is to that of the older, ſo is the " ſaid half remainder to the number of years purchaſe " required, when the life of B *is the older of the two.* But " if B *be the younger*; then to the number thus found, " add the value of an annuity on the longeſt of the lives " A and B, and ſubtract the ſum from the perpetuity, " for the anſwer in this caſe."

If the eſtate is 4*l. per annum*, the age of B 40, and of A 32, *intereſt* 4 *per cent.* the anſwer by this rule comes out *l.*14.35, which divided (as in the preceding Queſtion) by 104, gives *l.* 13.80, the value, as above, of 100*l.* in money. If B is 30 and A 40, the ſame value is 20*l.*

N. B. The value of the longeſt of two lives is always the *difference* between the value of the *joint* lives, and the *ſum* of the values of the two given *ſingle* lives. Thus; the value of a life at 40, is, by Table I, at the end of next vol. 13.2. The *ſum* of the values of two ſuch lives is 26.4. The value of two joint lives, whoſe common age is 40, is, by Table II, 9.82; and the difference is 16.58, or the value of the *longeſt* of two lives at 40.

all

all who become widowers marry again, and two fifths of theſe ſurvive, the anſwer will be *l.* 11.73.

This calculation ſuppoſes all marriages to leave children who ſurvive their parents. If this is conſidered as uncertain, the values now determined muſt be diminiſhed in the proportion of this uncertainty.—Thus; if one marriage in ſeven fails of leaving children (*a*) that ſurvive their parents; theſe values will be reduced a *ſeventh* part, or to *l.* 8. 28, if *half*, and *l.* 10.05, if a *quarter* of all widowers marry.

In this way may any other queſtions of the ſame kind be anſwered on any ſuppoſitions that may be thought moſt reaſonable.

QUESTION XIII.

"Let an eſtabliſhment be ſuppoſed which "takes in at once all the marriages in a "country, or all marriages among perſons "of a particular profeſſion within a given "diſtrict, and ſubjects them for perpetuity "to a certain equal and common tax, or an- "nual payment, in order to provide life an- "nuities for ſuch widows as ſhall reſult from "theſe marriages. What ought the tax to "be, ſuppoſing the annuity 20*l.* and calcu- "lating at 4 *per cent.* from Mr. *De Moivre's* "valuation of lives?"

(*a*) This for many years has been nearly the fact among the miniſters and profeſſors in *Scotland*.

ANSWER.

ANSWER.

Since, at the commencement of ſuch an eſtabliſhment, all the oldeſt, as well as the youngeſt marriages, are to be entitled equally to the propoſed benefit, a much greater number of annuitants will come immediately upon it, than would come upon any ſimilar eſtabliſhment, which limited itſelf in the admiſſion of members to perſons not exceeding a given age. This will check that accumulation of money, which ſhould take place at firſt, in order to produce an income equal to the diſburſements at the time when the number of annuitants comes to a *maximum*; and, therefore, will be a particular burden upon the eſtabliſhment in its infancy. For this, ſome compenſation muſt be provided; and the equitable method of providing it, is, by levying *fines* at the beginning of the eſtabliſhment, on every member *exceeding* a given age, proportioned to the number of years which he has lived beyond that age. But in the preſent queſtion, it is ſuppoſed, that ſuch fines cannot be conveniently levied, or that every payment muſt be equal and common, whatever diſparity there may be in the value of the expectations of different members. The fines, therefore, muſt be reduced to one common one, anſwering as nearly as poſſible to the diſadvantage I have mentioned, and

payable

payable by every member at the time when the eſtabliſhment begins. After this, the eſtabliſhment will be the ſame with one that takes upon it all at the time they marry; and the tax or annual payment of every member adequate to its ſupport, will be the annual payment during marriage, due from perſons who marry at the mean age at which, upon an average, all marriages may be conſidered as commencing.—There are then two points to be here determined. The *fines* neceſſary to be paid at firſt, according to the account I have juſt given; and the *conſtant annual payment*, neceſſary to be made by every member, as an equivalent for the expectation provided by the eſtabliſhment.—The *fines* to be paid at firſt are, for every particular member, the ſame with the difference between the value of the expectation to him at his preſent age, and what would have been its value to him had the ſcheme begun at the time he married? Or, they are, for the whole body of members, the difference between the value of the common expectation, to perſons at the mean age of all married perſons taken together as they exiſt in the world, and to perſons at that age, which is to be deemed their mean age when they marry.

Thus; let 33 for the man, and 25 for the woman, be the mean ages of all that marry annually. Let alſo 48 be the mean age of all the married men in the world, and 40 of

married

married women. (*a*)—Now, he that will calculate for theſe ages, in the manner directed in Queſt. IV. will find, that the value in *annual payments* during marriage, and beginning immediately, of the expectation of an annuity of 20*l.* *per annum* by a perſon 25 years of age, after a life whoſe age is 33, is *l.*6.64.—And that *l.* 8.04, is the value of the ſame expectation, the ages being 48 and 40.

The former, therefore, is the payment for perpetuity from every member of the eſtabliſhment; and the value of the *difference* between it and the latter, or of *l.*1.4. *per ann.* payable during two joint lives, whoſe ages are 40 and 48, that is, *l.*14.2, is the fine neceſſary to be levied on every married member at the beginning of the eſtabliſhment. (*b*)

It would be eaſy to extend the benefit of ſuch an eſtabliſhment, ſo far as to provide 100*l.* for the children of members, provided

(*a*) I muſt beg leave to refer to note (F) in the Appendix, for an explanation of what I mean by the mean ages of married men and women, and alſo for a confirmation of the anſwer I have given to this Queſtion.

(*b*) An annuity for ever, the firſt payment of which is to be made immediately, is worth 26 years purchaſe, intereſt being at 4 *per cent.* *l.*14.2 therefore is equivalent in value to 0.55*l.* or 11*s.* *per annum*, for ever. Add this to *l.*6.64, and it will appear, that *l.*7.19 *per annum*, beginning immediately, is the anſwer to this Queſtion, ſuppoſing the value of the *fine* to be provided for in the perpetual annual payments.

they

they leave no widows; and the neceſſary addition on this account to the perpetual annual payments, can ſcarcely, in the circumſtances this queſtion ſuppoſes, be much more than about 15*s.* payable during life, and excluding from all benefit ſuch as happen to be widowers at the commencement of the eſtabliſhment, and do not afterwards marry.

If, in ſuch an eſtabliſhment, all perſons of a particular denomination, whether married men, widowers, or batchelors, are ſubjected alike to the taxes and fines; they ought to be as much *leſs*, as the whole number of perſons ſubjected to them, is *greater* than the number of marriages conſtantly exiſting.

In carrying theſe ſchemes into execution, there cannot be a more eaſy, or equitable way of raiſing the neceſſary fines, than by providing, that none ſhall be entitled to any expectation for a few of the firſt years. Thus; an eſtabliſhment, entitling widows to 20*l.* *per annum* for life, and conſiſting of 667 married members, and 344 unmarried, always kept up at an average, ought to begin with a capital of *l.* 14.2 multiplied by 667, or 9471*l.* beſides one payment in hand of the conſtant annual payments. That is, (the proper annual payment of every member being in this caſe $\frac{667}{1011}$, multiplied by *l.* 6.64. or *l.* 4.38) it ought to begin with a capital

of

of 13.899 *l.* over and above the payment of *l.* 4.38, at the *end* of every year for ever afterwards. (*a*)—The excluſion of all the firſt members from any benefit, unleſs they ſurvive the firſt *two* years, or live to make *three* payments, would raiſe this capital nearly. And ſuch an excluſion for *three* or *four* years, would be an advantage ſo conſiderable, that it would probably give ſecurity and ſtability to the ſcheme for all ſubſequent time.

In theſe obſervations, I have had in view ſome ſchemes which have been eſtabliſhed in this kingdom; but more particularly, one eſtabliſhed by act of parliament among the clergy in *Scotland*; of which I ſhall have occaſion in the next chapter to take further notice.

I have choſen to calculate here only from Dr. *Halley*'s Table, or Mr. *De Moivre's hypotheſis* grounded upon it, becauſe the *London* Table is, by no means, adapted to the caſes in view.

It ſhould be further remembered, that when the mean ages, at which marriages commence, are ſuppoſed to be 33 and 25,

(*a*) Or, ſuppoſing the value of 9471*l.* (the fine) provided for in the annual payments, it ought to receive every year, at the *beginning* of the year, a contribution from each member of *l.* 4.74.

all

all ſecond and third marriages are included; and that it is to be expected, that almoſt all theſe marriages will begin after theſe ages; and likewiſe, that a conſiderable proportion of the firſt marriages will begin a much longer time *after* theſe mean ages, than any of the other firſt marriages will begin before them.—Probably, therefore, theſe mean ages ſhould not be taken younger. One or two years, however, more or leſs, in every ſuppoſition I have made, will make no difference of any conſequence.

QUESTION XIV.

"A perſon of a given age has an eſtate depending on the continuance of his life for a given term. What ought he to give for having it *aſſured* to him for that term?"

ANSWER.

From the value of an annuity certain for the given term, found by Table II. ſubtract the value of the life for the given term, found by Queſt. VI. and *reſerve* the remainder.—Multiply the value of 1 *l.* due at the end of the given term, (found by Table I.) by the *perpetuity*, and alſo by the *probability*, that the given life ſhall fail in the given term. The *product* added to the *reſerved* remainder, and the *ſum* multiplied by the given annuity, will

will be the required value of the aſſurance in one preſent payment. (*a*)

EXAMPLE.

An eſtate or annuity of 10*l.* *for ever*, will be loſt to the heirs of a perſon now 34, ſhould his life fail in 11 years. What ought he to give for the *aſſurance* of it for this term?—That is; What is the preſent value of ſuch an annuity to be entered upon at the failure of ſuch a life, ſhould that happen in 11 years.

The value of the life of a perſon whoſe age is 34 for 11 years, is, by Queſt. VI. (reckoning intereſt at 4 *per cent.* and taking Mr. *De Moivre*'s valuation of lives) 7.76; which, ſubtracted from 8.760, (the value of an annuity certain for 11 years) leaves 1*l.* the remainder to be reſerved.

The value of 1*l.* to be received at the end of 11 years, is, 0.6496, by Table I. Vol. II. The probability that the life of a perſon, aged 34, ſhall fail in 11 years, is, by Dr. *Halley*'s Table, $\frac{103}{499}$; and the pepetuity is 25. Theſe numbers multiplied by one another, and 1 added to the product, make 4.34, which, multiplied by 10, (the given annuity) gives *l.*43.4, the required value in a ſingle preſent payment.

(*a*) See the demonſtration in note (G) Appendix.

*l.*43.4.

l. 43.4, divided by 1.04, gives *l.* 41.7, the true value (by Scholium to Queſt. X.) of the aſſurance of an *equivalent ſum*, or of 250 *l.* for 11 years on the given life.

Again. 41.7, divided by 8.76, (the value of the given life for the given time with unity added to it) gives 4.76, the ſame value in annual payments beginning immediately, for 11 years, (*a*) ſubject to failure ſhould the life fail.

SCHOLIUM.

In a ſimilar way may the price of aſſurances on any two joint lives, or the *longeſt* of two lives for any given terms, be calculated; the rule being as follows:

" From the value of an annuity certain " for the *given term*, ſubtract the value of " the joint lives, or the longeſt of the two " lives for the *given term*, found by Scho- " lium to Queſt. VI. and *reſerve* the remain- " der.—Multiply the value of 1 *l.* to be re- " ceived at the end of the given term by the " perpetuity, and alſo by the probability " that the *joint lives*, or the *longeſt of the two* " *lives*, ſhall fail within the given term. This " product added to the reſerved *remainder*, " and the *ſum* multiplied by the annuity to be " aſſured, will be the value of the aſſurance " in a ſingle preſent payment."

(*a*) The laſt payment to be made at the end of the 11th year; or 12 payments in all.

EXAMPLE.

" What is the value of 10 *l. per annum*, to " be entered upon, ſhould *either* of two " perſons, one 40 and the other 30 years of " age, die in ten years, reckoning intereſt " at 4 *per cent.* and calculating from Dr. " *Halley*'s Table."

The value of two joint lives at theſe ages, for 10 years, (found by *Scholium* to Queſt. VI.) is 6.51; which, ſubtracted from 8.111, (the value of an annuity certain for 10 years, at 4 *per cent.*) leaves 1.60, the remainder to be *reſerved.*

The value of 1*l.* to be received at the end of 10 years, is, .6755, by Table I. in the firſt collection of tables, vol. ii.

The probability, that the lives of one or other of two perſons, aged 30 and 40, ſhall fail in 10 years, is, $\frac{185}{371}$ by Table V. (*a*) And the perpetuity 25. Theſe numbers, multiplied by one another, and 1.60 added to the product, make 7.48, which, multiplied by 10, (the given annuity) gives *l.* 74.8, the anſwer in a ſingle preſent payment.

£. 7.48, divided by 1.04, gives *l.* 71.92, the value of the aſſurance of an *equivalent ſum*;

(*a*) The probability taken from the Table, that a perſon aged 30, ſhall live 10 years, is, $\frac{445}{371}$. That a perſon, aged 40, ſhall live 10 years, is, $\frac{346}{445}$. That they ſhall *both* live 10 years, is, $\frac{346}{445}$, multiplied by $\frac{445}{371}$, or $\frac{346}{371}$. That they ſhall *not both* live 10 years, or that *one* or *other* of them ſhall die in this time, is $\frac{346}{371}$, ſubtracted from unity, or $\frac{185}{371}$. See note p. 23.

or

or of 250*l.*—*l.*71.92, divided by 7.51, (the value of the two joint lives for 10 years with unity added) gives 9.57, the value of the ſame ſum in annual payments beginning immediately, for 10 years, ſubject to failure ſhould the joint lives fail.

EXAMPLE II.

"What is the value of 10*l. per ann.* to be "entered upon, ſhould two perſons, one 30, "and the other 40, *both* die; that is, ſhould "the *longeſt* of the two lives fail in 10 years, "reckoning intereſt at 4 *per cent.* and cal-"culating from Dr. *Halley*'s Table?"

The value of the *longeſt* of the two lives for 10 years, (that is, the value of the joint lives for 10 years, ſubtracted from the ſum of the (*a*) values of the ſingle lives for 10 years) is, 7.91; which, ſubtracted from 8.111, the value of an annuity certain for 10 years, leaves .20 the remainder to be reſerved.—The value of 1*l.* to be received at the end of 10 years, is, 6755. The probability that the lives of two perſons, aged 30 and 40, ſhall fail in 10 years, is, by Table V. $\frac{86}{331}$, multiplied by $\frac{99}{445}$ or $\frac{85}{236}\frac{14}{295}$; and the perpetuity 25. Theſe numbers, multiplied by one another, and .20 added to the product, make .740, which, multiplied by 10, (the

(*a*) See Scholium to Queſt. VI.

given annuity) gives 7.4, the anſwer in a ſingle payment.

7.4, divided by 1.04, gives 7.11, the value of the aſſurance of 250*l.*

Remark I.

The values of ſingle lives for given terms, when theſe terms are leſs than ten years, muſt, in anſwering theſe Queſtions, and alſo in anſwering the following Queſtions, be found true to at leaſt 2 or 3 places of decimals. When they cannot be found to this exactneſs by any Tables, they muſt be calculated in the following manner:

"Multiply the probability, taken out of "the Table of Obſervations, that the life "ſhall exiſt 1, 2, 3, &c. years, by the value "of 1*l.* due at the end of 1, 2, 3, &c. years; "and the ſum of the products will be the "value of the life for 1, 2, 3, &c. years."

For Example. The probability, that a perſon whoſe age is 34, ſhall live a year, is, by Dr. *Halley*'s Table, $\frac{490}{499}$. The probability at the ſame age, of living 2 years, is, $\frac{481}{499}$; 3 years, $\frac{472}{499}-\frac{490}{499}$ multiplied by .9615, (the value, by Table I. of 1*l.* due at the end of a year, intereſt being at 4 *per cent.*) is, .942; or the value of the life for *one* year—$\frac{481}{499}$, multiplied by .245, (the value of 1*l.* due at the end of 2 years) is, .891. And this added

to

to the former product, gives 1.833; or the value of the life for 2 years.—$\frac{472}{499}$ multiplied by .8890, (the value of 1 *l.* due at the end of 3 years) is, .841; and this product, added to 1.833, makes 2.674, or the value of the given life for 3 years.

When the term exceeds 10 years, the rule in Queſt. VI. will give theſe values with ſufficient exactneſs; and it would do the ſame in all caſes, were the values of *lives* given true to 3 or 4 places of decimals (*a*), and in ſtrict agreement to the Tables of Obſervation uſed.

The remark now made is to be extended to the values of *joint* lives for given terms. For theſe values, like thoſe of *ſingle* lives cannot be found in ſolving theſe Queſtions with ſufficient accuracy, (when the terms are ſmall, and the values of lives are given only to one or two places of decimals) by any method, except the tedious one, of multiplying the probability that the 2 lives ſhall *both* continue, 1, 2, 3, &c. years, by the value of 1 *l.* due at the end of 1, 2, 3, &c. years, and taking the ſum of the products in the manner juſt deſcribed.

(*a*) Such tables are given in the preſent edition of this treatiſe; and therefore this remark is now leſs neceſſary than it was. See the tables of the values of ſingle and joint lives in the next volume, deduced from the regiſter of mortality at Northampton.

REMARK II.

If the annuity is to be entered upon, in caſe of the failure within a given time of any life or lives, *at the end of that time*; and not *at the end of the year in which the failure may happen*; its preſent value will be the product ariſing from the continual multiplication by one another of the perpetuity increaſed by unity; the value of 1 *l.* due at the end of the given time; the annuity, and the probability that the life, or lives, ſhall fail within the given time. And care ſhould be taken not to confound theſe two ſorts of Queſtions with one another.—Thus, the value in one payment of 10*l. per ann.* to be entered upon eleven years hence, in caſe a perſon aged 34 ſhould not live ſo long, is 26, (the perpetuity increaſed by unity, intereſt being at 4 *per cent.*) multiplied by .6496, and by 10*l.* and alſo by $\frac{103}{99}$; or 34.8.—This value, divided by 1.04, is 33.5, the value of an equivalent ſum, or of 250*l.* to be obtained on the ſame conditions.

The value of the *aſſurance* of any annuity on the whole continuance of any ſingle life is, by Queſt. X. the *exceſs* of the perpetuity above the value of the life, multiplied by the annuity. And in like manner; the value of the *aſſurance* of any annuity on the whole continuance of any two *joint* lives, or the *longeſt* of two lives, is the exceſs of the *perpetuity*

petuity above the value of the joint lives, or of the longeſt of two lives, multiplied by the annuity. This is very obvious; but no general method has been yet explained of finding the values of *aſſurances* on lives and ſurvivorſhips for terms of years leſs than the whole continuance of the lives. For this reaſon, I have been here more explicit than I ſhould otherwiſe have been; and, as ſuch aſſurances are now much practiſed, and may be very uſeful if their values are rightly determined, I have thought proper to add the two following Queſtions, which, when joined to Queſtion XI. and Mr. *Simpſon's* 33d *Problem* given in the note, p. 39, will, I believe, exhauſt this ſubject as far as two lives can be concerned.

QUESTION XV.

"B, expectant, will loſe a given ſum, "ſhould he ſurvive A, *within a given time*. "What ought he to pay for the *aſſurance* of "it?"—In other words: "What ought he "to pay for a given ſum to be received at "the death of A, ſhould he happen to ſur-"vive him within a given time?"

ANSWER.

Divide the *ſum* of the decrements of life in the Table of Obſervations from the age of A, for the given time, by the given time; and, by the *quotient*, divide the number of

the living in the Table at the age of A; and again, by this *ſecond* quotient (*a*), divide the given ſum, reſerving the *third* quotient.

Find the value of an annuity on the life of B, for the given time. To this value add the *quotient*, that will ariſe from dividing the value of an annuity certain, for the given time, by twice the *complement* of the life of B; and the *ſum*, multiplied by the *reſerved quotient*, will be the required value in a ſingle preſent (*b*) payment.

EXAMPLE.

Let the Table of Obſervations be Mr. *Simpſon*'s for *London* (ſee the Tables in the next volume). Let the rate of intereſt be 3 *per cent.* A, ſeven years of age. B, 30. The given time 14 years. The given ſum 100*l.* —The ſum of the *decrements*, for 14 years from the age of ſeven, is 73, which, divided by 14, gives 5.2. The number of the living at ſeven is 430, which, divided by 5.2, and 100*l.* divided by the quotient, gives *l.*1.21, the *quotient* to be *reſerved.*

(*a*) When the age of A is under 60, and the term ſo large as to exceed the difference between it and 70, it will be beſt when the *London* Table is uſed, to divide the given ſum, not by the ſecond quotient here mentioned, but by the *complement*, or double the expectation of A.

(*b*) See the demonſtration of this rule, and alſo of the rule that will be given for ſolving the next Queſtion, in the Appendix, note (H).

The

The value of an annuity for 14 years on the life of B, is, by Queſt. VI. 9.5.—The value of an annuity certain for 14 years, is, (by Table II. at the beginning of the next volume) 11.296, which divided by 94.4, (twice the *complement* of the life of B, by Table IX. (*a*) gives .12, which, added to 9.5, gives 9.62.; and this again multiplied by 1.21, the *reſerved quotient*, gives 11.64, the *preſent* value in *one* payment of 100*l.* payable at the death of A aged 37, to B aged 30, ſhould A die and leave B the ſurvivor within 14 years.

The preſent value for 14 years of two joint lives, one 7 and the other 30 years of age, may be found, by the help of Table XI, and the rule in the *Scholium* to Queſt. VI. to be nearly 9 years purchaſe; and, *l.* 11.64 divided by this value with unity added, or by 10, gives 1.164, the foregoing value in *annual payments* during the joint lives for 14 years, the firſt payment to be made immediately, and the *laſt* payment at the end of 14 years, ſhould the joint lives not fail.

SCHOLIUM.

It deſerves particularly to be remembered, that in this method likewiſe may be calculated, what ſums ought to be paid on any ſurvivorſhip, within a given time, of one life

(*a*) This table gives the *expectation* only, but it ſhould be remembered, that twice the *expectation* is always the *complement* of a life. See note, p. 37.

beyond

beyond another, in conſideration of any given ſum now advanced.—The following Example of this is a caſe which has offered itſelf in practice.

" A perſon, aged 30, has in expectation " an eſtate which is to come to him, pro- " vided he ſurvives a *minor*, aged 7, before " he is out of his minority; that is, pro- " vided he ſhould be himſelf living at the " time of the minor's death, ſhould that hap- " pen before he is 21.—In theſe circum- " ſtances, he wants to borrow 1000*l.* on his " *expectation.* What *reverſion* out of the " eſtate depending on ſuch a ſurvivorſhip, is " a proper equivalent for this ſum now ad- " vanced, intereſt being reckoned at 3 *per* " *cent.* and the probabilities of life being " ſuppoſed the ſame with thoſe in Mr. *Simp-* " *ſon*'s Table of *London* Obſervations?"

ANSWER.

It appears from what has been juſt determined, that for *l.* 11.64 now advanced, the proper equivalent in ſuch circumſtances, is, 100*l.* to be paid, in caſe the ſurvivorſhip ſhould take place; or, by the *correction* in page 34, as much of the eſtate as 100*l.* will buy at 3 *per cent.* ſuppoſing the firſt rent to be received immediately; (that is, ſuppoſing the eſtate worth 34.33 years purchaſe) or *l.* 2.912 *per annum.*—By the rule of proportion, therefore, for 1000*l.* the proper equi-

equivalent will be 859*l.* in money, or 250*l.* *per annum* out of the eſtate.

QUESTION XVI.

"100*l.* will be loſt to B's heirs, ſhould he "happen to die after A, *within a given time.* "What is the price of the *aſſurance* of it?— "That is: What is the preſent value of "100*l.* payable at the death of B, provided "his death ſhould happen *after* A's death, "*within a given time?*"

ANSWER.

Divide the ſum of the decrements of life in the Table of Obſervations from the age of B, for the given time, by the given time; and by the *quotient* divide the number of the living at the age of B; and again, by this *ſecond quotient* (*a*), divide the given ſum, reſerving the *third quotient.*

Find the value of an annuity on the life A for a number of years, leſs by *one* year than the given time, which ſubtract from the value of an annuity certain for the ſame number of years. Multiply the *remainder* by the *reſerved quotient*, and divide the *product* by the amount of 1*l.* for one year, and let this be a *ſecond* reſerved quotient.

(*a*) Or rather, if the *London* Table is uſed, by the *complement* of the life of B, when his age is under 60, and the term exceeds the *difference* between it and 70.

Again,

Again. Multiply into one another the *firſt* reſerved quotient, and the value of an annuity certain for the given time; and divide the product by twice the *complement* of A's life. This *laſt* quotient, added to the *ſecond* reſerved quotient, will be the *anſwer* in a preſent ſingle payment.

EXAMPLE.

Let the age of B be 40. Of A 30. The ſum 100*l.* Rate of intereſt 4 *per cent.* The given time 20 years. The Table of Obſervations, Mr. *Simpſon*'s, or Table VIII. in the collection of tables in the next volume.—The ſum of the decrements of life, in this Table from the age of 40 for 20 years, is 127, which, divided by 20, (the given time) gives 6.38.—The number of the living at 40 is 229, which, divided by 6.38, gives 35.8; and 100*l.* (the given ſum) divided by 35.8, gives 2.79, the *firſt* quotient to be reſerved.

The value of an annuity for 19 years on a life at 30 years of age, is 10.3; which, ſubtracted from 13.134, (the value of an annuity *certain* for 19 years, by Table II.) and the remainder multiplied by 2.79, gives 7.89. This product divided by 1.04, (the amount of 1*l.* in one year) gives 7.60; the *ſecond* reſerved quotient.

2.79 multiplied by 13.59, (the value of an annuity certain for 20 years) gives 37.916; and this *product* divided by 94.4, (twice the com-

complement of A's life by Table IX.) gives .401, which, added to 7.60, gives 8*l.* the *Anſwer*; or, the value of 100*l.* payable at the death of B, on the contingency of his ſurviving A aged 30, and *both* dying in 20 years.

It is plain, that this is likewiſe the ſum that ought to be lent to B now, on the expectation of 100*l.* at his death, provided it ſhould happen after A's death in 20 years.

This rule gives the juſt ſolution in all caſes, except when B, the expectant, is the *youngeſt* of the two lives, and at the ſame time the term of years *greater* than the complement of A's life. In this particular caſe the following rule muſt be uſed.

Find, by the preceding rule, the value of the aſſurance of the given ſum for a term of years, equal to the complement of A's life, and let this value be *reſerved*. Multiply by one another the given *ſum*; the *value* of 1*l.* to be received at the end of a number of years equal to the complement of A's life; and the value of an *annuity certain* for as many years as the given term exceeds this complement. And the *product*, divided by the complement of B's life, and the *quotient* added to the *value reſerved*, will be the true value ſought.

EXAMPLE.

Let the age of B be 30; of A 40. The term 47 years; and every thing elſe as in the

the laſt Example. The complement of A's life, is, by Table IX. 39.2. The value of 100*l.* to be received at the death of B, if he ſurvives A within 39 years, may be found by the preceding rule to be *l.*16.15; the value to be reſerved—The value of 1*l.* to be received at the end of 39 years is, by Table II. .2166. The value of an annuity *certain* for 8 years, (the exceſs of the given term above the complement of the life of B by Table IX.) is, 6.733.

And theſe two values multiplied by one another, and by 100*l.* give 145.83; which, divided by 47.2, (the complement of the life of B) and 16.15, added to the quotient, make *l.*19.23, the value ſought.

REMARK.

As after finding the preſent value of an eſtate, or annuity, it is neceſſary to *divide* that value by the amount of 1*l.* in one year; in order to find the preſent value of a *ſum* equivalent to the *annuity*; ſo, after finding the value of a ſum, it is neceſſary to *multiply* that value by the ſaid amount, in order to find from it the value of an equivalent annuity.

In the firſt example, therefore, the value of an eſtate of 4*l. per annum*, would be *l.*8.32. In the ſecond example 20*l.* And this is, as it ought to be, the value for the whole duration of the lives, agreeably to the Problem in the note p. 37.

In

In ſolving this Queſtion, care alſo muſt be taken not to forget the *firſt* Remark under the foregoing Queſtion.

In this chapter, rules have been given for finding the values of all aſſurances on ſingle lives, and any two lives, or any ſurvivorſhips between two lives, whether for *terms*, or their *whole duration*. In the ſame way rules may be inveſtigated for finding the values of all aſſurances on any three lives, or any ſurvivorſhips between them. But this is a work of more difficulty, and which requires great attention and ſkill. I can, however, with particular ſatisfaction acquaint the Reader, that it has been lately executed, in the compleateſt manner, by Mr. Morgan, in his Treatiſe on the *Doctrine of Annuities and Aſſurances on Lives and Survivorſhips*.

CHAP.

CHAP. II.

Containing an Application of the Queſtions in the foregoing Chapter to the Schemes of the Societies in Great Britain, *for making Aſſurances on Lives and Survivorſhips, and for granting Annuities to Widows, and to Perſons in Old Age.*

SECT. I.

Of the London Annuity, *and the* Laudable *Societies for the Benefit of Widows* (*a*).

THE ſcheme mentioned in Queſt. VIII. was nearly that with which the *London Annuity* Society ſet out in 1765. The *Laudable Society* was eſtabliſhed in 1761, and formed on a ſimilar plan. In both, the *annual contribution* of every member was five guineas, payable half-yearly; and for this a title was given to an annuity of 20*l.* to every widow during widowhood, if the huſband after admiſſion, lived *one* year according to the *firſt* ſcheme; or *three* years according to

(*a*) It muſt be remembered, that this ſection has in view the ſtate of theſe ſocieties in 1773, or at the time of the publication of the former edition of this tract.

the

the (*a*) *ſecond*; of 30*l.* if the huſband lived *ſeven* years, according to both ſchemes; and 40*l.* according to the *firſt* ſcheme, if he lived 15 years, or 13 years, according to the ſecond. —In both ſchemes alſo, there was no other premium or fine required, than five guineas extraordinary, at admiſſion, from every member whoſe age does not exceed 45. The *Laudable Society* admitted none above 45, and the *London Annuity* Society obliged every perſon between 45 and 55 to pay, at admiſſion, five guineas extraordinary, for every year that he was turned of 45.

Theſe were the main particulars in the ſchemes on which theſe Societies were formed; and, therefore, both of them, were the annuities to be enjoyed for life, received (ſuppoſing the members all under 46 at admiſſion, and of the ſame ages with their wives, and money at 4 *per cent.*) but little more than three-fifths of the true value of the annuities; or about one half, ſuppoſing wives, one with another, 10 years younger than their huſbands; as appears from Queſtion VIII.

It appears further in that Queſtion, that, ſuppoſing the annuities to be *life* annuities, and men and their wives of equal ages, the

(*a*) In this ſociety a member who lived but *one* year, was entitled to no more than an annuity of 10*l.* for his widow; if he lived two years, to 15*l.* if he lived three years, to 20*l.* four years, 25*l.* ſeven years, 30*l.* ten years, 35*l.* thirteen years, 40*l.*

expectation to which an annual payment of five guineas beginning immediately, entitles, is nearly 14*l.* if the contributor lives a year, and 20*l.* if he lives ſeven years (*a*), taking the medium between the *London* and the other Tables of Obſervations.

It is likely, that many perſons will be very unwilling to believe, that theſe ſchemes could have been ſo deficient as they have been now repreſented. I will, therefore, endeavour to prove this in a way which, though leſs ſtrict, is ſufficiently deciſive, and may be more likely to be intelligible to perſons unſkilled in mathematical calculation.—I ſhall here confine myſelf to the ſcheme of the *London Annuity* Society. The differences between it and the ſcheme of the *Laudable* Society are inconſiderable, and what ſhall be ſaid of the one will be fully applicable to the other.

According to this ſcheme, as it has been juſt deſcribed, all that live 15 years in the ſociety will be entitled to annuities of 40*l.* *per annum*, for their widows. Suppoſe the whole ſociety, at admiſſion, to be men of 40 years of age, taken one with another. A perſon of this age has an even chance of *living* 23 years; and he has an even chance of continuing with a wife of the ſame age, (that

(*a*) The ſame annual payment will, on the ſame ſuppoſitions, entitle to 14*l.* if a member lives a year, and 18*l.* if he lives *three* years.

is,

is, of continuing in the ſociety) 13 years and $\frac{1}{4}$ (*a*). Not much leſs, therefore, than half the members will continue in the ſociety 15 years; and, conſequently, not much leſs than half the widows that will come upon the ſociety will be annuitants of 40*l. per annum*. Theſe widows, however, being older than the reſt when they commence annuitants, will continue on the ſociety a ſhorter time; and, therefore, the number conſtantly in life together, to which they will in a courſe of years increaſe, will be proportionably ſmaller. Putting every thing as favourably as poſſible, let us ſuppoſe, that out of 20 annuitants conſtantly on the ſociety, *five* will be annuitants of 40*l. ſix* of 30*l.* and *nine* of 20*l.* To 20 annuitants then the ſociety will pay 560*l. per annum*, or the 20th part of this ſum, that is 28*l.* to *every* annuitant at an average. But ſuch an annuity for a life at 40, after another equal life, provided both ſurvive one year, is worth (by Queſt. VII. p. 24.) in a ſingle preſent payment, 85*l.* nearly, according to the *London*, and all the

(*a*) This is the exact truth according to Mr. *De Moivre*'s hypotheſis, and the *Norwich* Table. But according to Dr. *Halley*'s and the *Northampton* Table, a man 40 years of age has an even chance of living no more than 22 years, and of joint continuance with a wife of the ſame age, 13 years.—Forty muſt be more than the mean age of the members of the ſociety at admiſſion, and on this account the number of annuitants of 40*l.* muſt be proportionably greater. The mean age, therefore, has been taken very moderately.

Tables of Obſervations, intereſt being all along ſuppoſed at 4 *per cent.*

It cannot appear improbable to any one, that this ſhould be the true value of ſuch a reverſion. It is not probable, that there is any ſituation in which the decrements of life are ſuch as can make it a tenth part more or leſs.—85*l.* in preſent payment is the ſame with 3*l.* 8*s. per annum* for ever.—But is an annual payment of five guineas, which muſt ceaſe as ſoon as either of two lives each 40 fails, equal in value to ſuch a perpetuity? Every one muſt ſee, that there is a great difference.—A ſet of marriages between perſons all 40, will, according to the probabilities of life in Dr. *Halley*'s Table, laſt, one with another, 15 years (*a*); and an annual payment beginning immediately, during the joint continuance of the lives of two perſons of this age, is worth 10 years purchaſe (*b*).

(*a*) See the beginning of Eſſay I.

(*b*) The value of ſuch an annual payment, by Table XI. or the *London* Obſervations, is 9.1; and 10.8, by Mr. *De Moivre's* hypotheſis.—I have not taken into this account the five guineas *fine* paid at admiſſion, becauſe it is obviouſly of too little conſequence to make any conſiderable difference: The allowances I have made in favour of theſe ſchemes are more than equivalent to it. In particular; it ſhould be remembered, that the calculations ſuppoſe, that the payments required by theſe ſchemes, are yearly payments beginning immediately; (ſee p. 28.) and that the firſt payment of the annuity is not to be made till the end of the year in which the huſband ſhall die; and alſo, that the annuity is to be paid yearly, and nothing to be due for any part of the year in which the annuitant ſhall happen to die.

The

The compariſon then, in the preſent caſe, is between 3*l.* 8*s.* *per annum for ever*, and five guineas *per annum for* 15 *years*; or between an annuity of 3*l.* 8*s.* worth 25 years purchaſe, and an annuity of five guineas worth only 10 years purchaſe.

But, to throw this ſubject into another light. Let the number to which the ſociety is kept up be ſuppoſed to be 200. It has been demonſtrated in Queſt. II. that at leaſt half this number of widows will in time come to be conſtantly on the ſociety; and it has alſo been juſt now ſhewn, that the medium of annuities, payable to them, will be at leaſt 28*l.* After a courſe of years, then, the ſociety will have a conſtant expence to bear of 2800*l.* *per annum*.—But what will be its income?—In order to determine this, we muſt conſider, that there are two ſources from whence its income will be derived. Firſt, the annual payments of the members. And, ſecondly, the money accumulated, or the *capital* raiſed during the time the number of annuitants is coming to a *maximum*.—The firſt of theſe ſources affords 1000 guineas, or 1050*l.* *per annum*. This wants 1750*l.* of the annual expence juſt mentioned; and, therefore, in order that the income of the ſociety may be equal to the burden upon it, when the annuitants come to a *maximum*, there muſt be a fund raiſed in the mean time equal to 43,750*l.* or to an eſtate

in perpetuity of 1750 *l. per annum.*—But 1050 *l. per annum* beginning immediately, and forborn 25 years, and improved, without loſs or delay, all that time at 4 *per cent.* compound intereſt, will but juſt raiſe ſuch a capital (*a*). There is therefore, the fulleſt proof, that the ſcheme I am conſidering is extremely deficient. The truth is, that ſcarcely a *third* of ſuch a capital could be raiſed, as will appear from the following obſervations.

Out of 200 perſons, all 40 years of age, *more* than five, according to the *London* Table of Obſervations, and not ſo many by Dr. *Halley*'s Table, may be expected to die in a year. Suppoſe then five to be the real number of members that will die in the firſt year of the ſociety. In ſubſequent years the collective body of members will be continually growing older; and, therefore, the proportion of them that will die every year, will be continually increaſing, till it gets to a *maximum.* I will, however, ſuppoſe, that during the firſt 20 years no more than the number juſt ſpecified will die every year; and

(*a*) Every queſtion of this kind may be eaſily ſolved by Table III. in the next volume, which ſhews, that 1000 *l. per ann.* will, in 25 years, increaſe to 2666 *l. per ann.*; and, therefore, 1050 *l. per ann.* to 2800 *l. per annum.*

that

that, conſequently, no more than *five widows* will come every year on the ſociety. The ages of all theſe widows, when they commence widowhood, will, it is evident, be between 40 and 60. One with another then, they may be conſidered as having commenced widowhood at 50 years of age. Now, five widows left every year at this age, will, in 10 years, increaſe to 43 conſtantly in life together, according to the expectations of life in Tables V, VI, and VII; and, in 20 years, to 70 (*a*). Suppoſe the true number alive together at the end of 20 years to be only 62. The greater part of theſe will be annuitants of 30*l.* and 40*l.* *per ann.* and the reſt 20*l.* Were the former only equal to the latter, the medium of annuities payable to them would exceed 25*l.* Suppoſe then this medium to be no more than 26*l.* and it

(*a*) Every calculation of this kind is eaſily made by the rule in note (A) at the end of the next volume.—I have put the number living together at the end of 20 years at 62, not only that the reader may be better ſatisfied that I have kept low enough, but alſo to make an allowance for ſuch widows as will be left by thoſe members who die within a year after admiſſion, and who, therefore, according to theſe ſchemes, will be entitled to no annuities. This allowance is too large: For, after the firſt year of the ſcheme, it will not happen above once in 4 or 5 years, that the death of a member will be ſo circumſtanced, ſuppoſing the probability that a man at 40 will live a year, to be, as all but the *London* Tables make it, 50 to 1.

will follow, that, at the end of 20 years, the ſociety will have an annual rent to pay of 26 *l.* multiplied by 62, and 1612 *l.* and, if then able to bear ſuch an expence, it muſt, in the intermediate time, have acquired an increaſe of income equal to the difference between 1050*l.* and 1612*l. per ann.* That is; it muſt, with its ſavings, have accumulated a ſtock equal to 562*l. per ann.* and worth 14,050*l.* But as, during this time, there will be a number of annuitants conſtantly increaſing, to whom yearly payments muſt be made, the ſavings cannot certainly be one half of what they would have been had the ſociety been all the time free from all burdens. Suppoſe then the ſtock produced by theſe ſavings, to be equal to the ſtock that would ariſe from an income of 1050*l. per ann.* beginning immediately, and improved perfectly at 4 *per cent.* compound intereſt, for half the time I have mentioned, or for 10 years, without being ſubject to any checks or deductions: Such an income thus improved, would in 10 years produce an additional income of 504*l. per annum*, or a capital of 12,600*l.*—According to theſe Obſervations, therefore, the annual income of the ſociety at the end of 20 years, and before a third part of the higheſt annuitants could come upon it, would fall conſiderably ſhort of its expences. About that time

then

then it would neceſſarily run aground; and long before the number of annuitants could riſe to a 100, it would ſpend its whole ſtock, and find itſelf under a neceſſity of either doubling the annual payments of its members, or of reducing the annuities one half.

All I have now ſaid is meant on the ſuppoſition, that the ſociety begins with 200 members at 40 years of age, and is afterwards limited to that number, by admitting no more new members than will juſt ſupply the vacancies occaſioned by the loſs of old members. If it is allowed to increaſe, it may continue a longer time. And, for this reaſon, a ſociety that wants half the income neceſſary to render it permanent, may very well ſubſiſt, and even proſper for 30 or 40 years.—Thus, the *Laudable Society*, was it to keep to its preſent number of members, might poſſibly feel no deficiencies for 20 years to come; but if it ſhould continue to increaſe at the rate of 70 or 80 every year, it would, at the end of that time, poſſeſs a balance ſo much in its favour as might enable it to ſupport itſelf for 15 or 20 years more (*a*). But bankruptcy would

(*a*) What has been before demonſtrated in Queſt. III. ſhould be here recollected, that the number of annuitants on ſuch a ſociety as this, muſt go on to increaſe for more than 100 years after acquiring its greateſt number of members.

The

would come at laſt, and with the more weight the longer it had been deferred.

The rule in the *London Annuity* Society, which obliges every perſon between the ages of 45 and 55, to pay at admiſſion 5 guineas extraordinary for every year that he exceeds 45, is an advantage to it, but it is a very inadequate, and alſo a very unequitable advantage. For at the ſame time, that it obliges a perſon 55 years of age, to give *more* than the value of his expectation, it takes *above* two-fifths *leſs* than the value from a perſon who is 45 years of age.

If any perſons remain ſtill doubtful about what I have ſaid, I muſt beg their attention to one further argument.

The *Laudable* Society, I am informed, took its riſe from a calculation contained in a pamphlet entitled, *The Poſſibility and Probability of a* SCHEME *intended for the Benefit of Widows being able to ſupport itſelf.* The ſcheme here referred to, is the ſame with that which this Society has ſince followed; and I am afraid I ſhall not be credited, when I ſay, that the calculation to prove its capacity of ſupporting itſelf, is founded on the ſuppoſition, that a hundred married men, whoſe common age is 36, will leave but one widow every year, though at the ſame time it is ſuppoſed that two of them will die every year.

This miſtake has made the whole calculation one half wrong.—Nothing can be plainer than that, if the death of a married man does not leave a widow at the end of the year, the reaſon muſt be, that both himſelf and his wife have happened to die in the year. But it is always very improbable this ſhould happen.

It

It muſt be expected that every other member of theſe ſocieties, ſuppoſing them to conſiſt of perſons all of the ſame ages with their wives, will leave widows to whom, one with another (as already ſhewn), at leaſt 28*l.* *per ann.* muſt be allowed, for as many years as there have been payments from each member. For every 10 guineas then received they muſt ſometime or other hereafter pay 28*l.* But let it be well conſidered what can enable them to do this. Did money bear no intereſt, for any given ſum now received, they could not afford at any time hereafter to pay more than an *equal* ſum. That is; (ſince the duration of *ſurvivorſhip* is in the preſent caſe, by Queſt. II. equal to the duration of *marriage*) the proper conſideration for any given reverſionary annuity, to be allowed to *all* the ſurvivors of a ſet of marriages, would be, an equal annuity payable by each marriage during its exiſtence; and juſt *half* the reverſionary annuity, if it is to be allowed only to half the ſurvivors, or to widows excluſive of widowers. The annual payment then of *five* guineas, during marriage, can entitle widows to no more than an annuity of *ten* guineas, ſuppoſing money to bear no intereſt. But if money does bear intereſt, the ſame payment will entitle them to more, in proportion to the degree in which it is capable of being improved, during the time between

tween that in which the annual payments begin, and the commencement of widowhood. Now, it is eaſy to ſee, that unleſs money bears very high intereſt, this improvement cannot be likely in any circumſtances to produce a capital, the intereſt of which ſhall be equal to the annual payment itſelf. Any given annual payment perfectly improved at 4 *per cent.* compound intereſt, requires 17 years and a half to double itſelf; ſuppoſing the firſt payment made immediately (*a*). But no marriages are *likely* to laſt ſo long, except thoſe among perſons who are very young. A marriage between two perſons, both 40, will not *probably* laſt longer than 13 years, according to the probabilities of life in Dr. *Halley*'s Table. A marriage between two perſons, both 50, will not probably, by the ſame Table, laſt longer than *eleven* years; nor a marriage between two perſons, both 30, longer than 16 years. Such marriages, it is true, may poſſibly laſt 30 or 40 years. But this circumſtance is more than balanced by the fact, that no leſs poſſibly they may not laſt *one* year. The annual payments, then, being incapable of ſuch an improvement as ſhall produce an additional income equal to themſelves; it is ob-

(*a*) At 3 *per cent.* the period of doubling money by compound intereſt, is nearly 23 years and a half. At 5 *per cent.* 14 years.

vious,

vious, that no ſociety ought to go ſo far as to allow to widows annuities twice as great as thoſe which might be allowed, ſuppoſing no intereſt of money (*a*); ſo far, for inſtance, as to allow, inſtead of 10 guineas, 20 guineas for an annual payment of five guineas. In the circumſtances of moſt of theſe ſocieties three-fifths addition may be the full allowance. That is; ſuppoſing the annual payment of each member to be five guineas, time may be expected for gaining from hence a capital of 75 guineas, or that ſhall produce three guineas *per annum* intereſt; and the proper reverſionary annuity will be 16 guineas; or ſix guineas more than the proper reverſionary annuity, did money admit of no improvement (*b*).

The

(*a*) The money accumulated will not be exactly the ſame with that to which the annual payment would increaſe, if improved at compound intereſt for a number of years, equal to that which the joint lives have an *equal chance* of exiſting. Much leſs will the increaſe be the ſame with that which would ariſe from the annual payment forborn, and improved, for a number of years equal to the *expectations* of the joint lives. It will be leſs than either of theſe, for a reaſon explained in note (K) at the end of the next volume.

(*b*) To theſe accounts may be added the following ſhort and eaſy method of trying the ſufficiency of all ſchemes of this kind.

In an adequate ſcheme it can make no difference whether the annuities themſelves are paid, or the value of them in a ſingle payment at the time they become due.—

Suppoſe

The preceding obſervations have gone on the ſuppoſition, that the reverſionary annuities are to be *for life*. What difference in favour of theſe ſocieties ariſes from the circumſtance, that the annuities are to be paid only *for widowhood*, cannot be exactly determined. Some judgment, however, may be formed of it from what has been ſaid at the concluſion of Queſt. II. Were even one half of the widows to marry, ſtill the ſchemes I have

Suppoſe then a ſociety juſt eſtabliſhed, conſiſting of 600 members, all married men at the age of 40, each of whom, beſides one payment in hand, is to make an annual payment of five guineas. Suppoſe the age of their wives $39\frac{1}{2}$, and every widow to be entitled, on the day her huſband dies, to a life annuity of 20 *l.* the firſt payment to be made at the end of half a year.—Suppoſe further, that the ſociety is to be kept up for ever to 600 members by admitting new ones at the age of 40, as old ones drop off.—In the firſt year (according to Tables V, VI, and VII. in the next volume) twelve members, at leaſt, will die, and leave twelve widows, each intitled to 20 *l. per ann.* The value of ſuch an annuity to commence at the end of half a year, the age being 40, is $14\frac{1}{2}$ years purchaſe by Mr. *De Moivre*'s valuation of lives, (or Table I. in the laſt leaves of the next volume) reckoning intereſt at 3 *per cent.* The value, therefore, of 12 ſuch annuities; that is, the whole amount of the ſums becoming payable during the courſe of the firſt year, is 3480 *l.*—The annual contribution is 600 times 5 guineas, or 3150 *l.* and this, together with its intereſt for about half a year, or 3205 *l.* is all that ſuch a ſociety could be poſſeſſed of to bear an annual expence of 3480 *l.*—It appears, therefore, that, in order to ſupport the expence of the ſuppoſed annuities, the annual contribution of each member ought to have been more than five guineas.

A proof

have been conſidering would probably be inſufficient. But, in the circumſtances of theſe ſocieties, it cannot be expected, that above one in 10, or perhaps one in 20, will marry. The perſons moſt likely to enter into them, are ſuch as have not the proſpect or ability of making competent proviſions for their wi-

A proof of the ſame nature with that here given may be deduced, by conſidering theſe ſocieties as bodies of men united for the purpoſe of aſſuring to one another, from year to year, annuities for their widows; and the way of finding the value of ſuch an aſſurance is, to multiply the value of the annuity, by the probability that it will become payable in the courſe of the year.—For inſtance, Let the member's age, and alſo his wife's, be 40. Let the annuity be 20 *l. per ann.* for life to commence at the end of a year, or an annuity whoſe preſent value is (reckoning intereſt at $3\frac{1}{2}$ *per cent.*) 14 years purchaſe; that is, 280 *l.* The probability that a perſon at the age of 40 will die in a year, and that his wife of the ſame age will live a year: or, in other words, the probability, that ſuch a member will leave a widow in the courſe of the year, is, by the *Breſlaw* Obſervations, (or Table V, next volume) $\frac{9}{445}$ multiplied by $\frac{436}{445}$, or .0198. (See p. 18, and 23). That is; there will be the odds of nearly 49 to 1, againſt ſuch a member leaving a widow in the courſe of the year. The value of the aſſurance, therefore, is .0198, multiplied by 280, or the 50th part of *l.* 280: that is, 5 *l.* 11*s.*—In the ſame manner the value of a like aſſurance for a year at any other ages may be eaſily calculated. At the age of 35, it is 5 *l.* 7*s.* At the age of 45, it is 6 *l.* 7*s.* The value, therefore, increaſes continually with age; and, if given in an annual payment conſtantly the ſame, which is the caſe in theſe ſocieties, it ought to be greater than the annual payment due for one year at the commencement of the aſſurance.

Five guineas *per annum*, therefore, is, *demonſtrably*, an inſufficient payment from a married man for a life-annuity of 20 *l.* to his widow.

dows

dows in other ways. The widows left, therefore, will in general be unprovided for, and, being alſo left with families of children, it is quite unreaſonable to expect, that any conſiderable proportion ſhould marry. This is true of ſuch as may happen to be left young; but when a ſociety has ſubſiſted ſome time, the *greater* part will not be young when left, and theſe, at the ſame time that no advantage can be expected from their marrying will be in general the *higheſt* annuitants, and, therefore, the *heavieſt burdens*.—Moreover, the proſpect of the loſs of their annuities will have a particular tendency to check marriage among them.—For all theſe reaſons it ſeems to me likely, that the benefit, which theſe ſocieties will derive from marriages among their annuitants, will not be very conſiderable; or, at leaſt not *ſo* conſiderable as to be equal to the advantages I have allowed them, by calculating on the ſuppoſitions, that the money they receive will be *always improved perfectly, without loſs or delay, at the rate of 4 per cent. compound intereſt*; that the probabilities of life among males and females are the ſame, and all huſbands likewiſe of the ſame ages with their wives, and that conſequently the *maximum* of widows on ſuch ſocieties can amount to no more than half the number of marriages (*a*).—With reſpect to the

(*a*) Care ſhould be taken in theſe ſocieties, not to judge of the proportion of widows that will marry, from the

the laſt of theſe ſuppoſitions, it deſerves to be particularly obſerved, that from accounts taken annually with great care in *Scotland*, it appears, that the widows of the *miniſters* and *profeſſors* there (*a*), notwithſtanding the diminution occaſioned by their marrying, do exceed conſiderably half the number of marriages. And certainly it would be unreaſonable in theſe ſocieties not to reckon that the ſame will happen among them.—Indeed it ſeems certain, that notwithſtanding the hazards that attend child-bearing, the probability that the woman ſhall ſurvive in mar-

the proportion that may happen to marry during their firſt years. For moſt of the widows that will be left at firſt will be young; whereas the greater part will not be young when they commence widowhood, after a ſociety has ſubſiſted 30 or 40 years; and, therefore, though one in three or four ſhould marry at firſt, it will not be reaſonable to expect, that half ſo many ſhould marry after the affairs of the ſociety become ſtationary.

(*a*) The number of married miniſters and profeſſors, for 17 years, from 1750 to 1766, was at a medium 667. And from the enquiries that have been made, it appears, that from this whole body near 400 widows conſtantly living are derived. The medium of widows left annually has, for the laſt 36 years, been $19\frac{1}{10}$; and, for 10 years, ending in the year 1767, but nine of theſe had married.—Of the annuitants likewiſe (about 160 in number) on the fund eſtabliſhed among the Diſſenters in *London*, for relieving the widows of indigent miniſters, it is found that few ever marry. See Chap. 2. Sect. 2. See likewiſe the latter end of the 4th Eſſay; and note (A) at the end of the next volume.—In the *Laudable* Society during 19 years from 1761, the number of widows that came upon the Society was 167; and of theſe only 13 had married at Lady-day 1780. Fourteen had died.

riage, and not the man, is much greater (*a*) than is commonly imagined. It will be ſhewn in the laſt Eſſay, that it is not leſs than the odds of 3 to 2; and had I calculated agreeably to this fact, the values of annuities for widows, would have been given near a quarter greater than they have been given on the ſuppoſition, that the chance of ſurvivorſhip is equal between men and their wives.—It muſt be added, that I have made no account of any expences attending the execution and management of the ſchemes of theſe ſocieties. Some ſuch expences there muſt be, and ſome advantages ſhould be always provided in order to compenſate them.

There are in this kingdom many inſtitutions for the benefit of widows, beſides the two on which I have now remarked; and in general, as far as I have had any information concerning them, they are founded on plans equally inadequate, having being formed juſt as fancy has dictated, without any knowledge of the principles on which the values of re-

(*a*) Partly, as obſerved in page 8, on account of the greater mortality of males, but chiefly on account of the exceſs of age on the man's ſide.—The *Laudable Society*, for ſeveral years after its inſtitution, paid no regard to this exceſs of age; and the allowance required on this account by the *London Annuity* Society was ſo trifling as to deſerve no notice.

In March 1780, 167 huſbands had died in the *Laudable Society*, and only 138 wives.

verſionary

verſionary annuities ought to be calculated. The motives which influence the contrivers of theſe inſtitutions, may be *laudable*; but they ought, I think, to have informed themſelves better. This appears ſufficiently from what has been ſaid; but I will juſt mention one further proof of it.

The *London Annuity Society* promiſes that, if in 21 years; and the *Laudable Society* that, if in 25 years, it ſhall appear that there has been all along an annual ſurplus in favour of the ſocieties, it ſhall be employed in either raiſing the *annuities*, or in ſinking the *annual payments*. Now, they may be aſſured, that if, at the end of theſe periods, they ſhould not be poſſeſſed of a conſiderable ſurplus, the true reaſon will be, their having granted much higher annuities than the annual contributions are able permanently to ſupport: For it has been demonſtrated, that the number of annuitants, and conſequently the amount of the annual expences, will go on increaſing for a long courſe of years beyond theſe periods. The effect, therefore, of carrying into execution this regulation will be, precipitating that bankruptcy which would have come too ſoon had there been no ſuch regulation.

It has been ſaid in defence of theſe ſocieties, that the deficiencies in their plans cannot be of much conſequence, becauſe their rules oblige them to preſerve a conſtant equality

between their income and expences, by reducing the annuities as there ſhall be occaſion. And from hence it is inferred, that they can never be in any danger of a bankruptcy.—But it has appeared, that the time when they will begin to feel deficiencies is ſo diſtant, that it will be too late to remedy paſt errors, without ſinking the annuities ſo much, as to render them inconſiderable and trifling. All that is given too much to *preſent* annuitants is ſo much taken away from *future* annuitants. And if a ſcheme is *very* deficient, the firſt annuitants may, for 30 or 40 years, receive ſo much more than they ought to receive, as to leave little or nothing for any who come after them. Deficient ſchemes, therefore, are attended with particular injuſtice; and this injuſtice will be the ſame, if, inſtead of *reducing* the annuities, the annual payments ſhould be increaſed; for all the difference this can make will be, to cauſe the injuſtice to fall on *future contributors*, inſtead of *future annuitants*.

But what requires moſt to be conſidered here is, that, after either the annuities have been for ſome time in a ſtate of reduction, or the contributions in a ſtate of increaſe, it will be ſeen that theſe ſocieties have gone upon wrong plans, and, therefore, they will be deſerted and avoided; the conſequence of which will prove ſtill greater deficiencies in their

their annual income, and a more rapid deſertion and decline, till a total diſſolution and bankruptcy take place.—This will be the death of moſt of the preſent ſocieties for providing for widows, if they continue to be encouraged, and do not ſoon alter their plans; And at that period the number of *annuitants* will be greater than ever; whoſe annuities, having no other ſupport than the poor remains of a ſtock always inſufficient, will be ſoon left, without the poſſibility of relief, to lament that ignorance and credulity which gave riſe to theſe ſocieties, and which had ſo long ſupported them.

In the *London* Annuity Society, there is an encouragement to *batchelors* and *widowers* to join them, ariſing from the additional annuities to which they will be *immediately* entitled, when they marry, in conſequence of having made their payments a greater number of years; and it is imagined, that particular advantages will be derived from ſuch members. But even theſe will in general pay much leſs than the value of their expectations.—A perſon who begins an annual contribution of five guineas at the age of 24, will, ſhould he live 11 years, and marry a woman of the ſame age at the end of that time, entitle her immediately to 35 *l. per ann.* during ſurvivorſhip, and to 41 *l. per annum* ſhould he live four years after marry-

ing, (intereſt being at 4 *per cent.*) (*a*). In this particular caſe, therefore, a perſon will pay nearly the true value of his expectation. But *all* at all ages who *marry*, in leſs time than 11 years after admiſſion, will pay leſs than the value of their expectations.

SECT. II.

Of the Aſſociation among the London *Clergy, and the Miniſters in* Scotland, *for providing Annuities for their Widows* (*b*).

IN April, 1765, the clergy within the bills of mortality, and the county of *Middleſex*, at a general meeting in *Sion-College*, agreed to form themſelves into a ſociety for the ſupport of their widows and orphans. Many in

(*a*) The value of five guineas *per annum* (firſt payment made immediately) for 11 years, ſubject to failure ſhould a life now 24 fail; and, after 11 years, for the joint lives of two perſons both 35, is, by the Table of *London Obſervations*, *l.* 69.3—By Dr. *Halley*'s Table, *l.* 76.44—The preſent value of 35 *l. per annum* for life to the widow of a perſon now 24, ſhould he live 11 years, and marry a woman of the ſame age with himſelf at the end of that time; and alſo of 6 *l.* more, or 41 *l. per annum* in all, ſhould he live after marriage four years; is, by the Table of *London* Obſervations, *l.* 69.36.—By Dr. *Halley*'s Table, *l.* 76.03.

(*b*) This ſection, as well as the former, muſt be conſidered as written in 1773. Further accounts of theſe inſtitutions will be given in the ſupplement to this ſection.

this

this reſpectable body may be capable of doing, in a better manner, what I have attempted in this Treatiſe; and they are, perhaps, already ſenſible of the deficiencies in the plan which they have eſtabliſhed. I ſhall not, however, I hope, do wrong, in taking the liberty to recite briefly this plan, in order to introduce a few obſervations upon it.

According to the printed articles, every clergyman poſſeſſed of any benefice, lecture-ſhip, or licenſed curacy, within the bills of mortality and the county of *Middleſex*, who ſubſcribes annually one guinea, or two guineas, or more, ſhall entitle his widow to an annuity; or, if he leaves no widow, he ſhall entitle any ſuch children as he ſhall leave, to the ſame annuity for ſeven years as his widow would have had. And, in caſe a widow poſſeſſed of an annuity, ſhould either *die* or *marry* before the lapſe of 10 years, from the commencement of her annuity, ſuch children of her former huſband, as ſhall be then alive, are to be entitled to as many of the ten years payments of the annuities as ſhe ſhall not have received.—The annuity is fixed to no particular ſum, but inſtead of this, it is ordered, that a fourth part of the annual ſubſcriptions and intereſt ſhall be divided the firſt three years after the eſtabliſhment of the ſociety; half only the next four years; and three fourths the next five years; provided, however, that in no one of theſe 12 years the dividend

ſhall exceed 20 *l.* to the widows and orphans of the clergy ſubſcribing two guineas or more; and 10 *l.* to the widows and orphans of the ſubſcribers of one guinea. And, after the expiration of 12 years, the whole amount of the ſubſcriptions, and of the intereſt of the capital ſtock, is to be divided proportionably for ever.—It is further provided, that every clergyman who ſhall be married, or have children, at the time of his ſubſcription, ſhall pay a fine of two guineas towards a capital ſtock, if a ſubſcriber of two guineas or more, and 40 years of age or upwards. If 50 years of age or upwards, he ſhall pay a fine of three guineas; if 60 or upwards, five guineas. But, if not married at the time of his ſubſcribing, and ſhall afterwards marry, he ſhall pay a fine according to the age he ſhall be of at the time of his marrying. The obligation laid upon all, whether married or unmarried, to become ſubſcribers, is, an incapacity of being admitted members without the conſent of a general court, unleſs, within two years after becoming poſſeſſed of any eccleſiaſtical employment, they ſubſcribe.

Every one who has attended to the obſervations in this and the preceding chapter, muſt know what judgment to form of theſe regulations.

Let us ſuppoſe that all the clergy in *London* and *Middleſex* came into this aſſociation from

from the firſt; and that one with another they are ſubſcribers of two guineas annually; and that there are among them as many unmarried perſons as married.

In this caſe, it may be learnt from Queſt. XIII, that the annuity to which widows ſhould be entitled, (ſuppoſing no allowance to the children of any that die) ought not to exceed 10 or 11 guineas at moſt; and that, beſides the annual ſubſcriptions, there ought to have been a fine paid at the commencement of the ſcheme, by every married perſon, of ſix guineas at leaſt, or, by the whole number of ſubſcribers, three guineas. If the number of married members is double the unmarried, the annuity ought not to exceed eight guineas; and the fine from every member ſhould be about four guineas.—The order, that only a fourth part of the annual ſubſcriptions and intereſt ſhall be divided the firſt three years, half the next four years, and three quarters the next five, is without reaſon; becauſe the number of claimants, for the firſt 12 years of the ſcheme, will be ſo few, that it will not be poſſible, during that time, that there ſhould be occaſion for dividing any proportions ſo large of the annual ſubſcriptions and intereſt, unleſs they are indeed beyond all bounds too little. —After 12 years, the number of annuitants will go on increaſing for near 50 years, as appears from Queſt. III. The conſequence, therefore,

therefore, of dividing, after that time, the whole amount of the annual ſubſcriptions and intereſt, will be a conſtant yearly diminution in the dividends for near 50 years; and making the payments to the firſt claimants much more conſiderable than they ought to be, at the expence of all ſubſequent claimants—For theſe reaſons, it appears to me out of all doubt, that this ſcheme is by no means likely to anſwer the good ends propoſed by it; and that, therefore, it will be beſt to lay it aſide. At the time it was ſettled it was, I find, further agreed, that the annual ſubſcriptions of the *laity*, together with the intereſt of their benefactions, unleſs otherwiſe directed by the donors; and the annual ſubſcriptions of ſuch of the clergy as ſhall ſo direct, ſhall make a *charitable fund* to be applied to the relief of the diſtreſſed widows or children of all the clergy within the limits I have mentioned, whether ſubſcribers or not, provided that in no one year of the firſt twelve more than 20*l.* be given out of the fund to any one family.—This is an excellent deſign; and if the money ariſing from all the ſubſcriptions is thrown into this fund, an important means of relief may be provided, for ſuch of the more indigent widows and families as will accept the help of charity.

There is one more ſcheme of particular conſequence, which I muſt take notice of: I mean,

I mean, that which is eſtabliſhed by Act of Parliament, among the miniſters and profeſſors in *Scotland*, for making proviſion for their widows and orphans. The laſt mentioned ſcheme, and alſo ſeveral others of the ſame kind (*a*) in this kingdom, have been formed on the model of this: and the ſucceſs with which it has been hitherto attended, is one of the principle cauſes to which they have owed their riſe. It is, therefore, proper I ſhould give ſome account of it; and it will be ſufficient with this view to mention, "that for an annual payment, which "begun immediately, of *five guineas* from "1011 contributors, 667 of whom are mar-"ried perſons, beſides a tax on weddings, "producing about 142 *l. per ann.* it entitles "every widow to an annuity of 20 *l.* during "widowhood, and alſo every family of chil-"dren that ſhall be left by ſuch members "as die without leaving widows, to 200 *l.*" This ſcheme contains a variety of other particulars; but this is its ſubſtance—It commenced on the 25th of *March*, 1744; and from that time to the 22d of *November*,

(*a*) There is one among the Diſſenting Miniſters in the counties of *Cheſter* and *Lancaſter*; and another among the Diſſenting Miniſters in *Cumberland*, *Northumberland*, *Weſtmorland*, and *Durham*.—Even the *London Annuity* Society, though its plan is totally different, profeſſes to form itſelf on the principles of the *Scotch* eſtabliſhment, and to derive encouragement from it.

1770 (*a*), or in 26 years and near 8 months, 151 miniſters and profeſſors died, and left 151 families of children without widows; that is, 5.66 ſuch families were left annually; and the annual diſburſements to them have therefore been 1132*l.* Subtract this ſum from 5450*l.* the whole annual income; and the remainder, or 4318*l. per ann.* will be the ſtanding proviſion for bearing the expence of all the annuitants poſſible to be derived from 667 marriages. Such an annual payment, or 4.27 each from 1011 contributors, is the ſame with 6.55 each, from 667 contributors; and, conſequently, it appears, that in this eſtabliſhment a contribution is received equivalent to an annual payment beginning immediately, of *l.* 6.55 from every married man, in order to entitle his widow to an annuity of 20*l.* during her widowhood.

In the ſocieties mentioned in the laſt ſection, annuities increaſing from 20*l.* to 40*l.* are promiſed to widows for an annual payment of only 5 guineas (*b*). And, in all the ſocieties for the benefit of widows with which I am acquainted, there is an equal or a greater diſproportion between the contributions received, and the annuities promiſed.—

(*a*) In Nov. 1779, or 35 years and 8 months, 199 miniſters and profeſſors had died, and left 199 families of children without widows; that is 5.58 annually.

(*b*) See page 65.

With

With what ſtrange raſhneſs then has the plan of this eſtabliſhment been copied? And how abſurdly have the ſocieties in this kingdom pleaded it as a precedent which encourages and favours them?—It would be trifling to ſay more on this ſubject.

It may be obſerved that the annual income for the ſupport of this eſtabliſhment, ſuppoſing it to have only the benefit of widows in view, ought to be *l.*7.19 *per ann.* from every marriage, according to Queſt. XIII. p. 44. and *l.*7.44 *per ann.* according to the calculation in Note F, at the end of the next volume.

Theſe determinations exceed the income actually provided. But the exceſſes are by no means conſiderable enough, to afford any certain reaſon for concluding, that the fund of this eſtabliſhment will prove inſufficient. I was, however, once led to entertain ſome doubts on this ſubject. And in theſe doubts I thought myſelf confirmed by obſerving, that, in the calculations (*a*) made at the commencement of the ſcheme, the number

(*a*) See Table III. in a book printed at *Edinburgh* in 1748, entitled, Calculations, with the Principles and Data on which they are inſtituted, relative to a late Act of Parliament, entitled, *An Act for raiſing and eſtabliſhing a Fund, for a proviſion for the widows and children of the miniſters of the church, and of the heads, principals, and maſters of the univerſities of* SCOTLAND; ſhewing the riſe and progreſs of the Fund.

333 was ſtated, as the *maximum* of widows living at one time, likely to come upon it, or to be derived from 20 (*a*) widows left annually; and alſo, that 40 years was ſtated as the number of years neceſſary to bring on this *maximum*; whereas I was ſatisfied, that the *maximum* of widows would not prove much leſs than 400; nor the number of years neceſſary to bring it on, leſs than 60.—In the former editions of this work, I gave a diſtinct account of this. But I have lately received ſuch information (*b*) as has convinced me that my doubts have been in a great meaſure groundleſs. I have learnt, in particular, that there have been ſeveral calculations ſubſequent to thoſe I had ſeen; and that this eſtabliſhment has enjoyed advantages and proviſions for its ſupport which I was unacquainted with, and which give reaſon for expecting that it will indeed be able to bear the expence of 400 annuitants, ſhould ſo many come upon it. I ſhould only tire moſt of my readers, were I to enter into an account of theſe advantages and proviſions. It will be of more importance to take this opportunity to obſerve, that the probabilities of

(*a*) See the note A, among the notes at the end of the ſecond volume.—See likewiſe the note in p. 81.

(*b*) I owe this information to the kind and very obliging candour of the reverend and ingenious Dr. WEBSTER, of *Edinburgh*.

life

life from which the determinations I have mentioned are derived, though much lower than the probabilities of life among the miniſters and their wives in SCOTLAND (*a*), are yet ſuch as give the values of reverſions depending on ſurvivorſhips among them too high.

In order to underſtand this, it muſt be conſidered, that the difference between the probabilities of life in different ſituations, takes place much more in the firſt and middle than in the laſt ſtages of life; and that the effect of this muſt be to *increaſe* the duration of *joint lives*, and at the ſame time to *leſſen* the duration of *ſurvivorſhip* in thoſe ſituations which are moſt favourable to health. Or, in other words, to render the duration of marriage in ſuch ſituations, greater than it would otherwiſe be in proportion to the duration of widowhood; and, conſequently, to reduce the preſent value in annual payments during marriage, of any given annuity payable during widowhood. For inſtance. Were the probabilities of life among the miniſters and their wives in SCOTLAND the ſame that they are in Mr. *De Moivre*'s hypotheſis, or in the *Breſlaw* and *Northampton* Tables of Obſervation, the duration of marriages among them,

(*a*) More particular notice will be taken of this at the concluſion of the laſt of the following Eſſays.

taken

taken one with another, could not be more than 19 years. The duration of widowhood would be 22 years, and the *maximum* of widows living at one time derived from 667 marriages conſtantly kept up, would be conſiderably more than 400.—Were the probabilities of life among them the ſame that they are in LONDON, the duration of marriage would be ſtill *leſs*, and the duration of widowhood *greater*, and the *maximum* of widows derived from 667 marriages, could not be leſs than 500. But the fact is, that the duration of marriage among them is 22 years nearly; and that of widowhood about 20 years and a half (*a*). And it appears alſo, from accounts taken annually, that the number of widows living at one time, derived from the whole body of miniſters and profeſſors, does not exceed 400. It is, therefore, certain that a ſmaller income muſt be ſufficient for the ſupport of this ſcheme than would be neceſſary, according to the probabilities of life in the Tables juſt mentioned.—And upon the whole; after a careful review of all the circumſtances of this eſtabliſhment in its preſent ſtate, I am well ſa-

(*a*) See a note at the concluſion of the laſt Eſſay; and alſo note F, at the end of the next volume.—The *maximum* of widows (ſuppoſed 395) divided by the number left annually (or $19\frac{1}{10}$) gives $20\frac{3}{4}$, the expectation of widowhood. See page 81, and Note (A) at the end of the next volume.

tisfied,

tisfied that the ſucceſs with which it has been hitherto attended, is likely to continue; and that it will indeed prove a permanent foundation of that aſſiſtance to the *widow* and *fatherleſs* which is intended by it.—Caution, however, and vigilance, will for ſome time be neceſſary. Many more years muſt paſs before it can receive a deciſive confirmation from experience. Events have hitherto favoured it more than could have been reaſonably expected. They may perhaps hereafter try it; and deviations from probability may ariſe, which cannot be now foreſeen.—But I ought to aſk pardon for making theſe remarks. The venerable miniſters and profeſſors concerned will, I hope, excuſe me. They are eminently diſtinguiſhed by their abilities and knowledge: and can have little need of any information which I am able to give them.

SECT. III.

Of the beſt Schemes for providing Annuities for Widows.

INSTITUTIONS for providing widows with annuities would, without doubt, be extremely uſeful, could ſuch be contrived as would be *durable*, and at the ſame time *eaſy*

and *encouraging*. The natures of things do not admit of this in the degree that is commonly imagined. The calculations and rules, in the preceding chapter, will enable any one to determine in all caſes to what reverſionary annuities any given payments entitle, according to any given valuation of lives, or rate of intereſt. From Queſt. VII. and VIII. in particular, it may be inferred that (intereſt being at 4 *per cent.* and the probabilities of life as in the *Breſlaw*, *Norwich*, and *Northampton* Tables) for an annual payment beginning immediately of *four* guineas during marriage; and alſo for a guinea and half in hand, on account of each year that the age of the huſband exceeds the age of the wife; every married man, under 40, might be entitled to an annuity for his widow, during *life*, of 5*l.* if he lives a year, 10*l.* if he lives *three* years, and 20*l.* if he lives *ſeven* years.

If ſuch a ſociety chuſes, that thoſe who ſhall happen to continue members the longeſt time, ſhall be entitled to ſtill greater annuities, ſix guineas, additional to all the other payments at admiſſion, would be the full payment for an annuity of 25*l.* and 12 guineas for an annuity of 30*l.* if a member ſhould live 15 years.

All batchelors and widowers might be encouraged to join ſuch a ſociety, by admitting them on the following terms.—*Four guineas* to be paid on admiſſion, and *three guineas* every

every year afterwards, during celibacy; and on marriage, the ſame payments with thoſe made by perſons admitted after marriage; in conſideration of which, 1*l. per annum*, for every ſingle payment before marriage, might be added to the annuities to which ſuch members would have been otherwiſe entitled.

For example. If they have been members four years, or made five payments before marriage, inſtead of being entitled to life-annuities for their widows of only 5*l.* 10*l.* 20*l.* 25*l.* and 30*l.* on the conditions I have ſpecified, they might be entitled to annuities of 10*l.* 15*l.* 25*l.* 30*l.* and 35*l.* Or, if they have been members nine years, and made 10 payments, they might, inſtead of the ſame annuities, be entitled to annuities of 15*l.* 20*l.* 30*l.* 35*l.* and 40*l.*—In this caſe, the contributions of ſuch members as ſhould happen to deſert, or die in celibacy, would be ſo much profit to the ſociety, tending to give it more ſtrength and ſecurity.

This is one of the beſt ſchemes that I can think of, or would chuſe to recommend. But in the following ſcheme there is a ſimplicity and fairneſs which ſeem to give it a particular preference.

Every huſband, be his age what it will, for a ſingle payment at admiſſion, of 15*l.* with 1*l.* 10*s.* added for *every* year that his age exceeds his wife's, and an annual pay-

ment of 5*l.* during marriage, (the firſt to be made at the end of a year) might entitle his wife, ſhould he leave her a widow, to an annuity of 10*l.* for her life, if he lives *one* year; 11*l.* if he lives *two* years; 12*l.* if he lives *three* years; and ſo on; the annuity to increaſe continually at the rate of 1*l.* for every year that the huſband lives beyond one year.——Any addition to theſe payments might entitle to a proportionable addition to the annuity, and to its increaſe. ——And ſhould any huſband under 40 wiſh to ſecure a ſum for his children, *provided* he ſhould leave no widow, he might for every annual payment of nine ſhillings, *during life*, entitle them to 50*l.* payable among them at his death, whenever that ſhall happen. Making all theſe payments *guineas* inſtead of *pounds*, might probably be ſufficient, if the number of ſubſcribers is conſiderable, to defray the expences of management.

There is one particular advantage which ſocieties formed on plans of this kind would enjoy (*a*).—Perſons who know themſelves ſubject to diſorders which are likely to render them ſhort-lived, will have no great temptations to endeavour to gain admiſſion into ſuch ſocieties; and, if admitted, the danger

(*a*) See another advantage mentioned under Queſt. VIII.

from

from them will be leſs than on any other plan. Were it not for this danger, the following plan might be recommended.

In the plans hitherto mentioned it is implied, that, if either a member or his wife dies within any of the periods ſpecified, the additional annuities, that would otherwiſe have become due, will be loſt. But it would be much more agreeable to a purchaſer, that they ſhould be made certain to his wife, provided ſhe lives to the end of theſe periods, though in the mean time his own life ſhould fail. The value of ſuch annuities may be computed by the rule in Queſt. IX.

Suppoſe, for inſtance, the *ſcheme* to be "that a wife ſhall be intitled certainly to a "life-annuity of 20*l.* the firſt payment of "which ſhall be made at the end of 12 years, "provided ſhe ſhould be then alive, and her "huſband dead; or at the end of any year "beyond this term in which ſhe may hap-"pen to be left a widow." Suppoſe it alſo ſtipulated, "that ſhe ſhall be entitled to "10*l.* more, or 30*l.* in all, on the ſame "terms, provided ſhe ſhould live 16 years." —The value of ſuch an expectation (intereſt being at 3 *per cent.* and the probabilities of life as in the *Northampton* Table of Obſervations) will be, in the moſt convenient round ſums, ſuppoſing none admitted above 50 years of age, ſeven guineas in annual payments to be

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continued during marriage, and to begin immediately; beſides four guineas in preſent money for every year, as far as 15 years, that the huſband's age exceeds the wife's, if he is between 40 and 50, and three guineas on the ſame account if he is under 40; Or, if the whole value of the expectation is given in one preſent payment, 70*l.* added to a guinea and half for every year that the huſband's age falls ſhort of 50, beſides the payment juſt mentioned on account of diſparity of age.

The value of this expectation at 4 *per cent.* is ſix guineas in annual payments; beſides three guineas in preſent money, for every year that the huſband's age exceeds the wife's, if he is between 40 and 50: and two guineas, if he is under 40: Or, if the whole value of the expectation is given in one preſent payment, 56*l.* added to 1*l.* 5*s.* for every year that his age falls ſhort of 50, beſides the payment laſt mentioned on account of inequality of age (*a*).

He

(*a*) Suppoſing 16 years the only term, the annuity 20*l.* and intereſt at 4 *per cent.* the proper payments will be nearly, in the caſe of equal ages and *ſingle* payments, 46*l.*—40*l.*—29*l.* as the age of the man is 30, 40, or 50. Or, in *annual* payments, *l.* 3.80.—*l.* 3.66.—*l.* 3.13.—Suppoſing the woman's age 10 years leſs than the man's, the ſame values will be, in *ſingle* payments *l.* 58.92.—*l.* 56.56.—*l.* 53.66.—in *annual* payments *l.* 4.63.—*l.* 5.—*l.* 5.41.—It appears, therefore, that a ſociety, ſuppoſing money improved at the rate of 4 *per cent.* might entitle all married men *indiſcriminately*, who are under 50 years of age, to

He that will give himſelf the trouble to calculate, agreeably to the directions in the Queſtions to which I have referred, will find that, taking all particular caſes together, the rules now given come as near the truth as there is reaſon to deſire in an affair of this nature, the *defects* in ſome caſes being nearly compenſated by the *exceſſes* in others.

Theſe determinations are agreeable to the probabilities of living in Dr. *Halley*'s, as well as the *Northampton* Table of Obſervations, or Tables 5th and 6th in the next volume. Theſe Tables ſeem to give a proper *medium* between the different values of *town* and *country* lives. In the country the probabilities of living are much higher; but in *London*, and probably in all *great* towns and ſome *ſmaller* ones, they are much lower.

It is proper to add, that, according to the values of lives deduced both from the *London* and Dr. *Halley*'s Table, and taking intereſt as low as 3 *per cent.* all women whoſe huſbands are under 50 years of age, might be entitled to an annuity of 24*l.* during *life* (the firſt payment to be made at the end of the year in which they ſhall be left widows) for the ſum of 100*l.* ſuppoſing 3*l.*

to ſuch an expectation as this for their wives; for either 60*l.* in *one* payment, or five guineas in *annual* payments. —But equity requires, that different payments ſhould be made according to the different comparative ages of men and their wives.

additional given on account of every year that they are younger than their huſbands.—At 4 *per cent.* an annuity of 30*l.* might be granted on the ſame terms.

In the year 1690, the company of *Mercers* in *London*, adopted ſuch a ſcheme as that laſt mentioned. For 100*l.* in *one preſent payment*, they entitled every ſubſcriber to a *life-annuity* for his widow of 30*l.*; and this, at that time, (when money bore 8 *per cent.* intereſt) was conſiderably leſs than the value of the money advanced, ſuppoſing men and their wives of equal ages. As the intereſt of money ſunk, they ſunk alſo the annuity, firſt to 25*l.* and then to 20*l.* and 15*l.* But at laſt, after carrying on the ſcheme for above 50 years, finding the burden of the annuitants too heavy, and likely to go on increaſing, they were obliged to drop the ſcheme and to ſtop payment. In a little time, however, by a parliamentary aid of 3000*l. per ann.* they were reſtored to a capacity of making good all their engagements, and of paying their arrears.—Their failure, is, indeed, much to be lamented; for, in conſequence of it, the public has loſt the benefit of an inſtitution, that for many years promiſed the happieſt effects, by encouraging marriage, and affording relief to indigence. The rapid fall of the intereſt of money; their admitting purchaſers at too advanced ages; and, particularly, their paying no regard to the difference

difference of age between huſbands and their wives, muſt have contributed much to hurt them. Some of the principal cauſes, therefore, which have rendered them unſucceſsful, may be now avoided.

It muſt, however, be remembered, that the iſſue of the beſt ſchemes of this kind muſt be in ſome degree uncertain. For want of proper obſervations (*a*), it is not poſſible to determine what allowances ought to be made, on account of the higher probabilities of life among females than males. No prudence can prevent all loſſes in the improvement of money; nor can any care guard againſt the inconveniencies to ſuch ſchemes, which muſt ariſe from thoſe perſons being moſt ready to fly to them who, by reaſon of concealed diſorders, feel themſelves moſt likely to want the benefit they offer.

The ſocieties, therefore, on which I have remarked in the firſt ſection of this chapter, would have reaſon to take warning from what has happened to the *Mercer's Company*, were the ſchemes on which they are formed perfectly unexceptionable. But I have demonſtrated that theſe ſchemes are very defective; and that the longer they are carried on, the more miſchief they muſt produce. 'Tis vain (as appears from Queſt. III.) to form ſuch

(*a*) This defect will, I hope, be in ſome meaſure removed by the Obſervations and Tables in the next volume.

eſtabliſh-

eſtabliſhments with the expectation of ſeeing their fate determined ſoon by experience. If not more extravagant than any ignorance can well make them, they *will* go on proſperouſly for 20 or 30 years; and, if at all tolerable, they *may* ſupport themſelves for 40 or 50 years; and at laſt end in diſtreſs and ruin. No experiments, therefore, of this ſort ſhould be tried haſtily. An unſucceſsful experiment muſt be productive of very pernicious effects. All inadequate ſchemes lay the foundation of *preſent* relief on *future* calamity, and afford aſſiſtance to a *few* by diſappointing and oppreſſing *multitudes*.

As the perſons who conduct theſe ſchemes can mean nothing but the advantage of the public, they ought to liſten to theſe obſervations. At preſent their plans are capable of being reformed; but they cannot continue ſo always; for the greater number of exorbitant payments they now make to annuitants, the more they conſume the property of future annuitants, and the leſs practicable a retreat is rendered to a rational and equitable and permanent plan (*a*). They ſhould, therefore, *immediately* (*b*) either *reduce* their

(*a*) See p. 83, 84. Sect. I.

(*b*) Thus; was the *London* Aunuity Society to make their loweſt annuity 10*l.* the next 20*l.* and the higheſt 30*l.* they would probably be ſafe. But, after proceeding on their preſent plan ſome years longer, ſuch a reduction would by no means be ſufficient.

ſchemes,

ſchemes, or change them into one of thoſe which I have propoſed. But, I am afraid, this is not to be expected. The neglect with which they have received ſome remonſtrances that have been already made to them, gives reaſon to fear, that what has been now ſaid will be in vain; and that thoſe who are to come after them, muſt be left to *rue* the conſequences of their miſtakes.

SUPPLEMENT *to the preceding Sections; containing a further Account of the Societies for the Benefit of Widows.*

Of the LONDON ANNUITY SOCIETY.

IN the firſt Section, the Reader has ſeen on what very incompetent plans the two Societies, which are the ſubject of it, have been formed. Some changes have taken place in them ſince the laſt edition of this Treatiſe, of which it is neceſſary I ſhould here take notice.

The LONDON ANNUITY Society, conſiſting in January 1781 of 326 members, has ſo far reformed its plan, as to be now in little danger. Beſides ordering a compenſation for difference of age between huſbands and wives, it determined, in 1774, not to engage

to

to pay a higher annuity than 20*l.* to widows if their huſbands had lived a year after admiſſion, for a contribution of 10 guineas in hand, and five guineas *per ann.* afterwards. At the ſame time, however, room was left for expecting that ſome additional annuities might be paid to the widows of ſuch members as ſhould ſurvive 15 years in the Society; but what the additions ſhould be, was left to be determined at the end of 15 years from the eſtabliſhment of the Society. Accordingly, laſt year ſome able judges were conſulted; and, if I am rightly informed, the reſult has been, that the Society has agreed to promiſe for the ſame contributions an addition to the 20*l.* annuity juſt mentioned, of 4*l.* *per ann.* to widows, if their huſbands have been members 15 years or more.

He that will conſider the demonſtrations in the Firſt Section of this Chapter, or compute agreeably to any table of the decrements of life by the Rule in Queſt. X. may aſſure himſelf, that a contribution of ten guineas in *immediate payment* and five guineas every year after the firſt, is ſcarcely a ſufficient ſupport for an annuity of 20*l.* during life to widows, ſuppoſing huſbands and wives of the ſame ages, and money improved at an intereſt of 4 *per cent.*—But money may now be improved at a higher intereſt. Some advantages alſo muſt be derived

derived from making the annuities payable for widowhood only; and on theſe accounts, ſuch a contribution may ſafely enough be reckoned a proper payment for an annuity of 20*l.* as it is offered by this Society. But it cannot, without danger, offer more; particularly, as it is certain, that the lives of women in general, and more eſpecially of women in the advanced ſtages of life, are more durable than the lives of males.

The additional annuity, however, not being of more value than about three guineas and a half in a ſingle payment at entrance, the Society may poſſibly find itſelf capable of paying it, *provided* the contributions for ſupporting the ſcheme (namely ten guineas at entrance and five guineas *per ann.* from every member, beſides a juſt compenſation for the exceſs of his age above that of his wife) are not loaded with any of the expences of management.

Further Account of the Laudable *Society for the Benefit of Widows.* See the Firſt Section.

THIS Society affords a melancholy proof of the pernicious tendency of that diſpoſition to form annuity ſocieties which prevailed ſome time ago.—In conſequence of a petition to Parliament in 1774, from

from many of the moſt reſpectable members, it *reformed* its plan; but no arguments could engage the majority of the members to conſent to a reformation which was likely to be attended with any other effect than an increaſe of calamity by poſtponing it. For thirteen years from the time of its eſtabliſhment, it had overlooked the differences of age between men and their wives, and gone upon the plan mentioned in page 64.—In 1774, a compenſation for the wife's inferiority of age was ordered to be paid by all *new* members, and at the ſame time the following plan agreed to:—For an annual payment of five guineas, the firſt to be made immediately, every widow was entitled to an annuity during widowhood of

10 *l.* if her huſband had been admitted		2 years and a day
15 *l.* if	—	— 3 years
20 *l.* if	—	— 6
25 *l.* if	—	— 8
30 *l.* if	—	— 11
35 *l.* if	—	— 13
40 *l.* if	—	— 15

Any one who will calculate by the rule in Queſt. VIII. will find that the annual payment neceſſary to ſupport theſe annuities is nearly, by Dr. *Halley*'s Table of Obſervations, 7 *l.* ſuppoſing equality of age between huſbands and wives; money improved at

at 4 *per cent.* and the mean age of admiſſion *thirty-ſeven.*——This change, therefore, did not deſerve the name of a *reformation*; and an attention to the following account will ſhew, that inſtead of doing good, it has in fact only prolonged the exiſtence of the Society to do miſchief.

From the eſtabliſhment of the Society in 1761 to 1772, it had increaſed to 700 members; but in April 1780, it had gradually ſunk, by deaths and deſertions, to 550.—The whole number of widows which had come on the Society was then 168, of whom 84 had come upon it in ſix years, from 1774 to 1779 (*a*); that is, fourteen annually. *Thirteen* had died and *fourteen* had married, which had left 141 annuitants, the claims of 133 of whom amounted then to 3,310*l. per ann.* The claims of the remaining eight, reckoned at 30*l. per ann.* each, will make 240*l. per ann.*; and the expences of management (or 300*l. per ann.* nearly), added to theſe ſums, will make the whole annual expence of the Society in April 1780, 3,850*l.*——Its income, conſiſting of the intereſt of 49,090*l.* three *per cent.* ſtock, and the ſubſcriptions of 550 members at five guineas each, amounted at the ſame time to 4,360*l.* leaving a favourable balance of only 480*l.*—

(*a*) In 1780 fourteen more widows came upon the Society.

Suppoſing

Suppoſing the Society to preſerve its preſent number of members, and the number of annuitants to increaſe for ſix years to come only at the rate of ten annually, its expence at the end of the preſent year, or the beginning of the next, will be equal to its income; and afterwards it will find itſelf under the neceſſity of having recourſe to one of the three following expedients. It will be obliged either to run into its capital, or to increaſe its contributions, or reduce the annuities.—The conſequence of the firſt of theſe expedients will be, that the capital of the Society will be ſoon conſumed, and the annuitants left without ſupport.—The conſequence of the *ſecond* will be, that the contributions will be increaſing every year till, in 10 or 12 years, they are *doubled*, and at laſt almoſt *tripled*.—The conſequence of the *laſt* will be, that the annuities will ſink every year till they come to be leſs than half the annuities promiſed (*a*). Such are the affairs of this infatuated Society, nor is it eaſy to apply any remedy to them; for in conſequence of going on too long with an inſufficient ſcheme, the Society has large payments to make in order to compenſate paſt deficiencies; and a ſcheme at firſt adequate would now prove

(*a*) The annuitants (ſhould the number of members continue what it is) cannot increaſe to much leſs than double their number laſt year.

inadequate.

inadequate.—For example. (Suppoſing a juſt allowance required for the wife's inferiority of age) an annual payment of ſeven guineas from every member, begun in 1772, when the number of widows was only 42, would probably have been ſufficient to ſupport the reformed ſcheme mentioned in the laſt page; but *now* an annual payment of nine guineas from the *preſent* members, and of ſeven from all *future* members, would ſcarcely be ſufficient.

In ſuch circumſtances it ſeems beſt to break up, and to divide the preſent capital, as far as it will go, among the annuitants. Should this be done, the annuitants will indeed be great ſufferers; for, ſo miſerably circumſtanced is the Society, that its whole ſtock will not pay much more than *half* (*a*) the value of the annuities. But this only ſets in a ſtronger light the neceſſity of an *immediate* diſſolution: I ſay *immediate*; for the annuitants are increaſing faſt, together with the medium of the annuities due to them; and therefore the conſequence of delay muſt be, extending *greater* ſufferings to *greater* numbers.

(*a*) The mean age of the widows now on the Society is not probably more than 42 or 43. The value of annuities payable half-yearly during the lives of women at this age, is not really ſo little as 13 years purchaſe, reckoning intereſt at 4 *per cent.* Suppoſing therefore the Society to break up ſome time or other before the end of this year (1781), and the number of widows then upon it 154, the value of their annuities, at 350*l.* each, will be 53,900*l.* of which their ſtock, at its preſent price, will not pay ſo much as 28,000*l.*

POSTSCRIPT.

SINCE the preceding account was written, this Society, convinced at laſt of its miſtakes, has reſolved to reduce the annuities of ſuch widows as became claimants before the alteration in 1774, 35 *per cent.* and the annuities of all the other widows who are now claimants, 20 *per cent.* It was alſo reſolved by two general courts, that the annual payments ſhould be increaſed from five to ſix guineas. But this reſolution was revoked by the general court in July laſt; and a reſolution ſubſtituted in its room to change the plan deſcribed in p. 110, into one which makes it neceſſary that a member ſhould have been admitted 3 years to entitle his widow to an annuity of 10*l.* and 7 years to entitle her to 20*l.*; and 13 and 20 years to entitle her to 30*l.* and 40*l.*—— Poſſibly this reſolution, like the former, will be retracted by ſome future general court. Should it be confirmed, it will, in conjunction with the reduction juſt mentioned of the annuities payable to preſent claimants, conſtitute a reformation which (ſuppoſing the funds of the Society not incumbered with any expences of management) might have been *nearly* ſufficient *ten or twelve years* ago to ſave the Society. But it is far from being ſufficient *now*——The reduction of the annuities payable to preſent claimants is too little. It ſhould have been extended to the

the widows of the old members now living, and the new plan reſtricted to members *lately* admitted, and to *future* members; and a compenſation for paſt over-payments to widows ſhould have been provided.

Further Account of the Aſſociation *among the* London *and* Middleſex *Clergy; and of the* Eſtabliſhment *among the* Miniſters *and* Profeſſors *in* Scotland.

THE Clergy of London and Middleſex agreed, in 1775, to new rules and orders, by which ſuch fines were required (on account of a ſubſcriber's exceeding the age of 40, and being older than his wife) and ſuch reductions made in the annuities as would probably have rendered the contributions adequate to the expences of the aſſociation. But the event has been, that in conſequence of this neceſſary reformation, the Aſſociation has dwindled, and is now ſunk ſo low as not to be likely to ſubſiſt much longer.

On the contrary. The Eſtabliſhment among the miniſters and profeſſors in Scotland has proſpered to a degree which gives reaſon to believe that it cannot fail to anſwer the hopes of the venerable body intereſted in it. This has been owing chiefly to the great ability and faithful zeal of the Rev. Dr. Webſter, its founder and conductor.—To the account already given of it in the Second

Section, I will take this opportunity to add the following particulars.

Dr. Webſter (having, when the plan was firſt formed 38 years ago, no certain *data* to go upon) aſſumed 52 as the medium age at which the widows of miniſters would commence annuitants. By calculating on this ſuppoſition, and taking the chances of life as they are in Dr. *Halley*'s Table, he found that the number of annuitants on the ſcheme at Lady-day 1780 would be 310. —The fact is, that they were then 304; and that conſequently there was, even in this way of calculating, a difference of *ſix* in favour of the funds which ſupport the ſcheme. ——Since the eſtabliſhment of the ſcheme it has been diſcovered, that the medium age juſt mentioned does not probably exceed 47. Dr. Webſter, therefore, ſome years ago, in order to put the ſcheme to a ſeverer trial, inſtituted a new calculation, on the ſuppoſition that the medium age is no more than 44, and found that on this ſuppoſition the number of annuitants at Whitſuntide 1780 would be 328. This has made a ſtill greater difference in favour of the Eſtabliſhment, and gives a very encouraging proſpect of its ſtability; a ſufficient income having been in reality provided for bearing the expences, had the annuitants increaſed as in this laſt calculation.

Had 52 been the mean age of the widows when they commence ſuch, the *maximum* of widows

widows living at one time derived from 20 left annually, would be 334, according to Dr. *Halley*'s Table; but ſuppoſing it no more than 44, this *maximum* would exceed 400; and the enquiries which have been made, give reaſon to expect that it will not fall much ſhort of this number. Dr. Webſter, therefore, has in his laſt calculations, reckoned upon the increaſe of the annuitants to this number; and for this reaſon, and to ſecure more certainly the Eſtabliſhment, a new act of parliament was procured in 1779, by which, among other new proviſions, it was ordered, that the increaſe of the capital (then amounting to 75,088*l.*) ſhould not be diſcontinued till it roſe to 100,000*l.*——This capital, joined to the annual contributions, will probably be an ample ſupport to the Eſtabliſhment, ſhould the number of annuitants (which will go on to increaſe for near forty years more) become at laſt 400.—— Circumſpection and caution, however, continue to be neceſſary, becauſe ſtill unfavourable events may ariſe, which no human wiſdom can foreſee.

Having beſtowed a good deal of attention on this inſtitution, I cannot take leave of it without congratulating Dr. Webſter on his happineſs. By being the founder of this ſcheme, and by the care with which he has watched its progreſs, and conducted it to its preſent ſtate of maturity, he has entitled himſelf to the bleſſings of many indigent

widows and orphans, and made it impoſſible that he ſhould be ever remembered in the church of Scotland without gratitude and reſpect.

It is much to be wiſhed that inſtitutions of the ſame kind, could be eſtabliſhed in *England*. Some efforts have been made to this purpoſe. The reverend and ingenious Mr. *Gandy* of Plymouth, having with much labour and ability prepared a plan of this kind, endeavoured in 1774 and 1775 to get it eſtabliſhed in the dioceſs of *Exeter*. Had he ſucceeded, the benefits ariſing from it would have become in a little time very conſpicuous, and an example would have been given which would perhaps have been followed in other dioceſes. But he did not meet with ſufficient encouragement, and the ſcheme was given up.—Being unwilling that the time and pains which were employed in digeſting and calculating the tables for this ſcheme ſhould be entirely loſt; I have inſerted ſome of the principal of them among the Tables in the next volume.——The Rev. Mr. *Grant*, of *Henly* upon *Thames*, has been alſo lately engaged in ſoliciting encouragement to a ſimilar ſcheme; and I heartily wiſh him the ſucceſs his benevolence and abilities deſerve.

Account

Account of a Scheme eſtabliſhed among the Eaſt-India Commanders.

THE *Eaſt-India Commanders* ſix years ago entered into an aſſociation for the purpoſe of providing for their *nominees*, and did me the honour to deſire I would recommend a ſcheme to them. They approved the following, and have adopted it.——Every member is entitled to 500*l.* payable at his death to his *nominee*, in conſideration of 50*l.* at his admiſſion, and *eleven annual payments* of 25*l.* the firſt to be made at the beginning of the ſecond year, and the right to every payment to depend on the continuance of the life of the ſubſcriber. No ſubſcribers muſt be admitted whoſe ages are not leſs than 50; and their mean ages are reckoned at 40.

In calculating this ſcheme, intereſt was reckoned at $3\frac{1}{2}$ *per cent.* (*a*); but a much

(*a*) The value, by Table VI. in the next volume, reckoning intereſt at $3\frac{1}{2}$ *per cent.* of an annuity of eleven years, on a life aged 40, firſt payment to be made at the end of a year, is by Queſt. VI. Chap. I. 7.895.—The annuity, therefore, being 25*l.* its value is 197*l.* to which 50*l.* (the firſt payment) added makes 247*l.*; which is alſo, by Queſt. X. the value, reckoning the ſame intereſt, of 500*l.* payable at the death of a perſon aged 40.—The ſame contributions, ſuppoſing money improved at 5 *per cent.* would entitle a nominee to 600*l.*

higher improvement has for ſome years been made of money, and the ſcheme has eſcaped the danger of being too much loaded in its infancy. I am ſatisfied, therefore, that, without altering the contributions, the ſum payable to *nominees* may with perfect ſafety be increaſed to 550*l.* Indeed, I ſhould be in no pain were it even advanced to 600*l.* provided only the contributions were made in *guineas*, inſtead of *pounds*, and all ſavings made to accumulate in the ſhort annuity for 27 years from Chriſtmas laſt.——The progreſs of ſuch an aſſociation will be as follows.—Suppoſe it to conſiſt of 46 members, kept up from year to year by admitting, as old members die off, new ones at the mean age of 40. At firſt, according to Mr. *De Moivre*'s hypotheſis, only *one* member will die annually; but, after a certain period, *two* will die annually. During this interval there will be ſavings which will raiſe a ſtock, the intereſt of which, when added to the annual contributions, will be juſt ſufficient, when two members come to die annually, to pay two claims.—Suppoſing each claim 550*l.* the expence of the aſſociation, *when greateſt*, will be 1100*l. per ann.* The contributions at that period will be firſt, 50*l.* each (or 100*l.* in all) from *two* new members admitted every year; and 25*l.* each

(or

(or 450*l.* in all) from 18 * *other* members who had not been admitted more than eleven years. The remainder (or 550*l.*) neceſſary to make up 1100*l. per ann.* will be the intereſt of the capital, which, therefore, if lodged in the *three per cents*, muſt be 18,333*l.* When, therefore, any ſuch aſſociation conſiſting of 46 members has raiſed this ſtock; or, if it conſiſts of any other number of members, when it has raiſed a ſtock in the ſame proportion to it, that the ſtanding number of members bears to 46, it will become a reaſonable object of conſideration whether the increaſe of its ſtock ſhould not be diſcontinued, and all ſubſequent ſavings, ſhould any ariſe, be employed in either leſſening the contributions or increaſing the claims.

SECT. IV.

Account of ſome foreign Inſtitutions for the Benefit of Widows.

IN the Preface to the firſt edition of this Treatiſe, I took notice of an inſtitution for the ſale of annuities payable on

* Out of a body of new members derived from two admitted every year at 40 for 11 years, it may be expected that two will die before the end of the 11th year.

ſurvivor-

ſurvivorſhip, eſtabliſhed at *Amſterdam*, which ſeemed to be then much encouraged, and into which, I had been informed, many had entered from different parts of *Europe*. This was ſo wretched a deception that it was impoſſible it ſhould long ſtand its ground; and I am told that it now exiſts no more. I have, therefore, expunged the notice I took of it in that Preface; and I will not here give any further account of it.

In 1739, an inſtitution was eſtabliſhed in *Denmark* under the patronage and guaranty-ſhip of the King of Denmark, which, without regarding ages, promiſed penſions to widows at the rate of 40 rixdollars *per ann.* for life, from the commencement of widowhood, for every preſent payment of 110 rixdollars. This being leſs than the true value of ſuch penſions, the fate of this ſcheme has been the ſame with that of the *Mercers* Company mentioned in page 104. At the end of the year 1778, its whole fund was exhauſted, and the King of Denmark found himſelf burdened with the ſupport of 700 widows, and an obligation to ſupport as many more as would be derived from 1500 marriages then remaining undiſſolved.

At *Bremen*, an inſtitution was eſtabliſhed in 1760, which promiſed annuities to widows for a payment at admiſſion of a ſum equal to one yearly payment of the annuity purchaſed, and an annual contribution during

ing marriage of 15 *per cent.* (or a little more than a 7th) of the annuity. Theſe payments are not much more than half the proper compenſation for the annuities. The conductors of the ſcheme have therefore been obliged to reduce the annuities 10 *per cent.*; and they will ſoon be obliged to reduce them much more.

The ſtates of the dutchy of *Calenberg*, of which *Hanover* is the capital, eſtabliſhed in 1767 a like ſcheme, but on terms ſtill more deficient; for, though it differed from the two former ſchemes in paying a regard to the ages of married perſons, yet, notwithſtanding ſeveral augmentations, the contributions required by it did not two years ago come up to half the value of the annuities. Great numbers, influenced probably by the lowneſs of the terms and the authority of the ſtates, have been induced to encourage this inſtitution. In 1779, it had annuities to pay to 600 widows, and conſiſted of no leſs than 3800 members or ſubſcribers whoſe widows would be entitled to annuities. In conſequence of a rapid increaſe, its inſufficiency was not then become palpable enough to force either a diſſolution, or a timely and effectual reformation. It was, therefore, likely to lay the foundation of great confuſion and diſtreſs.

There are probably many other foreign bubbles of this kind, of which I have no knowledge.

knowledge. The information which has enabled me to give this account I owe to Mr. OEDER of *Oldenburg*; and it is with particular ſatisfaction, that I can from him on this occaſion add an account of one foreign inſtitution for the benefit of widows which is founded on juſt principles, and likely to be productive of great good.—The plan of this inſtitution has been formed and the calculations for ſettling its terms have been made by Mr. *Oeder*, who appears indeed to poſſeſs an acquaintance with this ſubject ſo extenſive and correct as to be perfectly qualified for ſuch an office.——This inſtitution is intended only for the benefit of the inhabitants of the dioceſs of *Lubec* and the dutchy of *Oldenburg*; and the ſovereign of this ſtate has himſelf given it the ſanction of a ſtatute, and guarantied to his ſubjects the advantages it promiſes. At prices deduced by calculations at 4 *per cent.* from Mr. *Suſmilch's* Table of Obſervations (See the Tables in the next volume), and agreeing nearly with the prices deduced by the rules in Queſt. I. and IV. from the *Northampton* Table (or Table VI. in the next volume), it offers to a married man any annuity for his widow not exceeding 500 rixdollars, (or about 88*l. per ann.*) (*a*) payable for life; but with a power reſerved to the huſband of directing that it ſhall be applied to the ſupport

(*a*) Six German rixdollars make about a guinea.

of

of his children in caſe his widow ſhould marry.

This inſtitution farther enables a parent to provide for his children annuities, (not exceeding 500 rixdollars) payable to them in the event of their ſurvivorſhip, till they are 25 years of age.

For example. To a huſband, aged 35, this inſtitution promiſes a life annuity of 10*l.* payable to his widow, for either an *annual* payment of 2*l.* 15*s.*—3*l.* 1*s.*—3*l.* 7*s.* 4*d.*, &c. or a *ſingle preſent* payment of 31*l.* 7*s.* 3*d.*—36*l.* 6*s.*—41*l.* 6*s.* &c. according as he is of the *ſame* age with his wife, or 5, 10, &c. years *older:*

And if he wiſhes to make a proviſion for any of his children, provided he ſhould leave them orphans under age, he may purchaſe annuities payable to them from the time they ſhall happen to ſurvive till they are 25 years of age, at the rate of an annuity of 10*l.* for every *annual* payment during the joint lives, of 1*l.* 17*s.* 4*d.*—1*l.* 14*s.*—1*l.* 7*s.* 4*d.* or a *ſingle* payment of 19*l.* 14*s.*—18*l.* 12*s.* 9*d.*—12*l.* 7*s.* according as the child's age is two, five or ten (*a*) years.—

(*a*) It is obvious that the values in this caſe for every annuity of 1*l.* are, in a *ſingle payment*, the *exceſs* of the value of the life of the child for as many years as his age is leſs than 25, above the value of the joint lives for the ſame time found by the *Scholium* to Queſt. VI.; and, in *annual payments* beginning immediately, the *Quotient* ariſing from dividing the ſingle payment by the value juſt mentioned of the joint lives, with unity added.

Theſe values are greater or leſs as the age of the parent is greater or leſs; and all the prices of ſuch annuities, and alſo of annuities for widows, are ſpecified in Tables, for all ages and all *differences* of ages.

The ſufficiency of the receipts to anſwer the expences in this inſtitution, as far as it provides annuities for widows, has been proved by Mr. *Oeder* in the cleareſt manner from accounts which have been collected in the dutchy of *Oldenburgh* of the duration of 1273 marriages, and compared with accounts of the duration of the widowhoods derived from theſe marriages.——One circumſtance in theſe accounts deſerves particular notice.

The ages of the men, one with another, when the marriages juſt mentioned commenced, was 32½ years; of the women 28 years. The men lived after marrying 27¾ years; the women, 31 years and nine tenths. The former, according to Mr. *Suſmilch*'s Table (and alſo nearly according to the *Northampton* Table of Obſervations), ſhould have lived only 27 years and one tenth; and the latter, 29 years and eight tenths. The former, therefore, having exceeded the duration of life exhibited in the Tables only *ſix tenths of a year*, but the latter having exceeded it above *two years*, it follows that women, notwithſtanding the hazards of the critical periods and of child-bearing, live longer than men.

I will

I will add, that by examining 154 of theſe marriages, I find their duration to have been, one with another, 21 years and a quarter, and the duration of the ſurvivorſhip of the widows derived from them, 19 years. Had Mr. *De Moivre*'s hypotheſis of an equal decrement of life been juſt, the latter would have been *longer* than the former. The reaſon why the contrary happens has been given in p. 95; &c.

In theſe marriages (if I may judge from examining only 140 of them) *four* widows were left to *three* widowers, which ſhews a chance of ſurvivorſhip in favour of the wife in marriage, greater than could have taken place, had their been no other reaſon for it than inferiority of age.

At *Hamburgh*, an annuity ſcheme has been lately eſtabliſhed of a more comprehenſive nature than any of the ſchemes which have been hitherto mentioned; but the account of it will be more properly given at the concluſion of the laſt Section of this Chapter.

SECT. V.

Of Schemes for providing Annuities for Old Age.

A GENERAL diſpoſition has lately ſhewn itſelf, to encourage ſchemes for granting *annuities* to perſons in the latter ſtages

of

of life; and this has occaſioned the 6th Queſtion in the former chapter; and, as a further and more particular direction in caſes of this kind, I have thought it neceſſary here to give the following Table.

Values of 1*l.* *per ann.* for life, after 50, to perſons whoſe ages are	Values in *one preſent payment*, intereſt 4 *per cent.*	Intereſt 3 *per cent.*	Values in *annual payments*, till 50, to begin at the end of a year, intereſt 4 *per ct.*	Intereſt 3 *per cent.*
10	1.235	2.015	.0789	.113
15	1.583	2.444	.106	.146
20	2.028	2.989	.146	.193
25	2.594	3.644	.203	.259
30	3.369	4.508	.297	.366
35	4.446	5.667	.466	.559
40	5.953	7.232	.822	.950
Values of the ſame annuity, after 55, to ages			Values in annual payments till 55.	
30	2.114	2.937	.197	.211
35	2.722	3.632	.241	.297
40	3.732	4.708	.394	.464
45	5.088	6.115	.703	.803
Values of the ſame annuity, after 60, to ages			Values in annual payments till 60.	
35	1.667	2.290	.135	.168
40	2.234	2.923	.203	.245
45	3.043	3.811	.327	.384
50	4.255	5.061	.600	.679

The numbers in the 2d and 3d columns of this Table, multiplied by any annuity, will give the value of that annuity in a *ſingle* payment,

payment, to be enjoyed for life, by the ages correſponding to thoſe numbers in the firſt column, *after* the age at the head of that column.—And in the ſame manner; the numbers in the 4th and 5th columns will give the values in *annual* payments.—Thus: The value of 44*l.* *per annum*, to be enjoyed for life, after 50, by a perſon now 40, (intereſt at 4 *per cent.*) 5.95, multiplied by 44, or *l.* 261.9, in a *ſingle* payment; and .822, multiplied by 44, or *l.* 36.16, in *annual* payments till 50, the firſt payment to be made at the end of a year.

In order to find the ſame values, partly in *annual payments*, and partly in any given *entrance* or *admiſſion money*; ſay, "As the va-
" lue of the *given annuity* in a *ſingle* payment,
" (found in the way juſt mentioned) is to the
" given *entrance money*; ſo is its value in *an-*
" *nual* payments, to a fourth proportional;
" which, ſubtracted from the value in *annual*
" *payments*, the *remainder* will be the annual
" payment due, over and above the given
" entrance money."

EXAMPLE.

Suppoſe a perſon now 40, to be willing to pay 200*l.* entrance-money, *beſides* ſuch an annual payment for 10 years as ſhall, together with his entrance-money, be ſufficient to entitle him to a life-annuity of 44*l.* after 50. What ought the annual payment to be?

ANSWER.

*L.*8.55.—For, *l.*261.9, is to 200*l.* as *l.*36.16, to *l.*27.61; which, ſubtracted from *l.*36.16, the remainder is *l.*8.55.

This Table has been calculated from the *probabilities* of living in Table V. at the beginning of the next volume, and Mr. *De Moivre's* valuation of lives.—The probabilities of life among the inhabitants of *London*, are (as I have often had occaſion to obſerve) much lower than among the generality of mankind; and the values in the preceding Table, had they been given agreeably to the *London* Obſervations, would have been leſs. But, certainly, an office or ſociety, that means to be a permanent advantage to the public, ought always to take higher rather than lower values, for the ſake of rendering itſelf more ſecure, and gaining ſome *profits* to balance *loſſes* and *expences*.

There have lately been eſtabliſhed, in *London*, ſeveral ſocieties for granting ſuch annuities as thoſe now mentioned; and he that will compare their true values, as they may be learnt from the preceding Table, with the *terms* of admiſſion into theſe ſocieties, as given in their printed *Abſtracts* and *Tables*, muſt be ſurpriſed and ſhocked. They

are all impoſitions on the public, proceeding from ignorance, and encouraged by credulity and folly.

It has been ſhewn; that the proper payment (allowing compound intereſt at 4 *per cent.*) for an annuity of 44*l.* to be enjoyed by a perſon now 40, for what may happen to remain of his life after 50, is 200*l.* in *admiſſion-money*; beſides *l.* 8.55, or 8*l.* 11*s.* in annual payments till he attains 50, the firſt of theſe payments to be made at the end of a year.—The conditions of obtaining this annuity, according to the preſent Tables of the *Laudable Society of Annuitants* (*a*) *for the Benefit of Age*, conſiſting of about 1300 members, are (*a*) 76*l.* 17*s.* in *admiſſion-money*; and 6*l.* 14*s.* in *annual payments*.—According to the Tables of the ſociety of *London Annuitants for the Benefit of Age*, the conditions of obtaining the ſame annuity are 30*l.* in *admiſſion-money*, and 10*l.* in *annual payments*.—The *Equitable* Society of Annuitants requires for the ſame annuity 38*l.* 10*s.* in *admiſſion-money*, and 13*l.* in *annual* payments. The true value is, over and above the *admiſſion-money* juſt mentioned, an *annual payment* of 30*l.* 17*s.* (intereſt reckoned at 4 *per cent.*) or an *annual payment* of 36*l.* 15*s.* (intereſt reckoned at 3 *per cent.*)—The *London Union Society for the*

(*a*) The firſt members of this Society have paid no admiſſion-money; and are now expecting 44*l.* *per ann.* for contributions which do not entitle them, one with another, to 10*l.* *per ann.*

comfortable ſupport of aged members promiſes an annuity of no leſs than 50 guineas for life, after 50, to a perſon now 40, for 40*l.* 10*s.* admiſſion-money, and 7*l.* in annual payments.

The *Amicable Society of Annuitants for the benefit of age*, promiſes an annuity of 26*l.* *per annum*, for life, to a perſon now 40, after attaining 50, for 28*l.* 16*s.* in *admiſſion-money*, and 6*l.* in *annual payments*.—The true value of this annuity is 28*l.* 16*s.* in *admiſſion-money*, and 17*l.* 8*s.* in *annual payments*, (intereſt ſuppoſed at 4 *per cent.*); or the ſame ſum in *admiſſion-money*, and 20*l.* 18*s.* in *annual payments*, intereſt ſuppoſed at 3 *per cent.*

The *Provident Society for the benefit of age*, conſiſting of 1280 members, promiſes an annuity of 25*l.* to a perſon now 40, after the age of 50, for 34 guineas in admiſſion-money, and eight guineas in annual payments. The true value is, 34 guineas in *admiſſion-money*, and 15*l.* 12*s.* in *annual* payments, intereſt at 4 *per cent.*; or, the ſame ſum in *admiſſion-money*, and 19*l.* in *annual* payments, intereſt being at 3 *per cent.* (*a*).

But I will not tire the reader, by going, in this manner, through the ſchemes of all theſe

(*a*) The account here given of the terms on which a perſon whoſe age is 40, is admitted into theſe ſocieties, I have taken from their printed Tables as they ſtood at the end of the year 1770.—In the younger ages, the deficiencies are greater.

ſo[illegible]s.

ſocieties. The contrivers of them, it is certain, can know nothing of the principles on which the rule in Queſt. VI. and the demonſtration of it in Note (B) at the end of the next volume, is founded; and, therefore, if unwilling to be guided by the authority of mathematicians, it may not be poſſible to convince them of their miſtakes. I will however, offer to them the following demonſtration, which will be underſtood without difficulty, by every one who knows how to compute the increaſe of money at compound intereſt.

The value of a life at 50, (intereſt being at 4 *per cent.*) is $11\frac{1}{3}$ years purchaſe by the firſt Table in the laſt leaves of the next volume. For an annuity, therefore, of 44*l.* *per annum* for life, to be enjoyed by a perſon at this age, 498*l.* ought to be given. *Two* in *three* of a number of perſons at the age of 32 will (by Tables V, VI, and VII, at the beginning of the next volume) live to 50; and therefore, in order to be able to pay an annuity to them of 44*l.* for life, after 50, the money now advanced by every *three*, ought to be ſuch as will, in conſequence of being laid up to be improved, increaſe in 18 years to double 498*l.* that is, to 996*l.*—From the preceding Table it may be learnt, that the money which ought to be advanced by every ſingle perſon is 165*l.* or by *three* perſons 495*l.* and this, in 18 years, will (as may be learnt from Table III. in the next volume) double

itſelf, or increaſe to juſt the ſum that will then be the value of the annuities to be paid.—But the money required in this caſe by the *Laudable Society*, is 14*l.* 11*s.* 9*d.* from each member at admiſſion, beſides an *annual* payment of 4*l.* The admiſſion-money, therefore, of two members, being 29*l.* 3*s.* 6*d.* may be increaſed to twice this ſum, or to 58*l.* 7*s.* An annual payment of 4*l.* for 18 years will, if perfectly improved at 4 *per cent.* compound intereſt, increaſe to 102*l.*; and two ſuch annual payments will increaſe to 204*l.* as may be learnt from Table IV. in the next volume.

The whole pay, therefore, of *two* members will produce at the end of 18 years 262*l.* 7*s.*—A third part, I have ſaid, will die before 50 years of age, and theſe will live one with another 9 years. An annuity of 4*l.* for this time, will produce a capital of 42*l.* 6*s.* See Table IV. in the next volume; and this capital improved for nine years more will increaſe to 60*l.* The whole profit, therefore, from the member who will die is, his admiſſion-money doubled and added to 60*l.* or 89*l.* 3*s.* 6*d.* And this ſum added to 262*l.* 7*s.* makes 351*l.* 10*s.* 6*d.* the *whole* money with which the ſociety can be provided, at the end of 18 years, to bear the expence of *two* life-annuities, worth together 996*l.*

By a ſimilar computation it may be [illegible], that the improvement of money at only [illegible] per cent.

cent. will *ſink* the *former* ſum to 324*l.* at the ſame time that the values of the *annuities* will be *raiſed* to 1100*l.*

The deficiencies in the ſchemes of moſt of the other ſocieties, are no leſs conſiderable.—What confuſion then muſt they produce ſome time or other? How barbarous is it thus to draw money from the public by promiſes of advantages that *cannot* be obtained? Have we not already ſuffered too much by *bubbles?*

I have ſaid, that theſe ſocieties are "impoſitions on the public, proceeding from "ignorance, and ſupported by credulity and "folly." But this is too gentle a cenſure. There is reaſon to believe, that worſe principles have contributed to their riſe and ſupport. The preſent members, conſiſting chiefly of perſons in the more advanced ages, who have been admitted on the eaſieſt terms, believe that the ſchemes they are ſupporting will laſt *their* time, and that *they* will be gainers. And as to the injury that may be done to their *ſucceſſors*, or to *younger members*, it is at a diſtance, and they care little about it. Agreeably to this principle, the founders of theſe ſocieties begin ſo low as not to require perhaps a *fourth* or a *fifth* of the values of the annuities they promiſe. Afterwards they advance gradually, juſt as if they imagined, that the value of the annuities was nothing determinate, but increaſed with every increaſe of the ſociety. But, as no

ignorance can believe this, the true deſign appears to be, to form ſoon as large a ſociety as poſſible, by leading the unwary to endeavour to be *foremoſt* in their applications, leſt the advantage of getting in, on the eaſieſt terms, ſhould be loſt.—It is well known, that theſe arts have ſucceeded wonderfully; and that, in conſequence of them, theſe ſocieties now conſiſt of perſons who, for the *ſame* annuities, make higher or lower payments according to the time when they have been admitted; and the generality of whom, therefore, muſt know, that either more than the values have been required of the members laſt admitted; or if not, that they are themſelves expecting conſiderable annuities, for which they have given no valuable conſideration, and which, if paid them, muſt be *ſtolen* from the pockets of ſome of their fellow-members (*a*).

(*a*) If any perſon wants more information than I have given him concerning theſe ſocieties, he ſhould conſult a work of great merit, publiſhed ſince the ſecond edition of this treatiſe, and entitled, CALCULATIONS *deduced from firſt Principles, in the moſt familiar Manner, by plain Arithmetic, for the Uſe of the Societies inſtituted for the Benefit of old Age; intended as an Introduction to the Study of the Doctrine of Annuities.* By Mr. *Dale*, a Member of one of the Societies. In this Treatiſe there is not only a very ample account given of the inſufficiency and iniquity of the ſchemes of theſe Societies, but the principles on which the values of all annuities on ſingle lives are determined, and the method of calculating them, are explained with the greateſt clearneſs.

I do not, however, mean to condemn all inſtitutions of this kind. They may be very uſeful, if the full values are taken, and proper care is uſed in the *improvement* of money. Intereſt, in theſe caſes, ought not to be reckoned higher than 3 *per cent.* and, ſuppoſing money improved at this rate, a perſon, for a ſingle payment of 50*l.* before he is 40, might be entitled to a life-annuity of 10 guineas *after* 55; or, if he chuſes it, to a life-annuity of 17*l.* after 60. But if he pays the ſame ſum before he is 34, he might be entitled to a life-annuity of 14*l.* *after* 55, or 22*l.* *after* 60. 25*l.* would purchaſe for him *half* theſe annuities; and 100*l.* *double*.

A ſociety or office that would go on this plan, might do great ſervice. Perſons in the lower ſtations of life might be brought to a habit of induſtry, in the beginning of life, by ſtriving to get 25*l.* or 50*l.* beforehand in order to purchaſe ſuch annuities, and thus to make proviſions for themſelves in the more advanced parts of life, when they will be incapable of labour (*a*).

It

(*a*) The benevolent Mr. *Howard*, in his *State* of the Priſons in *England* and *Wales*, p. 60. Octavo Edit. gives the following account of an inſtitution at *Harlem* in *Holland*. "In this city," he ſays, "there is a noble hoſpital, "airy and ſpacious, called the *Proeveniers*, in which the "perſons admitted are decently provided with meat, "drink, and lodging, during their lives; and a burying-"place at their deceaſe. Perſons of all ages, from 20 "to

It is proper to obſerve here, that inſtitutions of this kind would furniſh one of the *ſafeſt* ways of providing for widows.—A married man might, by paying 100*l.* before his wife attained 40, intitle her, after 55, or 60, to a life-annuity of 21*l.* or 34*l.* Or, by paying the ſame ſum before ſhe attained 34, he might intitle her, after the ſame ages, to a life annuity of 28*l.* or 44*l.* (*a*); and in this caſe he would have a chance of ſharing himſelf in the benefit of the annuity.

I have called this the *ſafeſt* way of providing for widows, becauſe attended with none of the dangers ariſing from diſproportion of age between men and their wives, and from the admiſſion of perſons labouring under concealed diſtempers.

" to the moſt advanced periods, are admitted, though it " is not common for any under 40 to apply for admiſ- " ſion. At entrance each perſon pays a greater or ſmaller " ſum, according to his age. If his age is 30, he pays " 4500; if 40, he pays 3900; if 50, he pays 3300; if " 60, 70, &c. he pays 2700, 2100, &c. florins; and " in proportion to theſe ſums, at all the intermediate " ages.—A common table is provided for all that are not " ſick or infirm.—It often happens that there are not va- " cant places when perſons deſire admiſſion. But many " ſecure places before a vacancy, by paying 200 florins in " advance of their entrance-money, in conſequence of " which their names are inſcribed in a liſt, and the " money deducted from the ſum required when they " enter.—Thoſe who chuſe to leave the houſe, receive, " during the remainder of their lives, a certain but ſmall " intereſt for the ſum which they paid at admiſſion."

(*a*) The ſame payment before 30, would entitle to an annuity of 22*l.* after 50.

I cannot

I cannot conclude this Section, without mentioning the following plan of a proviſion for Old Age.

Let 13 guineas be given as *entrance-money*; and let beſides 1*l.* 2*l.* 3*l.* 4*l.* &c. be given at the beginning of the 1ſt, 2d, 3d, 4th, &c. years, as the payments for theſe years reſpectively; and let the laſt payment be 16*l.* at the beginning of the 16th year. All theſe payments put together will, according to the probabilities of life in the 5th or 6th Tables in the next volume, (intereſt being at 4 *per cent.*) entitle a perſon, whoſe age was 40 when he begun them, to an annuity, after 15 years, beginning with 15*l.* and increaſing at the rate of 1*l.* every year, till at the end of 15 years, or (*a*) when he has reached to 70, it becomes a ſtanding annuity of 30*l.* for the remainder of his life.

If the addition of three guineas is made to the *entrance-money*, for every year that any life between 30 and 40 falls ſhort of 40, the value will be obtained nearly, of the ſame annuity to be enjoyed by that life, after the ſame number of years, and increaſing in the ſame manner, till it becomes *ſtationary* and *double*.——This plan is particularly inviting, as it makes the *largeſt* payments

(*a*) According to the probabilities of life in the *London* Table, this annuity ſhould be greater.—A *Theorem* for finding what the annuity ought to be in theſe caſes is given in Note (I), at the end of the next volume.

become

become due, when the *near* approach of the annuity renders the encouragement to them *greateſt*; and as, likewiſe, the annuity is to increaſe continually with age, till it comes to be higheſt (*a*), when life is moſt in the decline,

(*a*) The lower part of mankind are objects of particular compaſſion, when rendered incapable, by accident, ſickneſs, or age, of earning their ſubſiſtence. This has given riſe to many very uſeful ſocieties among them, for granting relief to one another, out of little funds ſupplied by *weekly* contributions. A ſociety of this kind, formed on the following plan, would probably thrive.

Let the ſociety, as its firſt eſtabliſhment, conſiſt of 100 perſons, all between 30 and 40; and whoſe mean age may therefore be reckoned 36; and let it be ſuppoſed to be always kept up to this number, by the admiſſion of new members, between the ages of 30 and 40, as old members die off. Let the contribution of each member be four-pence *per* week, making, from the whole body, an annual contribution of 85 *l.* 17 *s.*—Let it be further ſuppoſed, that ſeven of them will fall every year into diſorders, that ſhall incapacitate them for ſeven weeks.—30 *l.* 12 *s.* of the annual contribution will be juſt ſufficient to enable the ſociety to grant to each of theſe 12 *s.* *per* week, during their illneſſes. And the remaining 55 *l.* *per annum*, laid up and *carefully* improved at 3½ *per cent.* will increaſe to a capital that ſhall be ſufficient, according to the chances of life in Tables V, VI, and VII. in the next volume, to enable the ſociety to pay to every member, *after* 67 years of age, or *upon* entering his 68th year, an annuity, beginning with 5 *l.* and increaſing at the rate of 1 *l.* every year for ſeven years, till, at the age of 75, it came to be a ſtanding annuity of 12 *l.* for the remainder of life.

Were ſuch a ſociety to make its contribution *ſeven-pence per* week, an allowance of 15 *s.* might be made, on the ſame ſuppoſitions, to every member during ſickneſs; beſides the payment of an annuity beginning with 5 *l.*

when

cline, and when therefore it will be moſt uſeful.—It is further a recommendation of this

when a member entered his 64th year, and encreaſing for 15 years, till, at 79, it became fixed for the remainder of life at 20*l.*

If the probabilities of life are lower among the labouring poor, than among the generality of mankind, this plan will be ſo much the more ſure of ſucceeding.

In 1773, a pamphlet was publiſhed, entitled, *A Propoſal for eſtabliſhing Life Annuities in Pariſhes, for the Benefit of the induſtrious Poor.*—"It is," ſays this writer, "a "common (*a*) obſervation that the money annually raiſed "for the poor, amounts to, *at leaſt*, a million a year; "and that yet in many places they are but indifferently "provided for. To make proviſion for one's old age is "ſo natural a piece of prudence, that it ſeems at firſt "ſight wonderful, that it ſhould not be generally practiſed by the labouring poor, as it is almoſt univerſally "by perſons in the higher paths of induſtry: Nor can "their negligence in this reſpect be accounted for, in "any other way ſo naturally, as by aſcribing it to their "wanting proper opportunities of employing the money "they might ſave, in ſome ſafe and eaſy method that "would procure them a ſuitable advantage from it in "the latter periods of their lives. They know, for the "moſt part, but little of the *public funds*; and when it "happens that they are acquainted with them, the ſmallneſs of the ſum they would be entitled to receive, as the "intereſt of the money they could afford to lay out in "them, is no encouragement to them to diſpoſe of it "in that way. What inducement, for inſtance, can it "be to a man who has ſaved ten pounds out of his year's "wages, to inveſt it in the purchaſe of 3 *per cent. Bank* "*annuities*, to conſider that it will produce him ſix or "ſeven

(*a*) The amount of the poor-rate for one year at the end of the reign of king Charles II. was 665,362*l.* See *Davenant's* works, Vol. I. p. 8.
——In 1777, it was 1,556,804*l.* according to the returns made in that year to parliament by the overſeers of the poor.

this plan that leſs depends in it on the *improvement* of money than in moſt other plans.

" ſeven ſhillings a year? It is but the wages of three " days labour.—And if they lend their money to tradeſ- " men of their acquaintance, as they ſometimes do, it " happens not unfrequently that their creditor becomes a " bankrupt, and the money they had truſted him with is " loſt for ever; which diſcourages others of them from " ſaving their money at all, and makes them reſolve to " ſpend it in the enjoyment of preſent pleaſure. But if " they ſaw an eaſy method of employing the money they " could ſpare in ſuch a manner as would procure them " a conſiderable income in return for it at ſome future " period of their lives, without any ſuch hazard of loſing " it by another man's folly or misfortune, it is pro- " bable they would frequently embrace it: And thus a " diminution of the poor rate on the eſtates of the rich, " and increaſe of preſent induſtry and ſobriety in the poor, " and a more independent and comfortable ſupport of " them in their old age, would be the happy conſe- " quences of ſuch an eſtabliſhment. Now this might " be effected in the following method."

Firſt, "Let the church-wardens and overſeers of every " pariſh be impowered, by act of parliament, to grant " life-annuities to ſuch of the inhabitants of the pariſh " as ſhall be inclined to purchaſe them, to commence at " the end of one, two, or three years, or ſuch other fu- " ture period of time as the purchaſer ſhall chuſe, and " to be paid out of the poor rates of the pariſh, ſo that " the land and other property in the pariſh that is " chargeable to the poor-rate, ſhall be anſwerable for " the payment of theſe annuities.—This circumſtance " would give theſe annuities great credit with the poor " inhabitants, by ſetting before them a ſolid and ample " ſecurity for the payment of them."

Secondly, "Let the annuities, thus granted to the " poor inhabitants, be ſuch as ariſe from a ſuppoſition " that the intereſt of money is 3 *per cent.* or ſome higher " rate

plans.—But I muſt leave theſe hints to be purſued by others.

The

"rate of intereſt, if the church-wardens and overſeers of "the poor think fit to make uſe of ſuch higher intereſt."

Thirdly, "But at the rate of 3 *per cent.* the purchaſer "ſhould have a right to an annuity, and the church-"wardens and overſeers of the poor ſhould be compell-"able to grant it."

Fourthly, "No annuity depending on one life ſhould "exceed 20 *l.* a year."

Fifthly, "No leſs ſum than 5 *l.* ſhould be allowed "to be employed in the purchaſe of an annuity.——— "This is to avoid intricacy and multiplicity in the ac-"counts."

Sixthly, "An exact regiſter of theſe grants ſhould be "kept, by the church-wardens and overſeers of the "poor, in proper books for the purpoſe, in which the "grants ſhould be copied exactly, and the copy of each "grant ſubſcribed by the perſon to whom it is granted. "And this copy, in the regiſter-book of the pariſh, "ſhould be good evidence of the purchaſer's right to "the annuity, in caſe the original deed of grant to the "purchaſer, which was delivered to him at the time of "the purchaſe, ſhould be afterwards loſt."

Seventhly, "The money thus paid to the church-"wardens and overſeers of the poor for the purpoſe of "life-annuities, ſhould be employed in the purchaſe of "3 *per cent.* Bank-annuities in the joint names of all "the church-wardens and overſeers, and by them tranſ-"ferred at the expiration of their offices to their ſuc-"ceſſors, and ſo on to the next ſucceſſors for ever, ſo "as to be always the legal property of the church-"wardens and overſeers of the poor for the time being, "in truſt for the perſons who ſhould be entitled to the "ſeveral life-annuities, granted in the manner above-"mentioned; and the intereſt of this money ſhould "be received every half year, and inveſted in the pur-"chaſe of more principal continually, ſo as to make a "perpetual fund for the payment of the annuities, &c.

&c.

The body of diſſenting miniſters in London had under conſideration ſome time ago a plan of this kind; and a ſet of Tables were compoſed for them. The deſign was dropped; but as it is poſſible it may be taken up again, and the Tables may be of uſe, I have thought fit to preſerve them by inſerting them among the additional Tables and Obſervations in the next volume.

Additional Account of the Societies for the Benefit of old Age.

SINCE the publication, in three former editions, of the Obſervations in the preceding Section, almoſt all the ſocieties mentioned in it, convinced of the inſufficiency and pernicious tendency of their plans, have diſſolved themſelves, and diſtributed among their ſubſcribers the money they had paid, with ſuch intereſt or profit as remained after

" &c. Deficiencies, if any ſhould ever happen, to be " made good by the poor-rates, &c. &c."

The very able and public ſpirited and worthy writer of the pamphlet from which I have taken this quotation, now *Curſitor Baron* of the *Exchequer*, took great pains to carry into execution the deſign he has explained in it. With this view, a bill, with ſuitable Tables annexed, was brought into the Houſe of Commons and ſupported by the excellent Sir *George Saville*, the late Mr. *Dowdeſwell*, and many others of the moſt reſpectable members. It paſſed that Houſe without much oppoſition, but was rejected in the Houſe of Lords.

deducting

deducting the expences of management; and there are now left within my knowledge only two of theſe ſocieties which require any particular notice. I mean, the *Amicable* and the *Laudable* Societies *for the Benefit of Age*. The firſt of theſe Societies, mentioned in p. 132, finding upon examination that, inſtead of an annuity of 26*l.* they could not in reality afford to pay a higher annuity than 8*l.* determined, with great fairneſs, to leave it to the option of all their members, either to continue their contributions with a view to this reduced annuity, or to take back all they had paid and withdraw. Near two hundred members having choſen the former, the Society now conſiſts of *them* only, and therefore can ſcarcely be in any danger.—— The other Society, mentioned p. 131, has alſo ſunk the annuity it promiſes from 44*l.* to 24*l.*; but it is certain, that it cannot permanently pay to all its members a greater annuity than 15*l.* I ſhould loſe too much time were I to give an account of the calculations which prove this. He that would ſee it demonſtrated with all poſſible clearneſs, ſhould conſult a Tract publiſhed in 1777, by Mr. *Dale*, entitled, *A* SUPPLEMENT *to Calculations deduced from firſt Principles*, &c. No perſon who underſtands common arithmetic can avoid being convinced by the evidence offered in this Tract, nor can any *honeſt* man avoid being ſhocked by the narrative it

contains of the obſtinacy with which the majority in this Society have perſevered in error, contrary to the efforts of the more reſpectable part of the Society; and in defiance of reaſon, juſtice, and humanity. I cannot, in ſhort, ſpeak more properly on this ſubject than in the words of Mr. *Morgan* in his Treatiſe *on the Doctrine of Life Annuities and Aſſurances*, p. 47. "There is ONE Society "for the benefit of old age ſtill left, on "which, none of the calls of juſtice and "humanity have been able to make any pro-"per impreſſion. I mean the *Laudable So-"ciety* of Annuitants, whoſe office is held at "the bottom of *Bartholomew-Lane*. In op-"poſition to the *plaineſt* evidence, this So-"ciety goes on to offer *double* the annuity it "can afford to pay; and the late tranſac-"tions in it (as related by Mr. *Dale* "in his *Supplement*) exhibit an inſtance "of ſuch an obſtinate and wilful per-"ſeverance in impoſition as has ſeldom "been equalled.——I am ſorry to add", ſays Mr. *Morgan*, "that this cenſure is applica-"ble to another Society called alſo LAUDA-"BLE, but in reality PERNICIOUS, as many "ſuffering widows will ſome time or other "experience."

It is here ſaid, that this Society promiſes *double* the annuity it can pay; that is, 24*l.* when in reality it can pay only 12*l.*; whereas I have ſaid that it may pay 15*l.* In order

order to explain this difference, it is neceſſary to obſerve that Mr. *Dale* has ſhewn that 15*l.* nearly is the annuity which the Society can afford to pay according to the chances of living in Dr. *Halley*'s or the *Breſlaw* Table; but that the chances of living in the Society had for eight years before 1776 (*a*), (when the number of the Society was above 1300, and its *ſtock* near 96,000*l.*) been found by particular enquiry to be not much leſs than *double* to thoſe at the ſame ages in the Table; from whence it follows, that the annuity payable by the Society being more valuable, it ought not to be ſo high as 15*l.* nor probably more than 12*l.*—The fact now mentioned is important, but not ſingular; for it has been found to take place in other ſimilar ſituations (*b*); and the reaſon is, that Dr. *Halley*'s and moſt other Tables of Obſervations give the chances of living as they exiſt in towns among men of all ſorts taken in the groſs; whereas ſuch Societies as thoſe for the benefit of old age, and in general all purchaſers of life-annuities for *themſelves*, muſt conſiſt of a *ſelection* of the beſt lives.

(*a*) I know not what the rate of mortality in the Society has been ſince this year.

(*b*) See Additional Obſervations on Civil Liberty, p. 135.

POSTSCRIPT.

Since the preceding Obſervations were written, I have been informed that this Society has reduced its annuity from 24*l.* to 20*l.* This reduction, together with the high intereſt at which money may be now improved (particularly in the ſhort annuity), will prolong conſiderably the duration of the Society; but, unleſs it is favoured by uncommon events, cannot make it permanent.

SECT. VI.

Of the Amicable Society for a perpetual Aſſurance Office: And the Society for Equitable Aſſurances on Lives and Survivorſhips.

THE 10th Problem has been given, with a particular view to the corporation of the *Amicable Society*, for a perpetual Aſſurance-Office on ſingle lives, kept in *Serjeant's-Inn*. This Society was eſtabliſhed in 1706, and is the only one I am acquainted with, which has ſtood any conſiderable trial from time and experience. It is limited by its charter to the diſpoſal of

ſhares

ſhares or numbers (not to exceed 2000) held by *ſingle* lives, and entitling to *claims* when the lives drop. For each of theſe ſhares every purchaſer pays at entrance 7*l.* 10*s.* beſides 1*l.* 11*s.* as the firſt *quarterly* payment of 6*l.* 4*s.* *per ann.* to be continued during life. An annual dividend of 1*l.* 4*s.* for each ſhare is allowed to every purchaſer out of the profits of the corporation, which reduces the *annual* payment for each ſhare to 5*l.* The nett annual income ariſing from all the annual payments (making 10,000*l.* when the Society is full, and all the ſhares are diſpoſed of), is equally divided among the nominees of ſuch members as die within the year; which dividend, therefore, is more or leſs at the end of every year as a ſmaller or greater number of the members happen to die in that year. In 1757, the Society engaged that this dividend, though it might be more, ſhould not be leſs on each ſhare than 125*l.* and in 1770 (*a*), that it ſhould not be *leſs* than 150*l.*——No one perſon is allowed to purchaſe more than *three* ſhares; nor are any admitted to be purchaſers whoſe ages exceed 45, or fall ſhort of 12; and all

(*a*) In 1757, the Society had accumulated by its ſavings 25,300*l.* three *per cent.* ſtock, which in 1770, had been increaſed to 33,300*l.*; and a part of this ſtock was in theſe years appropriated to the payment of claims, whenever the number of them in any year ſhould happen to be ſo great as to render the annual contributions inſufficient to make them up to the guarantied ſums.

between theſe ages are admitted on the ſame terms.

This Society has, I doubt not, been very uſeful to the public; and its plan is ſuch, that it cannot fail to *continue* to be ſo. It might, however, certainly have been much more uſeful, had it gone from the firſt on a different plan. It is obvious, that regulating the dividends among the *nominees* by the number of members who die every year, is not *equitable*; becauſe it makes the benefit which a member is to receive to depend, not on the value of his contribution, but on a *contingency*; that is, the number of members that ſhall happen to die the ſame year with him. This regulation muſt alſo have been diſadvantageous to the Society; as will appear from the following account of the natural progreſs of the affairs of ſuch a Society, when eſtabliſhed on a right plan.

Suppoſe a *thouſand* perſons, whoſe common age is 36, to form themſelves into a Society for the purpoſe of *aſſuring* a particular ſum at their deaths, to ſuch perſons as they ſhall name, in conſideration of a particular annual contribution to be continued during their lives. Suppoſe the annual contribution to be 5 *l.* and the firſt payment (*a*) to be made immediately. Suppoſe, like-

(*a*) Such payment, it has been ſhewn, Queſt. VIII. p. 28, are better than any *half*-yearly or *quarterly* payments, and at the ſame time they ſave ſome trouble.

wiſe,

wiſe, the original number of the Society to be conſtantly kept up by the admiſſion of new members, at 36 years of age, in the room of ſuch as die.—In Queſt. X. p. 31, it appears, that an annual payment, beginning immediately, of 5*l.* during a life now at the age of 36, ſhould entitle, at the failure of ſuch a life, to 172*l.* reckoning intereſt at 4 *per cent.* and taking Mr. *De Moivre*'s valuation of lives.—A *thouſand* perſons, all 36 years of age, will die off at the rate of 20 every year. The diſburſements, therefore, of ſuch a Society will be, the firſt year, 20 times 172*l.* or 3440*l.* and its income will be 5000*l.* It will, therefore, at the end of the year, have a ſurplus of 1560*l.* to put to intereſt.—In conſequence of the yearly acceſſions to ſupply vacancies, the number dying annually will be always increaſing after the firſt year. In 50 years (*a*) it will get to a *maximum*; and then the affairs of the Society will become *ſtationary*, and the number dying annually will be 40, and its annual expence will be 6,880*l.* exceeding the annual contribution, 1,880*l.* But, in the mean time, by improving its ſurplus monies, it will have raiſed a capital equal to this ex-

(*a*) This period will (by Queſt. III.) be longer if the Society is any time in filling, and admits members at younger ages than 36. It will, for inſtance, be 84 years, if the Society is ten years in filling, and admits at all ages between 12 and 45.

ceſs, and, conſequently, its affairs will be fixed on a firm baſis for all ſubſequent times.

Suppoſe now that ſuch a Society, at its eſtabliſhment, ſhould reſolve to divide its whole yearly income among the *nominees* of deceaſed members. The effect of this would be, that no capital could be raiſed; that the dividends payable to *nominees* would diminiſh continually, till, at the time that the greateſt number of members came to die *annually*, (or at the end of 50 years), they would be reduced to *half*; and that all claimants, after this period, would receive too little, becauſe the firſt claimants had received too much (*a*).

(*a*) The reverſe of this will take place, if ſuch a Society *begins* with admitting all at all ages, and afterwards changes its plan, and *limits* the age of admiſſion. In this caſe, the number of *yearly deaths* will be *greateſt* at firſt, and the *dividends ſmalleſt*. In conſequence of altering its plan, the *yearly deaths* will leſſen gradually, and the *dividends* riſe; but in time *both* would return again to their original ſtate.

The following facts incline me to ſuſpect, that this remark may be applicable to the *Amicable Corporation*.

Firſt. In their *original charter*, as it is given in their printed abſtracts, there is no limitations of age mentioned; but 31 years afterwards, I find a bye-law made againſt admitting any perſon who ſhould be above the age of 45, or under 12.—Secondly. In their printed advertiſements in 1770, it is ſaid, that in 59 years they had paid, among 3643 claimants, 378,184*l.* from whence it follows, that though the average of their dividends, for 17 years before 1773, has been 154*l.* the average, for 59 years, has been only 104*l.*

At

At the time of the inſtitution of the *Amicable Corporation*, the intereſt of money was at 6 *per cent.* and, as they admit none whoſe ages are not under 45, the mean age of admiſſion cannot be much greater than 36. It appears, therefore, that had they avoided the error now mentioned, and gone from the firſt on the plan I have deſcribed, they might have all along paid to each *nominee* 172*l.* beſides raiſing a capital much greater in proportion to the number of members, than that I have ſpecified, from the premiums at admiſſion, forfeitures, and other advantages which they have enjoyed (*a*). Indeed, I cannot doubt but that on this plan, and with theſe advantages, they might have found themſelves always able to pay at leaſt 200*l.* to each *nominee* (*b*).

I have already mentioned one inſtance in which the plan of this Society is not equit-

(*a*) A ſurplus from a *thouſand* members of only *five ſhillings per annum*, duly improved at 4 *per cent.* would, in 41 years, produce a capital of 25,000*l.*

(*b*) It ſhould be remembered, that all this was ſaid in the former editions, on the ſuppoſition that proper care has been taken to keep out unhealthy perſons; that the ages of admiſſion have never exceeded 45; and that the probabilities of life among the members of this Society, are the ſame with thoſe in the 5th, 6th, and 7th Tables, in the next volume. But I have lately found the truth to be, agreeably to the ſuſpicion expreſſed in the laſt note but one, that for many years after the firſt inſtitution of the Society, members were admitted at all ages.

able.

able. Another inſtance of this is, their requiring the ſame payments from all perſons under 45, without regardng the differences of their ages; whereas, the annual payments of a perſon admited at 45, ought to be double the annual payment of a perſon admitted at 12.

Further. The plan of this Society is ſo narrow, as to confine its uſefulneſs too much. It can be of no ſervice to any perſon whoſe age exceeds 45. It is, likewiſe, far from being properly adapted to the circumſtances of perſons, who want to make aſſurances on their lives, for only ſhort terms of years.—Thus; the true value of the aſſurance of 150*l.* for 10 years, on the life of a perſon whoſe age is 30, is by Queſt. XIV. (intereſt being at 3 *per cent.*) 2*l.* 13*s.* in annual payments (for 10 years), to begin at the end of the firſt year; and ſubject to failure when the life fails. But ſuch an aſſurance could not be made, in this Society, without an annual payment of 5*l.*—Neither is the plan of this Society at all adapted to the circumſtances of perſons, who want to make aſſurances on particular ſurvivorſhips.—For example. A perſon poſſeſſed of an eſtate, or ſalary, which muſt be loſt with his life, has a perſon dependent upon him, for whom he deſires to ſecure a ſum of money, payable at his death. But, he deſires this only as a proviſion againſt the danger of his dying

firſt,

firſt, and leaving a wife, or a parent, without ſupport. In theſe circumſtances, he enters himſelf into this Society; and by an annual payment of 5*l.* entitles his *nominee* to 150*l.* In a few years, perhaps, his *nominee* happens to die; and, having then loſt the benefit he had in view, he determines to forfeit his former payments, and to withdraw from the Society. In this way, probably, this Society muſt have gained ſome advantages. But the right method would have been to have taken from ſuch a perſon the true value of the ſum aſſured, "on the "ſuppoſition of non-payment, provided he "ſhould ſurvive." In this way he would have choſen to contract with the Society; and had he done this, he would have paid for the *aſſurance*, (ſuppoſing intereſt at 3 *per cent.* his age 30, the age of his *nominee* 30, and the probabilities of life as in the 5th, 6th, and 7th Tables) 3*l.* 8*s.* (*a*) in annual payments to begin immediately, and to be continued during the *joint* continuance of his own life, and the life of his *nominee*.

(*a*) The value of 150*l.* payable at the death of a perſon, aged 30, *provided* he ſurvives another perſon of the ſame age, is, by Queſt. XI. Chap. I. *l.*45.65; and this value divided by 13.43 (the value increaſed by unity, of two joint lives both 30), gives *l.*3.4, or 3*l.* 8*s.*—The value of the ſame reverſion, according to the probabilities of life in *London*, is, *l.*49.19, in *one* payment; and *l.*4.16, in *annual* payments, during the joint lives, the firſt payment to be made immediately.

Further

Further Account of the Amicable Corporation.

THE affairs of this *Corporation*, have lately taken a very favourable turn. The dividends, from the annual contributions of 5*l.* for each number, which for eight years, ended in 1769, had not been 150*l.* on each claim, have, for eight years ended at 1779, been nearly 200*l.* (*a*)——The ſubſiſting ſhares have increaſed from 1120, (their number in 1769) to 1990 (their full complement nearly) in 1780; and in the ſame time the *ſtock* of the Society has been increaſed from 33,300*l.* to 51,300*l.* in the 3 *per cent.* annuities; in conſequence of which it finds itſelf now poſſeſt, after diſcharging all expences, of a clear ſurplus of about 1350*l.* *per ann.*——In theſe circumſtances a propoſal has been made to the Society to diſcontinue the increaſe of its ſtock, in order to make uſe of the ſurplus in increaſing the dividends on claims. This being an inviting propoſal, it is not ſurpriſing that the Society in general has ſhewn itſelf

(*a*) This dividend for 1780, was 192*l.* 6*s.* 1*d.*$\frac{1}{4}$; but made up to 193*l.* in conſequence of a diſcretionary power given the directors to take what ſums they think reaſonable from the ſavings to increaſe the dividends.

diſpoſed

diſpoſed to accede to it.——The imprudence however of ſuch a ſtep will be evident from the following obſervations.——It ſhould be conſidered, that the reaſon of the late increaſe of the annual dividends, has been the late increaſe of the Society by the influx of young members. So great has been this increaſe, that the Society has been nearly *doubled* in twelve years, and *tripled* in the laſt 30 years.——It could not, therefore, but happen that the number of deaths ſhould become much leſs in proportion to the number of members, than they were *before* the increaſe. The Society being now full, and admitting of no farther increaſe, the collective age of the members, and, together with it, the annual deaths, will be for ſome time increaſing, till that part of the Society which conſiſts of the late additions (*a*) come to

(*a*) An addition of near 400 to the ſubſiſting numbers or ſhares of the Society was made from 1749 to 1768; and of above 900 more, from 1769 to 1779.—— Even that part of the increaſe of the Society which conſiſts of members to whom the firſt of theſe additions has been owing, is at preſent far from dying off ſo faſt as it will. In truth, 50 or 60 years at leaſt muſt elapſe before this can happen. See Queſt. III. in the Treatiſe on Reverſionary Payments. I am told it has been aſſerted by ſome belonging to the Society, that a perſon about 40 or 45 is not leſs likely to die than a perſon 20 or 30 years older, from whence, I ſuppoſe it is inferred, that the deaths now in the Society may be as numerous as they will ever be. It would be doing an injury to the Society to ſuppoſe that it

to die off *twice* as faſt as they did at firſt. During this interval the dividends will be growing leſs and leſs, till at laſt they will fall below the dividend which the Society has guarantied (or 150*l.*); and produce a neceſſity of entering into the capital in order to make it up. And the reduction of the capital, once begun will proceed faſter and faſter till it is all ſpent; and when ſpent, the Society will be thrown back into the ſtate it was in before its increaſe, when frequently it could not afford a dividend of 100*l.* on each number.——Such will be the certain effect of adopting the meaſure I have mentioned. It is, therefore, imprudent in the higheſt degree; and I will add, that the *injuſtice* of it is equal to its imprudence; for it is benefiting the *older* part of the preſent members at the expence of the *younger* members, and all *preſent* members at the expence of *future* ones. For a few years the dividends will probably, with the

it can be influenced by ſuch aſſertions. According to all obſervations on human mortality, a body of men at 65 or 70 will die off twice as faſt as a body of men at 40; and a body of men at 75 or 80 will die off *four* or *five* times as faſt as a body of men at 50. The human frame, after the age of 12 or 15, is continually wearing out and becoming leſs capable of combating the cauſes of mortality; but more or leſs ſlowly according to the degrees of firmneſs with which it was built, and the favourableneſs or unfavourableneſs of the ſituations into which it happens to fall.

help

help of the addition of the annual profits, exceed 200*l.* which will be paid chiefly among the claimants derived from the older members. But after a courſe of years they will ſink to little more than half; and at that period the Society will conſiſt of the younger part of the preſent members, and ſuch *new* members as ſhall be hereafter admitted to fill up vacancies, who will therefore be great *loſers*, becauſe their predeceſſors, by neglecting to improve the eſtate, made themſelves too great *gainers*.

In order to ſet this in a clearer light, I would deſire it may be conſidered that, according to the mean probabilities of the duration of life, a body of people at the age of 36, will, one with another, live 25 years. It muſt, therefore, be expected that a *twenty-fifth* part will die annually of a Society which has ſubſiſted any conſiderable time, and the members of which are admitted at this mean age. Suppoſing, therefore, 36 the mean age of admiſſion in the *Amicable Corporation*, the time muſt come when (if kept (*a*) up to its full complement) a 25th part of the members will die annually, or when 80 numbers (the 25th of 2000) will drop, and produce 80 claims annually.——The whole

(*a*) Should the Society decline, more than a 25th part will die annually, and the period when this will happen will arrive ſooner.

income

income of the Society (conſiſting of 10,000*l.* *per ann.* from the charter contributions, and 1350*l.* *per ann.* profits), when divided equally among 80 claimants, will give 142*l.* for each claim. The dividend gaurantied in 1770 being 150*l.* there will be a deficiency of 8*l.* in each claim, or of 640*l.* in the total of claims; which ſum procured by ſelling ſtock in one year will leave a neceſſity of ſelling more the next year, and ſtill more the following, and ſo on through every ſucceſſive year, till the whole ſtock falls rapidly to nothing.——It is proper to obſerve here, that this deficiency of 640*l.* *per ann.* is a *maximum* to which the deficiencies of many preceding years had increaſed gradually; and that, therefore, the whole ſtock may be conſumed even before the period arrives when the greateſt deficiencies will happen.

All this reaſoning ſuppoſes that no more than a 25th part of the members of this Corporation will hereafter come to die annually; or that they are admitted at the mean age of 36, and live after admiſſion, one with another, 25 years—But theſe are probably too favourable ſuppoſitions. Perhaps, the mean age at which members are admitted (and particularly the purchaſers of two or three numbers) may be above 40; and perhaps alſo, on account of the difficulty there muſt be in excluding from *ſuch inſtitutions* all bad lives,

lives, the duration of the lives of the members may be ſomewhat leſs than is common among perſons at the ſame ages. Should this be true, their duration of life after admiſſion will not exceed 22 years. A 22d part will hereafter die annually. The claims from 2000 numbers will be 91; and the dividend on each claim, with the addition of the ſavings, will be only 124*l.* 12*s.* which will produce a deficiency of 2311 *l. per ann.*

If I may judge from what has hitherto happened in the Society, even theſe laſt ſuppoſitions favour it too much; for, I find, that before the increaſe (*a*) which begun in 1750, a *nineteenth* part of the exiſting numbers dropped annually, which made the di-

(*a*) This increaſe ſeems to have been owing to the limitation of age in 1737. In a few years after this, the greateſt part of the members who had been admitted at advanced ages having died off, and the Society conſiſting chiefly of younger members admitted in their room; the dividends roſe, which occaſioned a quick *increaſe* of young members, raiſed the dividends ſtill higher, till, in 1757, the Society thought itſelf capable of guarantying a dividend of 125*l.*; and, in 1770, a dividend of 150*l.* Theſe augmentations contributed yet more to increaſe the Society, and conſequently the dividends. It became, therefore, ſoon full, and now finds itſelf in the proſperous ſtate deſcribed at the beginning of theſe Obſervations; a ſtate, which the Society may render *really* and *permanently* proſperous, if they are properly attentive to its cauſes, and will avail themſelves of the opportunity it gives them to rectify the faults in their original plan by increaſing their guaranteid dividend in the manner I ſhall preſently propoſe, inſtead of diſcontinuing the increaſe of their capital.

vidends then fall frequently below 100*l.* This, however, muſt have been owing to the admiſſion, for ſome years after the eſtabliſhment of the Society, of members at too advanced ages, and the neglect of proper care to exclude bad lives. Much uncertainty in this inſtance would be removed, and the beſt guide obtained in conducting the affairs of the Society, by taking an exact account of the mean age at which, for the laſt ten or twelve years, all members (diſtinguiſhing particularly ſuch as have two or three numbers dependent on their lives) have been admitted; and alſo of the number of years which all admitted, till within the laſt 30 years, have lived after admiſſion.

Upon the whole. Till new light is given by ſuch an enquiry as this, I muſt think that, however proſperous the affairs of the Society ſeem at preſent, it cannot prudently act on any other expectation than that a period will come when a 22d or 23d (*a*) part of

(*a*) If Mr. *Brand*, the clerk of this Society, is right, the members of it are uncommonly ſhort-lived; for, according to an account which he has given of the duration of the lives of 3826 perſons who have been admitted into it, and on whoſe deaths claims have been paid; they do not, one with another, live after admiſſion 16 years. See *Smart*'s Tables republiſhed by Mr. *Brand*, p. 189. See likewiſe his Treatiſe on Aſſurances and Annuities, p. 68.

This, were it true, would be very threatening to the Society. But Mr. *Brand*, in other parts of the *little* that is his own in theſe Treatiſes, has erred ſo palpably, and

ſhewn

of its members will die annually, and when, therefore, it will want at leaſt an *additional* income of 2000*l.* *per ann.* to enable it to make good its engagements. A greater additional income might be acquired by continuing to lay up all its ſavings after dividing the charter contributions—But were it to lay up the *whole of its income* above what may be neceſſary to divide 170*l.* on every claim, it might advance its *guarantied* dividend from 150*l.* to *this ſum.* And this, in my opinion, is the moſt rational and equitable meaſure it can adopt.

I know, indeed, that there is a clauſe in the charter which limits the increaſe of its eſtate to 2000*l.* *per ann.* But this clauſe ſhews that the charter was framed with too little foreſight; and it muſt be repealed, or the conſequences will be that *danger* to the Society and *injuſtice* to its future members which I have repreſented.

The preceding Obſervations are offered very reſpectfully to the conſideration of this Society. Should any of the members or directors think them worth their attention, I

ſhewn himſelf ſo wonderfully ignorant, that, however poſſeſſed in this inſtance of the means of information, he deſerves no credit.—The truth is, in the preſent caſe, that he has included in his account ſuch members as have been admitted and have died *lately*; and this muſt neceſſarily lead to a wrong concluſion concerning the duration of the lives of the members taken at large.

hope they will reflect, that having no intereſt to ſerve, I can mean nothing by them but the preſervation of the credit and uſefulneſs of the Society.

SECT. VII.

Of the Society for Equitable Aſſurances on Lives and Survivorſhips.

THE Society which is to be the ſubject of this Section, has juſtly ſtiled itſelf, "*A Society for Equitable Aſſurances on Lives and Survivorſhips.*" The buſineſs of it is carried on at its office, in *Chatham-Square*, near *Blackfriars Bridge*. It was founded in 1761, in conſequence of lectures recommending ſuch an inſtitution, which had been read by Mr. *Simpſon*, a name that can never be forgotten while there is any mathematical or philoſophical knowledge left in the world. Mr. *Dodſon* alſo, the author of the *Mathematical Repoſitory*, was active in recommending the plan of this Society, and compoſed Tables for its uſe.—It aſſures any ſums or reverſionary annuities on any lives, for any number of years as well as for the whole continuance of the lives, at rates ſettled by particular calculation; and in any manner that may be beſt adapted to the views of the perſons

ſons aſſured. That is, either by making the aſſured ſums payable *certainly* at the failure of any given lives, or on *condition* of ſurvivorſhip; and alſo, either by taking the price of the aſſurance in *one preſent payment*, or in *annual payments* during any ſingle or joint lives, or any terms leſs than the whole continuance of the lives.—In ſhort; the plan of this Society is ſo extenſive and ſo important, that I cannot ſatisfy my own mind, without offering to the gentlemen concerned in the direction of it, the following obſervations, hoping they will not think them impertinent.

Firſt. They ſhould conſider what diſtreſs would ariſe from the failure of ſuch a ſcheme in any future time; and what dangers there are, which ought to be carefully guarded againſt in order to ſecure ſucceſs. I have already more than once obſerved, that thoſe perſons will be moſt for flying to theſe eſtabliſhments, who have feeble conſtitutions, or are ſubject to diſtempers which they know render their lives particularly precarious; and it is to be feared, that no caution will be ſufficient to prevent all danger from hence.

Again. In matters of chance, it is impoſſible to ſay, that an unfavourable run of events will not come, which may hurt the beſt contrived ſcheme. The calculations only determine probabilities; and, agreeably

to theſe, it may be depended on, that events will happen on the whole. But at particular periods, and in particular inſtances, great deviations will often happen; and theſe deviations, at the commencement of a ſcheme, muſt prove either very favourable, or very unfavourable.

But further. The calculations ſuppoſe, that all the monies received are put out immediately to accumulate at compound intereſt. They make no allowance for loſſes, or for any of the expences attending management. On theſe accounts, the payments to a Society of this kind, ought to be more than the calculations will warrant, and the intereſt of money ought to be reckoned low. Mr. *Dodſon*, I find, has paid due attention to all this, by reckoning intereſt, in his calculations for this Society, at 3 *per cent.* and taking the loweſt of all the known probabilities of life, or thoſe deduced from the *London* bills of mortality (*a*). There is, beſides,

(*a*) It ought, however, to be remembered here, that in ſelling life-annuities to commence either immediately, or after given terms; and alſo in ſome other caſes, the values come out *leſs* in conſequence of *lower* probabilities of life. Would it in *ſuch* inſtances, be taking an unfair advantage, to eſtimate the values by Tables which give the *higheſt* rather than the *loweſt* values? Thus; was the Society to ſell 20*l.* *per annum*, for life, to a perſon now 30, after 50, the value, according to Dr. *Halley*'s Table, would, reckoning intereſt at 3 *per cent.* be 90*l.* in a ſingle

beſides, a liberty provided of making a call on all the members, in caſe of any particular emergency. It is, therefore, highly probable, that this Society muſt be ſecure. The laſt expedient, however, would be a very diſagreeable one, ſhould there be ever any occaſion for having recourſe to it; and, in order to guard ſtill more effectually againſt danger, it would not, I think, be amiſs to charge a profit of 6 *per cent.* on all the payments.—Should the conſequence of this prove, that in ſome future period the Society ſhall find itſelf poſſeſſed of too large a capital, the harm will be trifling, and future members will reap the advantage. But this leads me to repeat an obſervation of particular conſequence.

As this Society is guided, in every inſtance, by ſtrict calculation, it is not to be expected that it can meet with any difficulties for many years; becauſe not till the end of many years after it has acquired its *maximum* of members, will the *maximum* of yearly claimants and annuitants come upon it. Should it, therefore, through inattention to this remark, and the encouragement ariſing from the poſſeſſion of a large ſurplus, be led to check or

gle payment; but according to the *London* Table, the value would be only 70*l.*

But in reality the value, even by Dr. *Halley*'s Table, is leſs than the Society, in ſuch a caſe, ought to take, for the reaſon mentioned in page 147.

ſtop the increaſe of its ſtock too ſoon, the conſequences might prove pernicious.

Again. I would obſerve, that it is of great importance to the ſafety of ſuch a Society, that its affairs ſhould be under the inſpection of able mathematicians. Melancholy experience ſhews, that none but mathematicians are qualified for forming and conducting ſchemes of this kind.—In ſhort; dangerous miſtakes may ſometimes be committed, if the affairs of ſuch a Society are not managed frugally, carefully, and prudently. One inſtance of this I cannot avoid mentioning.

A perſon, who deſires to aſſure a particular ſum to be paid at the failure of his life, on condition of the ſurvivorſhip of another life, may chuſe to pay the value in annual contributions during the continuance of his own ſingle life rather than during the continuance of the joint lives; becauſe the annual contributions, in this caſe, ought to be much leſs. But a Society that would practiſe ſuch a method of *aſſurance* would hurt itſelf; for as ſoon as the life, on whoſe ſurvivorſhip the aſſurance depends, is extinct, the perſon aſſured, if then living, would have no longer any benefit in view; and, therefore, would make his payments with reluctance, and in time, perhaps, entirely withdraw them; the conſequence of which would be that the Society would ſuffer a loſs by being

being deprived of the juſt value of the expectation it had granted. The plan of a Society ought always to be ſuch, as that the loſſes ariſing from diſcontinuance of payment, ſhould fall on the purchaſer, and never on the Society.

I muſt not forget to add, that it is neceſſary that ſuch a Society ſhould be furniſhed with as complete a ſet of Tables as poſſible. This will render the buſineſs of the Society much more eaſy, and alſo much more capable of being conducted by perſons unſkilled in mathematics. It will alſo contribute much to its *ſafety*. For in all caſes to which Tables can be extended, there would be no occaſion for imploying any calculators; and, conſequently, a danger would be prevented to which, though it is not *now*, it may *hereafter* be expoſed; I mean, the danger of happening to truſt unſkilful or careleſs calculators.—It is indeed furniſhed with Tables, by which a great part of its buſineſs is tranſacted; but there are ſome important Tables which it wants, and with which it ſhould be ſupplied; and theſe when compoſed, together with all its other Tables, ſhould be ſubject to the reviſal and examination of the beſt judges, and afterwards publiſhed; with a minute account of the principles aſſumed and the method taken in compoſing them. Such a publication would be a valuable addition to this part of ſcience; and it would

would alſo be the means of increaſing and eſtabliſhing the credit of the Society.

In Queſtions 4th, 6th, 10th, 11th, 14th, 15th, and 16th, I have, with a particular view to this Society, given rules by which may be formed every Table it can want, for ſhewing the values of aſſurances on the *whole duration*, or any *terms*, of any *one* or *two* lives; and nothing but care and attention can be neceſſary to enable any good arithmetician to calculate from them.

Further Account of the Equitable *Society, with an Account of an Inſtitution for the Sale of Life-Annuities at* Hamburgh.

I Have juſt referred to the queſtions in the firſt Chapter of this Treatiſe, for the rules by which the values of aſſurances on any *one* life, or any *two* lives, may be computed. Since the laſt publication of this Treatiſe, inveſtigations of this kind have been carried much farther, and this ſubject, as far as it reſpects aſſurances on any number of lives not exceeding *three*, has been nearly exhauſted by Mr. *Morgan*, the actuary of this Society, in the Treatiſe referred to at the end of the Firſt Chapter.——In this work, Mr. *Morgan* has given a diſtinct account of the ſtate of this Society as he had made it out to January 1777; and to that account,

account, and to the Obſervations addreſſed to the Society in the Introduction to Mr. *Morgan*'s Treatiſe, I muſt refer for the fulleſt information that can be given of the plan and progreſſive increaſe of the Society, and of the methods employed to keep in conſtant view the ſtate of its accounts. I ſhall here only add, that its increaſe has been going on ever ſince with rapidity; that, in the laſt five years (or ſince 1776), its *annual income* has been nearly *doubled*, and its *capital* (conſiſting now in part of land ſecurities) more than *tripled*; that the rate of mortality among the perſons aſſured has continued much below that in the Tables by which it has hitherto made its calculations (*a*); and that, upon the whole,

(*a*) The ratio of the decrements of life or rate of mortality *in the Society* has been, for *twelve* years, from 1768 to 1780,

		to thoſe in *London*		to thoſe at *Breſlaw*
from 10 to 20	—	as 1 to $1\frac{1}{4}$	—	as 1 to 1
20 to 30	—	as 3 to 7	—	as 2 to 3
30 to 40	—	as 4 to 9	—	as 2 to 3
40 to 50	—	as 3 to 5	—	as 8 to 9
50 to 60	—	as 2 to 3	—	as 6 to 7
60 to 70	—	9 to 10	—	9 to 8

The profits of the Society by aſſurances only on ſingle lives during four years, from 1775 to 1778, were, on an average, 6446 *l. per ann.* In 1779, the Society (in order better to improve a part of their capital) ſold 27,000 *l.* three *per cent.* ſtock, yielding an intereſt of 810 *l. per ann.* and with the produce of the ſale (amounting to 16,696 *l.* 7 *s.* in money) purchaſed 1334 *l.* of the ſhort annuity for 30 years from the 5th of January 1778; by which change it added to its preſent income 200 *l. per ann.* after appropriating

whole, it appears at preſent to poſſeſs ſuch a *ſurplus* of income and ſtock as places it (if no miſmanagement takes place) above danger; except from events the moſt extraordinary.

In theſe circumſtances, the Society, not willing to raiſe an exorbitant capital, or to take unreaſonable profits, came to a reſolution at the beginning of the laſt year, or 1781, to make ſuch abatements in its demands as its preſent circumſtances render ſafe, and to ſettle ſuch new arrangements in its buſineſs as may contribute to make it as great a benefit as poſſible to the public.

In the preceding Section, but more particularly in the Introduction to Mr. *Morgan*'s Treatiſe, I have expreſſed my wiſhes that the Society would order new Tables to be calculated from Obſervations more adapted to the general ſtate of mortality among mankind than thoſe given by the *London* bills of mortality.——I can now inform the public, that ſuch an order has been given and lately

priating 324 *l. per ann.* to accumulation in this ſhort annuity, the effect of which accumulation will be, that if the ſtocks continue the ſame, the annuity, with its increaſe, will always ſell for more than the original purchaſe money; and, if never ſold, will, in the two laſt half-yearly payments, become at leaſt one *half* more than the purchaſe money.——If the ſtocks *riſe*, the profit from a ſale will be increaſed. If they *fall*, the accumulation will be increaſed. In every ſtate, therefore, of the funds, the Society will be gainers.

carried

carried into execution——Theſe new Tables are,

Firſt. A Table exhibiting the values of ſingle lives for their whole duration.

Secondly. A Table of the values of ſingle lives for any terms of years not exceeding ſeven.

Thirdly. A Table of the values, in ſingle and annual payments, of aſſurances on ſingle lives for terms, and for their whole duration.

Fourthly. A Table of the values of two joint lives for all ages.

Fifthly. A Table of the values, in ſingle and annual payments, of aſſurances of groſs ſums, and life annuities payable on the ſurvivorſhip of one life beyond another.—— The moſt material parts of theſe Tables will be found among the other Tables in the next volume. They have been calculated by Mr. *Morgan* with incredible care and induſtry; and are correct and complete to a degree never before attempted in any Tables of this kind. They are to form the baſis of the future buſineſs of the Society, and muſt conduce much to its growing credit and uſefulneſs.

The ſecond, third, and fifth Tables, have been calculated by the rules in Chap. 1ſt of this Treatiſe, Queſtion 6th, 10th, 11th, and 14th. And the other Tables, or the 1ſt and 4th juſt mentioned, in the method deſcribed by Mr.

Mr. *Morgan* in the IId. Section of the ſecond Chapter of his Treatiſe on the Doctrine of Annuities and Aſſurances; a method which, at the ſame time that it leſſens the labour of theſe calculations, prevents the poſſibility of falling into any miſtakes (*a*).

They are all founded on a Table of the probabilities of the duration of human life at *Northampton*, which will be inſerted among the other Tables in the ſecond volume of this work. This Table made a part of all the former editions of this work; but it is, in the preſent edition, much improved, and gives, I believe, more correctly than any other, the mean probabilities of the duration of human life; and, therefore, as I ſhall obſerve again hereafter, ſeems to be properer than any other for general uſe.

I had, in the Introduction to Mr. *Morgan*'s Treatiſe, recommended to the Society the obſervations on human mortality at *Cheſter*; and I had procured a copy of them from Dr. HAYGARTH, the ingenious founder of them. But the directors of the Society have judged very rightly, that they carry the probabilities of life too high for their buſineſs—Theſe Obſervations, however, are not on this account leſs important. I have been enabled

(*a*) *Specimens* of the ſhorteſt and eaſieſt method of making theſe calculations by *Logarithms*, and grounded on Mr. *Morgan*'s method here mentioned, will be given in a Poſtcript to the ſecond of the Eſſays at the end of this volume.

by

by them to make the *Northampton* Table of Obſervations more complete; and Tables of the decrements and expectations of lives, deduced from them, for both ſexes, will be given in the next volume.

The intereſt of money in calculating the new Tables of the Society has been reckoned, as it was in the old Tables, at 3 *per cent.* This gives the Society (more eſpecially at preſent when money may be improved at near double this intereſt) a very great advantage. It likewiſe poſſeſſes the two following advantages.

Firſt. The intereſt of a large and faſt increaſing capital, the greateſt part of which is a SURPLUS over and above all that is neceſſary to enable it to make good its engagements.

Secondly. The profits ariſing from higher probabilities of living among the members of the Society than are exhibited, even in the new Table of Obſervations, by which its demands are for the future to be governed. This Table differs but little from the *Breſlaw* or Dr. *Halley*'s Table of Obſervations, which, as may be ſeen in the Note, p. 171, gives probabilities of living near a *third* lower than thoſe which have hitherto taken place in the Society.

I believe the Society might now ſafely truſt itſelf to the ſecurity ariſing from theſe advantages, and take the payments for aſſurances

ances in ſtrict conformity to its new Tables, without any charge upon them; and the conſequence of this would be, that theſe payments, which four years ago were reduced a *tenth*, will be further reduced about *two tenths*, or in the whole about 30 *per cent.* (*a*).

But as it has been the cuſtom of the Society (in conformity to the recommendation in p. 167) to make an addition of 6 *per cent.* to all the payments required by the old Tables, it may, I think, be excuſed, if, for the ſake of greater ſafety and to provide better for the expences of management, it ſhould make an addition of 3 or 4 *per cent.* to the payments required by the new Tables.

There ſtill remain a few Tables, which perhaps ſome time or other the Society may think proper to furniſh itſelf with.——I will mention the two following.

Firſt. A Table of the ſame kind with that mentioned in p. 125, ſhewing the values of ſums payable *at* a given age, and of annuities payable *till* a given age, to a child, ſhould he loſe his parent.

Secondly. A Table containing the values of aſſurances of annuities for the remainder of life after given terms.

(*a*) It ſhould be remembered, that the reduction here mentioned, does not extend to the *ſingle* payments for aſſurances of *life-annuities* payable on *ſurvivorſhip*, the *London* Table of Obſervations giving theſe almoſt as low as the *Northampton* Table.

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This would inform the public what advantages could be reaſonably offered to perſons who wiſh to be purchaſers of ſuch annuities; and the avidity with which the deceptions in this way have been encouraged renders ſuch an information particularly proper.——The public, indeed, has been led by theſe deceptions to entertain ſuch wrong ideas of the terms on which theſe annuities may be ſold, that probably no terms which the Society can afford will appear ſufficiently encouraging. There are, however, annuities of this kind which, at the ſame time that they have the moſt uſeful tendency, might poſſibly invite purchaſers.——Suppoſe, in particular, an annuity of 1*l.* to commence at 56, and to increaſe at the rate of 1*l.* every year afterwards, (ſo as to become 15*l.* at 70, 25*l.* at 80, and 35*l.* at 90), was offered for a given ſum payable at the age of 30, with proper abatements for every year that the purchaſe was made before this age—Would not ſuch a propoſal be likely to engage attention? And might it not be extremely uſeful, by holding forth an incitement to induſtry in the beginning of life, and providing a way of laying out ſmall ſavings to the beſt advantage?——According to a high valuation the ſum in this caſe would be about twenty guineas; and the proper abatement about twenty-five ſhillings for every year

that a purchaſer's age, if not leſs than 20, falls ſhort of 30.

I muſt not conclude theſe Obſervations on annuity ſchemes without taking particular notice of an excellent example in this way, which is given by a general annuity inſtitution lately eſtabliſhed at *Hamburgh*; and to which I have referred at the end of the fourth Section, p. 127.

Having received, through Mr. *Oeder* at *Oldenburgh*, an account of this inſtitution, and finding that the conductors of it wiſh to extend its advantages beyond the limits of *Hamburgh*; I embrace with pleaſure this opportunity of recommending it, and reciting the following particulars in its plan.

Perſons of all ages who may deſire to increaſe their incomes by purchaſing annuities for their own ſingle lives, or for the longeſt of any two lives, may in this inſtitution purchaſe ſuch annuities.——A perſon at 50 may receive during his life $7\frac{5}{8}$ *per cent.* for any ſum; at 60, he may receive $10\frac{1}{16}$ *per. cent.*; at 70, 15 *per cent.*——Perſons who depend for a ſubſiſtence on the permanency of their capacities for ſervice or labour, may with ſuch ſavings as they may be able to make in their years of vigour, purchaſe for themſelves a competence for old age. A perſon at 40 may with 100*l.* purchaſe 18*l.* *per ann.* for his life after 55; or, for a payment of 17*s.* 6*d.* every half year till he is 60, he may purchaſe for his life after that age

age 6*l.* 12*s.* *per ann.*——Young perſons whoſe fortunes do not produce a ſufficient income may, by ſinking a part of them, procure the means of a future ſettlement in life. A boy, for inſtance, aged 10, may with 100*l.* purchaſe an annuity of 8*l.*$\frac{3}{4}$ dependent on his life till he is 25 years of age, when it may be ſuppoſed he will be provided with other means of ſupporting himſelf.

Perſons who have friends or relations dependent upon them may purchaſe for them, either by a ſingle preſent payment, or by half-yearly payments, any annuities to commence at the time of their ſurvivorſhip, ſhould that happen, and to be continued during the remainder of their lives. In the caſe of widows an abatement is made, if the purchaſer chuſes that the annuity ſhould be paid only during widowhood.

Parents wiſhing to provide *portions* for their children, or ſums for putting them out to apprenticeſhips when grown up to a certain age, may purchaſe (by either half-yearly contributions or ſingle payments), ſuch portions or ſums to be paid them at that age, ſhould they live to it.

The plan of this inſtitution includes in it ſeveral other particulars; but I will only add, that the money received by the conductors of it is lodged in the chamber of *Hamburgh*; that the prices or contributions are diſtinctly ſpecified for every age in

a ſet of Tables which have been publiſhed at *Hamburgh*; and that theſe Tables have been calculated at an intereſt of 3 *per cent.* from ſome of the beſt regiſters of mortality, and (as far as I have examined them) with ſkill and correctneſs.

CHAP.

CHAP. III.

Of PUBLIC CREDIT, *and the* NATIONAL DEBT.

THE *National Debt* is a ſubject in which the public is deeply intereſted. Some obſervations have occurred to me upon it, which I think important; and for this reaſon, though foreign to my chief purpoſe in this work, I ſhall beg leave to offer them to public attention.

The practice of raiſing the neceſſary ſupplies for every national ſervice, by borrowing money on intereſt, to be continued till the principal is diſcharged, muſt be in the higheſt degree detrimental to a kingdom, unleſs a plan is ſettled for putting its debts into a regular and certain courſe of payment. When this is not done, a kingdom, by ſuch a practice, obliges itſelf to return for every ſum it borrows infinitely greater ſums; and, for the ſake of a preſent advantage, ſubjects itſelf to a burden which muſt be always growing heavier and heavier, till it becomes inſupportable.

This ſeems to be now the very ſtate of this nation. At the REVOLUTION, an æra in other reſpects truly glorious, the practice I have mentioned begun. Ever ſince, the public debt has been increaſing faſt, and every new war has added much more to it than was taken from it during the preceding period of peace. In the year 1700, it was 16 millions. In 1715, is was 55 millions. A peace, which continued till 1740, ſunk it to 46 millions; but the ſucceeding war increaſed it to 78 millions; and the next peace ſunk it no lower than 75 millions. In the *laſt* war it roſe to 146 millions and a half. During a peace which has laſted now 10 years, it has been reduced to 138 millions: and at a ſum not much leſs than this, it will, perhaps, be found at the commencement of another war, which may poſſibly raiſe it to 200 millions (*a*).—One cannot reflect on this without terror.—No reſources can be ſufficient to ſupport a kingdom long in ſuch a courſe. 'Tis obvious, that the conſequence of accumulating debts

(*a*) It ſhould be remembered that this was written in 1773.—In 1774 and 1775, two millions of the capital of the 3 *per cent.* annuities were paid off, which reduced the capital of the public debt to 136 millions; and at this ſum, nearly, (ſuppoſing the long annuity then worth 27 years purchaſe) it ſtood at the beginning of the *American* war. The addition made to it by this war, and its amount in the preſent year (1790), may be learnt from the account at the end of the Second Volume of this work.

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ſo rapidly, and of mortgaging poſterity, and funding for eternity, in order to pay the intereſt of them, muſt, in the end, prove deſtructive. Rather than go on in this way, it is abſolutely neceſſary, that no money ſhould be borrowed, except on annuities, which are to terminate within a given period. Were this practiſed, there would be a LIMIT beyond which the national debts could not increaſe; and time would do that *neceſſarily* for the public, which, if truſted to the œconomy of the conductors of its affairs might poſſibly *never* be done.

This, therefore, is one of the propoſals to which on this occaſion, I wiſh I could engage attention.—I am ſenſible, indeed, that the *preſent* burdens of the ſtate would, in this caſe, be increaſed, in conſequence of the greater preſent intereſt which would be neceſſary to be given for money: but I do not conſider this as an objection of any weight. For let the annuity be an annuity for 100 years. Such an annuity is, to the preſent views of men, nearly the ſame with an annuity for ever; and it is alſo nearly the ſame in calculation, its value at 4 *per cent.* being 24½ years purchaſe, and therefore only half a year's purchaſe leſs than the value of a *perpetuity*. Suppoſing, therefore, the public able to borrow money at 4 *per cent.* on annuities for ever, it ought not to give above 1 *s.* 7 *d.*

per cent. more for money borrowed on annuities for 100 years: But ſhould it be obliged to give a *quarter*, or even an *half per cent.* more (*a*), the additional burdens derived from hence, would not be ſuch as could be very ſenſibly felt; and the advantages, ariſing from the neceſſary annihilation of the public debts by time, would abundantly overbalance them.

Theſe advantages would be, indeed, unſpeakably great. By ſuch a method of raiſing money, the expence of one war would, in time, come to be always diſcharged before a new war commenced; and it would be impoſſible that a ſtate ſhould ever have upon it, at any one time, the expence of many wars; or any larger debts than could be contracted within the limited period of the annuities; and, conſequently, it would enjoy the invaluable privilege of being rendered, in ſome degree, independent of the manage-

(*a*) The annuities might be kept 18 years without being much diminiſhed in value; for, ſuppoſing intereſt at 4 *per cent.* an annuity for 82 years, is, within a 49th part, or 2 *l.* in 98 *l.* worth as much as an annuity for a 100 years.

Perhaps, in this way of raiſing money, it might be beſt to offer a higher intereſt at firſt, which ſhould fall to a lower, at the end of given intervals. Thus, though $4\frac{1}{2}$ for 100 years is equal in value to 5 *per cent.* for 17 years, and after that 4 *per cent.* for 83 years, yet the latter might appear more inviting.

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ment of its finances by ignorant or unfaithful ſervants.

I muſt add, that it is by no means neceſſary, that the limited period of the annuities ſhould be ſo long as I have mentioned, or 100 years; and that, at any time before the expiration of this period, the public might employ any ſurplus monies, in extinguiſhing part of the annuities, by purchaſing them for itſelf at the market price; and thus it might aid the operations of time, and keep its debts within any bounds that its intereſt rendered neceſſary. Our government has, I know, in ſome inſtances adopted the plan now propoſed; but it is to be wiſhed that, inſtead of retracting (*a*) it, as was once done, it had been carried much further.

I am, however, far from intending to recommend this plan as the beſt a ſtate can purſue. There is another method of gaining the ſame end, which is, on many accounts preferable to it. I mean, "by providing an "annual ſaving, to be applied invariably, "together with the intereſt of all the ſums "redeemed by it, to the purpoſe of diſcharging the public debts: Or, in other words "by the eſtabliſhment of a permanent SINKING FUND."

(*a*) In the year 1720, the nation was put to the expence of above three millions, in order to reduce ſeveral long and ſhort annuities then ſubſiſting to redeemable *perpetuities*.

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It is well known, that this plan has been alſo adopted by our government; but, though capable of producing the *greateſt* effects in the *eaſieſt* and *ſureſt* manner, it has never been carried into execution. It will abundantly appear, from what follows, that this obſervation is juſt.

Suppoſe the annual ſaving to be 100,000*l.* This ſum, applied *now* to diſcharge an equal debt, bearing intereſt at 4 *per cent.* will tranſfer to the public, from its creditors, an annuity of 4000*l.* The annual ſaving, therefore, would be increaſed to 104,000*l.*; and this ſaving would transfer to the public another annuity of 4,160*l.* and make the ſaving at the beginning of the 2d year, to be 108,160*l.*—Thus, the original fund would go on increaſing, at the ſame rate with money improved at 4 *per cent.* compound intereſt —At the beginning of the 3d year it would be 112,486*l.* At the beginning of the 18th year, 202,581*l.* Of the 36th year, 410,393*l.* and of the 95th year, 4,151,138*l.*—In 94 years, then, the nation might be eaſed of above 4 millions *per annum* in taxes; and above 100 millions of its debts would be diſcharged, gradually and inſenſibly, at no greater expence than 100,000*l.* *per annum*; and without interfering with any of the reſources of government; or making any other difference, than cauſing *funds* to be engaged for

for a courſe of time to the *public*, which would have been otherwiſe neceſſarily engaged to its *creditors*, and which, therefore, muſt have been entirely uſeleſs to it.

It is an obſervation that deſerves particular attention here, that on this plan, it will be of leſs importance to a ſtate what intereſt it is obliged to give for money: For the higher the intereſt, the ſooner will ſuch a fund pay off the principal. Thus; a 100 millions borrowed at 8 *per cent.* and bearing an annual intereſt of eight millions, would be paid off by a fund, producing annually 100,000 *l.* in 56 years; that is, in 38 years leſs time than if the ſame money had been borrowed at 4 *per cent.* (*a*).

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(*a*) What is here ſaid, ſuppoſes the *ſame* fund applied to the diſcharge of debts bearing *different* intereſts. If different funds are applied, bearing to one another the ſame proportion with the intereſts of the debts which they are to diſcharge, the benefit derived from borrowing on lower rather than higher intereſts, will be reduced to almoſt nothing; for the diſburſements of the public on account of all equal loans, will, in this caſe, be nearly the ſame.

The following example will explain and demonſtrate this:

Let a million be borrowed at 3 *per cent.* and let a fund be charged with it, bringing in *ſix ſhillings per cent. per ann.* more than the intereſt; or 33,000 *l.* inſtead of 30,000 *l. per ann.* This *ſurplus*, *unalienably* applied, together with all the intereſts diſengaged by it, will annihilate the *principal* in 81 years, as may be gathered from Table IVth. in the next volume. And the diſburſements, on account of the loan, will be 81 multiplied by 33,000 *l.* that is, 2,673,000 *l.* Let us ſuppoſe again, a million borrowed at

It follows from hence, that reductions of intereſt would on this plan, be no great advantage to a ſtate. They would, indeed, lighten its preſent burdens; but this advantage would be, in ſome meaſure, balanced by the addition which would be made to its *future* burdens, in conſequence of the longer time during which it would be neceſſary to bear them.—I mean this on the ſuppoſition, that the ſavings produced by reductions of intereſt, are immediately applied to the relief of the ſtate, by annihilating taxes equivalent to them. But if that is not the caſe; and if, likewiſe, there is either no plan eſtabliſhed for putting the public debts into a certain courſe of payment, or it is not faithfully carried into execution; in theſe circumſtances, reductions of intereſt may prove hurtful. For, firſt, They would only furniſh funds for contracting further debts, and with more money for ſupplying the deficiencies ariſing from profuſion and bad management. And, ſecondly, As, in ſuch circumſtances, they would only *retard*, and not *prevent* the increaſe of the burdens occaſioned by the public debts, a period would come when the affairs of the ſtate would

at 6 *per cent.* and let a fund be charged with it, producing a ſurplus of *twelve ſhillings per cent. per ann.* ſuch a fund, beſides paying the intereſt, will diſcharge the *principal* in 41 years; and the diſburſements, on account of the loan, will be 66,000 *l.* multiplied by 41; that is, 2,706,000 *l.* or nearly the ſame with the diſburſements on account of an equal loan at 3 *per cent.*

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get near to a *criſis*; and at ſuch a period, its danger would be increaſed, in proportion to the reductions of intereſt that had been made.

In order to underſtand this; let us ſuppoſe that a debt, bearing an annual intereſt of five millions, is the whole debt which a ſtate can bear without being ſo much oppreſſed as to be near ſinking. Let it, however, be ſuppoſed to have ſtill ſome laſt reſources left, which may enable it to bear, for 23 years to come, this load, together with every additional load, which, during this time, may be neceſſary to be thrown upon it.—Let it further be ſuppoſed, that at this time, the ſtate, urged by the fear of an approaching bankruptcy, reſolves upon entering into ſome effectual meaſures for preſerving itſelf.—— Certain it is, that in ſuch circumſtances, no meaſure *ſo* effectual can be purſued, as the eſtabliſhment of a *ſinking fund*, and ſuch a faithful application of it as I have explained. Let that then be the meaſure entered upon; and let the ſtate be ſuppoſed capable of providing a fund, producing a million annually. If all the debts bear intereſt at 6 *per cent.* this fund would pay off three-fifths of them, within the time I have mentioned; or, in 23 years; and the ſtate might be ſaved. But if, in conſequence of reductions, they bear intereſt at no more than 3 *per cent.* the ſame fund would not give the ſame relief, in leſs than

than *double* that time; and therefore, a bankruptcy might prove unavoidable (*a*).

I wiſh I could think, that there is nothing in this repreſentation, that can be applied to the preſent ſtate of this nation. The intereſt of the public debts has been reduced, at different periods, from 6 to 5, from 5 to 4, and from 4 to 3 *per cent.*; but ſtill they have grown with rapidity; and we now ſee ourſelves overloaded, and in no way of gaining relief. Had there been no reductions of intereſt, we ſhould, indeed, have been in the ſame condition ſooner; but, we might have been relieved alſo ſooner, and with leſs difficulty and danger.

In ſhort. Reductions of intereſt are advantageous chiefly when made to gain additions to ſuch a *ſinking fund* as I have deſcribed.—When made with other views, they are only *palliatives*, which give *preſent* relief by increaſing *future* danger; or *expedients* which *poſtpone* a public bankruptcy, by rendering it a calamity more *unavoidable* and *dreadful*. As managed therefore among us, they have been indeed the effects of too narrow a policy, and deſerve none of the *encomiums* which have been beſtowed upon them——The preceding obſervations prove this ſufficiently; but there is one farther

(*a*) It ſhould not be forgotten, that the operation of ſuch a ſcheme would ſoon raiſe the 3 *per cent.* debts nearly to *par*, and render the diſcharge of them at a diſcount impracticable.

proof

proof of it which I cannot help mentioning. —Suppoſe 200,000*l. per ann.* to have been gained in 1716, by the reduction which was then made of the 6 *per cents.* to 5 *per cents*; or, in other words, by ſaving 1 *per cent. per ann.* on a capital of 20 millions. This ſaving, in conſequence of being applied unalienably in the manner I have repreſented, to the paymant of the public debts, would, in 37 years, have diſcharged a debt of 20.325,000*l.* bearing 5 *per cent.* intereſt. But if applied every year to current ſervices, in order to avoid levying new taxes, the benefit derived from it in the ſame period, would be 37 times 200,000*l.* or 7.400,000*l.* but at the ſame time, a debt would have been continued of 20 millions, which muſt have been otherwiſe paid. The effect, therefore, in this caſe, of the reduction, would be to prevent an incumbrance on the public of 200,000*l. per ann.* by leaving upon it an incumbrance of a million *per ann.* rendered more difficult and unlikely than ever to be removed (*a*).

But

(*a*) There is an inconvenience ariſing from reductions of intereſt, which I did not attend to when this Chapter was compoſed, but which is greater than any here mentioned. They have given riſe to the extravagant *douceurs* or *premiums* which have been annexed to our public loans, by cauſing the lenders of money to conſider whatever intereſt could be offered above 3 *per cent.* as no more than a ſhort annuity:—Had there been no reduction of intereſt before the war which begun in 1755, the price of the *four per*

But to return to the ſubject I have principally in view.

per cent. ſtocks would have been during the whole war either *above* par or not much *below* it; and government might have borrowed at this or a lower intereſt without *premiums.* But the reduction which had juſt taken place of 56 millions to an intereſt of 3 *per cent.* made it unavoidable for monied men to conſider this ſtock as only a 3 *per cent.* ſtock with 1 *l. per ann.* added till peace was eſtabliſhed; and this made it neceſſary for government, either to promiſe intereſt extravagantly high, or to offer long, or ſhort, or life-annuities, or additional capitals, as rewards for lending money at 3 *per cent.* The laſt method has conſtantly been adopted; and in conſequence of it, the public has been put to an enormous expence, which the avoiding of reductions and a little management would have rendered entirely unneceſſary.

I have endeavoured to explain this in the ſecond Tract on Civil Liberty, Part II, Sect. III, and particularly in the Supplement to that Tract, where regulations are propoſed for raiſing the value of ſtocks bearing higher intereſt than 3 *per cent.* in order to avoid the neceſſity of offering high *premiums* and creating artificial debts.—The following example will ſhew what occaſion there is for ſome ſuch regulations.——The 3 *per cent.* ſtocks were (Oct. 25th, 1781,) at 55 $\frac{1}{4}$; and the price of the 4 *per cent.* ſtocks, which, in order to bear a proportionable price, ought to have been 73$\frac{3}{4}$, was (deducting a fortnight's intereſt) 70$\frac{5}{8}$. One *per cent.* added to make a *five per cent.* ſtock, would have been ſtill more undervalued; and ſuch a ſtock, even with the advantage of being declared irredeemable for 15 or 20 years, would not probably have ſold for more than 82 *l.* or 83 *l.* Had, therefore, government attempted to borrow by offering 5 *per cent.* it could not have ſucceeded without giving a *premium* of near 20 *l.* for every 100 *l.* advanced. But, by ſuch regulations as thoſe propoſed in the *Supplement* to which I have juſt referred, it would, I imagine, have become practicable to borrow at intereſts correſponding nearly to the prices of the 3 *per cents.* without *premiums.*

What

What I have ſaid implies, that a ſtate always diſcharges its debts, whatever intereſt they bear, by paying the original ſum borrowed. It may, perhaps, be imagined, that when a loan is under *par*, it may be diſcharged at a leſs expence. But this is by no means ſo practicable as it may ſeem; for it ſhould be conſidered, that a public loan, now under *par*, would not long keep ſo, after being put into a courſe of payment: And for this reaſon, as a ſtate can never be obliged, in redeeming its debts, to pay *more* than the original ſum borrowed, ſo neither ought it to expect, in general, to be able to redeem them by paying *leſs*. I have ſaid, *in general*; for I am ſenſible that, at the beginning of the operations of a fund, when its produce is ſmall, and alſo in a time of war, a ſtate might derive great advantages from the low price of its debts. And I am ſenſible alſo, that conſiderable advantages might be derived from *lotteries*, in paying the public debts: But *lotteries* do great miſchief in a ſtate, by foſtering the deſtructive ſpirit of gaming. It is wretched policy to make them familiar, by recurring to them in the ordinary courſe of government. There are great occaſions on which they may be neceſſary, and for ſuch occaſions they ſhould be reſerved.

The advantages of putting the public debts into ſuch a courſe of payment as I have deſcribed, are ſcarcely to be imagined. It would give a vigour to public credit, which would enable a ſtate always to borrow money eaſily, and on the beſt terms. And the encouragement to lenders might be always improved, without any inconvenience, by making every loan irredeemable, during the firſt 20 or 30 years; for, there could ſeldom be any occaſion for beginning to diſcharge any *one* loan ſooner.

It might be eaſily ſhewn, that the faithful application, from the beginning of the year 1700, of only 200,000 *l.* annually, would long before 1790, notwithſtanding the reductions of intereſt, have paid off above a hundred millions of the public debts. The nation might, therefore, ſome years ago, have been eaſed of a great part of the taxes with which it is loaded. The moſt important relief might have been given to its trade and manufactures; and it might now have been in better circumſtances than at the beginning of the laſt war; its credit firm; reſpected by foreign nations, and dreaded by its enemies. A new account might alſo ſome years ago have been begun; and another fund, not much more conſiderable, applied in the ſame way, would, in a ſhort courſe of years, have paid all that would have been now unpaid

of

of our debts. And thus, without any expence that could be ſenſibly felt, its debts, as ſoon as they began to grow heavy, might have been conſtantly reduced to a *half*, or a *third*; and not only all *danger*, but all conſiderable *inconvenience* from them prevented.

All I have now ſaid, ſuppoſes a *ſingle* fund with a *general* appropriation to the payment of the public debts. The ſame ends might be anſwered by *particular* funds, with ſmall ſurpluſſes, appropriated to *particular* debts. In the wars of King *William* and Queen *Anne*, an intereſt of ſix, and ſometimes ſeven and eight *per cent.* was given for loans. It would have been eaſy to have annexed to each loan a *fund* producing a *ſurplus* of 1 *l. per cent.* after paying the intereſt; and ſuch a *ſurplus* would have been ſufficient to annihilate the principal of every loan in 33 years. Had this plan been followed, the diſengagement of the public funds, and the relief attending it, would have begun 60 years ago; and the debts contracted, during the reigns of King *William* and Queen *Anne*, would have been all cancelled near 30 years ago, without any of that trouble, tumult, and diſtreſs, which have been occaſioned by reductions of intereſt, and by the various ſchemes which have been tried for leſſening the debts (*a*).—

A

(*a*) The ſums to be laid out would, in this caſe, be ſo ſmall at firſt, that it would be proper to employ them in

A fund, yielding 1 *l. per cent.* ſurplus, annexed to a loan at 5 *per cent.* would diſcharge the principal in 37 years (*b*). At 4 *per cent.* in 41 years. At 3 *per cent.* in 47 years.

Theſe obſervations relate only to what *might* have been the ſtate of the nation with reſpect to its debts, had a right plan been purſued from the firſt. But it will be aſked, What can be done with them as they *are?*— I wiſh I was able to give a more ſatisfactory anſwer to this enquiry. Every one muſt ſee our proſpect to be diſcouraging, and our ſtate hazardous. Some have thought, that a good method might be found out of diſcharging the national debt, by ſhort annuities, and life annuities. The following obſervations will ſhew how groundleſs an imagination this is.

purchaſing part of the loan to be annihilated at the prices in the public market; and this, as far as it can be carried, is the moſt eaſy and quiet and ſilent way poſſible of extinguiſhing the public debts.

(*b*) I have all along ſuppoſed the produce of the public funds to come in yearly. The truth is, that it comes in *half*-yearly; but this gives no advantage in the payment of the public debts worth taking into account. 1 *l. per annum*, together with its growing intereſts, at 4 *per cent.* taken *yearly* out of 100 *l.* will reduce it to nothing in 41 years; if taken *half*-yearly, it will annihilate the ſame capital only four months and 12 days ſooner.

Short

Short annuities and life-annuities have been always undervalued by the public; and were they offered to ſale to ſuch an amount as would be neceſſary to make any conſiderable reduction in the national debt, they would probably fall to a very low price. Let the three *per cent.* ſtocks be ſuppoſed at 86. 100*l.* of this ſtock would not be given up for a life-annuity leſs than 6*l.* or a ſhort annuity for a leſs term than 30 years; for this is valuing them at 14½ years purchaſe, and life-annuities in particular have never yet been diſpoſed of at ſo high a price.—In order, therefore, to diſcharge in this way a capital in the 3 *per cent.* annuities of 33 millions and a third (the intereſt of which is a million *per ann.*) it would be neceſſary to provide a ſurplus producing another million *per ann.* for 30 years. But this ſurplus, employed during the ſame time as a ſinking fund, would redeem 47 millions and a half at *par*, and 51 millions and a half at 86*l.* in *money* for every 100*l. ſtock*. It would, therefore, be great folly to employ ſuch a ſurplus in the former way rather than the latter.

But I will beg leave to detain the reader here a little longer——The obſervations now made may be of uſe in ſhewing what the beſt method is of incurring debts as well as redeeming them.

Suppoſe a million raiſed by *annuities* on a ſet of lives, all at 30 years of age. The purchaſers of ſuch annuities cannot reaſonably be reckoned to have an expectation of leſs than 30 years. That is; the duration of their lives, taking them one with another, will be 30 years; and they will be entitled, ſuppoſing intereſt at 4 *per cent.* to 7*l.* *per annum*, for every 100*l.* advanced. For a million then, the public would make 30 payments of 70,000*l.*—Let us ſuppoſe next, that a fund producing this ſum annually, inſtead of being engaged to pay theſe life-annuities, is engaged for 30 years, to pay the principal and intereſt of a million, borrowed on *redeemable* perpetuities, at 4 *per cent.* There will, at the end of the firſt year, be a ſurplus of 30,000*l.*—In conſequence of applying this to the extinction of the principal, it will be reduced to 970,000*l.* on which, at the end of the ſecond year, the intereſt due will be 38,800*l.* There will, therefore, be a ſaving of 1200*l.* Inſtead of employing this ſaving in further ſinking the *principal*, which would cauſe the fund to accumulate in the ſame manner with money at compound intereſt, let it be taken and employed in any other way: And let the ſame be done with all the ſubſequent ſavings, reſerving only 30,000*l.* annually, for the purpoſe of ſinking the principal. At the end of the ſecond year, the principal will be 940,000*l.*;

and

and the ſaving of intereſt upon it, at the end of the third year, 2400*l.* At the end of the 30th year, the principal will be reduced to 100,000*l.* The ſaving of intereſt that year will be, 1200*l.* multiplied by 29, or 34,800*l.* and the ſum of all the ſavings will be 522,000*l.*—Deduct from hence 100,000*l.* remaining then undiſcharged of the principal; and 422,000*l.* will be the loſs the public would ſuſtain, in the circumſtances I have ſuppoſed, by raiſing money on life-annuities. But if we ſuppoſe the ſavings, as they ariſe, as well as the conſtant ſum of 30,000*l.* to be applied to the diſcharge of the principal, inſtead of being ſpent on current ſurvices; the whole million will be annihilated in 21 years and a half; and the loſs to the public by preferring life-annuities, will be 8 $\frac{1}{2}$ years purchaſe of the annuities; or 595,000*l.*—By ſimilar deductions it may be eaſily found, that the loſs, in *younger* lives, is greater; in *older* lives leſs; but never inconſiderable, except in the *oldeſt* lives.

It appears, therefore, that in conſequence of ſuch a way of raiſing money, the public muſt always pay much more in intereſt than there is any occaſion for; and *waſte* a ſum equal to more than half the principal borrowed (*a*). This, however, though ſo waſteful,

(*a*) It is obvious, that the obſervations here made, may be applied to the common methods of raiſing money on life-annuities, for building churches, paving ſtreets,

ful, is a more frugal way of procuring money than by borrowing on *perpetuities*, without putting them into a courſe of redemption; for in this caſe the loſs may be *infinite*.

But to return.

making navigations, &c. &c. And, in general, to all caſes where the money received is not laid up to be improved. —For, to view this ſubject in another light, let us ſuppoſe 10,000*l.* borrowed for any public work, on perpetuities, at 4 *per cent.* And, if that will afford more encouragement, let them be made irredeemable for any number of years leſs than ſeventeen. Let us further ſuppoſe ſuch rates, or tolls, eſtabliſhed for the payment of the intereſt and principal, as ſhall produce *double* the intereſt of the ſum borrowed; or 800*l. per annum*, inſtead of 400*l. per annum*. Let the *ſurplus*, as it comes in *half-yearly*, be laid up to accumulate in the public funds. In 17 years and a half, reckoning intereſt at 4 *per cent.* a capital will be raiſed, equal to the whole ſum borrowed; and, therefore, at the end of that time, the whole debt may be diſcharged, and the whole tranſaction finiſhed.—But if the ſame ſum had been borrowed on annuities, for the lives of a ſet of perſons 50 years of age, at 8 *per cent.* which is 1*l. per cent.* leſs than the true value of ſuch annuities: Had this, I ſay, been done, *half* the annuitants would have been alive at the end of the term I have mentioned; and the whole tranſaction, together with the expences and trouble attending the management of it, could not have been finally cloſed till the extinction of all the lives; that is, not in leſs time, moſt probably, than 35, or, perhaps, 40 years.—It is a neceſſary obſervation here, that, if public credit maintains its ground, much will not depend, in the plan now propoſed, on the riſe and fall of STOCKS. If a *war* ſinks them, the money laid out. While the war laſts, will accumulate faſter. If a peace raiſes them, the money that had been previouſly laid out will be proportionably increaſed.

The

The enquiry which has occaſioned this digreſſion, muſt be intereſting to every perſon who wiſhes well to this country.—All ſchemes for diſcharging the public debts by life-annuities, have been ſhewn to be abſurd and extravagant.—In general it may be obſerved, that it is far from probable, that any money which the nation can ſpare, if applied ſo as to bear only *ſimple* intereſt, can be capable of reducing its debts within due bounds; or of doing us, in our preſent circumſtances, any eſſential ſervice. A fund, producing a ſurplus of even two millions annually, would, when thus applied, pay no more than 40 millions in 20 years; and, in that time, a war might probably come, which would interrupt the application of it; and increaſe our debts much more than ſuch a fund had leſſened them.

Certain it is, therefore, that if our affairs are to be retrieved, it muſt be by a *fund* increaſing itſelf in the manner I have explained. The ſmalleſt *fund* of this kind is, indeed, *omnipotent*, if it is allowed time to operate. But we are, I fear, got ſo near to the limits of the reſources of the nation, that it cannot be allowed much time: And, in order to make amends for this, it is neceſſary that it ſhould be *large*.

In redeeming the public debts, conſiderable advantages might be derived from their low price, and this advantage, in a time of war,

war, would be nearly the ſame in whatever manner the redemption was made: That is, whether it was always made at *par*, or at the market prices below *par*. The latter method would be the beſt; but ſuppoſing the former adopted, an equivalent advantage would ariſe from the power it would give to borrow money on lower terms.—Suppoſe, for inſtance, that in a time of war, four millions muſt be borrowed for the ſervice of the year; and let the produce of the fund be then increaſed to two millions; and the intereſt of money in the *ſtocks*, above 4 *per cent*. In theſe circumſtances, it would be the intereſt of the lenders of money, to take 3½ *per cent*. for the ſums they advanced, in conſideration of having their 3 *per cents*. paid off at *par*, to the amount of half theſe ſums.—War, therefore, would accelerate the redemption of the public debts; and it would do this the more, the longer it laſted, and the higher it raiſed the intereſt of money. Or if, in conſequence of paying always at *par*, this could not happen; an equivalent effect would be produced in the way juſt mentioned. The ſtocks would be always kept up by the operations of the fund; and, in proportion to the ſums yielded by it, the public would be able to borrow money more advantageouſly, and leſs would be added to its burdens.—This ſeems to me an obſervation of particular conſequence. It demonſtrates,

ſtrates, that the invariable application, in *war* as well as *peace*, of the produce of the fund I am ſuppoſing, to the payment of the national debts, rather than to any current ſervices, would, independently of its effect in redeeming theſe debts, be attended with great advantages to the public. But this is a ſubject on which I ſhall have occaſion to ſay more preſently.

The *ſinking fund*, in its preſent ſtate, and, after ſupplying the deficiencies of the peace eſtabliſhment, yields, I ſuppoſe, a conſiderable part of a million and a half *per ann.* (*a*). And I cannot doubt but that ſuch ſavings might be made in the collection and expenditure of the national revenue, as would cauſe this fund to yield the *whole* of this ſum, without impoſing any new burdens on the public. But, were there, indeed, no way of providing any part of it, but by creating new funds, or impoſing new taxes;

(*a*) A more careful enquiry has ſhewn me that the true ſurplus of the public income was at the time when this was written much leſs than I imagined. For five years after 1763 it was no more than 30,000 *l. per ann.* For five years before 1775 it was 338,729 *l.*; and the medium for the whole duration of the peace between the two laſt wars was 196,000 *l. per ann.* according to the accounts in the ſecond Tract on Civil Liberty, Part 3d, Sect. 4th. And from the commencement of the preſent peace to the preſent year 1790, the ſtated revenue has produced *no* ſurplus, as I fancy will appear from the ſtatements at the end of this work.

it

it *ought* to be done, becauſe it *muſt* be done, or the nation ſink.

The evils and dangers, attending an *exorbitant* public debt in this country, are ſo great, that they cannot be exaggerated.—Without repeating, what has been ſo often ſaid, of its increaſing the dependance on the crown; rendering us tributary to foreigners; and raiſing the price of proviſions and labour; and, conſequently, checking population, and loading our trade and manufactures; I will only take notice of the following evils which attend it.

Firſt. The execrable practices of the *Alley*. Theſe cannot be mentioned in language too ſtrong. They are increaſing every day; and the national debt, by giving occaſion to them, is likely ſoon (with the aid of annual lotteries) to ruin all honeſt induſtry among us, and to turn us into a nation of gamblers.

Secondly. It muſt check the exertions of the ſpirit of liberty in the kingdom. The tendency of every government is to deſpotiſm; and in this it muſt end, if the people are not conſtantly jealous and watchful. Oppoſition, therefore, and reſiſtance, are often neceſſary. But they may throw things into confuſion, and occaſion the ruin of the public funds. The apprehenſion of this muſt influence all who have their intereſt connected with

with the preſervation of the funds, and incline them always to acquieſcence and ſervility.

But further. It expoſes us to particular danger from *foreign* enquiries by inviting inſults, and rendering us incapable of meeting war, however neceſſary, without the hazard of bringing on terrible convulſions, by overwhelming public credit.

All theſe are evils which muſt increaſe with every increaſe of the national debt; and there is a point at which, when they arrive, the conſequences muſt be fatal (*a*).

But to proceed to ſome further obſervations.

What has been ſaid, has all along ſuppoſed a *ſacred* and *inviolable* application of the fund I have deſcribed, and of all its earnings, to the purpoſe of ſinking the national debt.

(*a*) "Either the nation (Mr. *Hume* ſays, Eſſays Vol. II. p. 145) muſt deſtroy public credit; or public "credit will deſtroy the nation." Mr. *Gordon*, in the Preface to Cato's Letters, tells us, that the great and good Mr. TRENCHARD had two things much at heart, namely, keeping *England* free from foreign broils, and *paying off the public debts*. He thought that one of theſe depended on the other, and that the *being* of the ſtate depended on the latter. Mr. *Gordon* adds, that he believed no one, who thought at all, could think Mr. TRENCHARD miſtaken.

fund

The whole effect of it depends on its being allowed to operate, WITHOUT INTERRUPTION, a proper time. But it may be aſked, how this can be ſecured? Or, by what method an object, that muſt be continually growing more and more tempting, can be defended againſt invaſion and rapine?—I might here mention the ſuperintendency and care of the repreſentatives of the kingdom, the faithful guardians of the ſtate, to whom miniſters are reſponſible for the uſe they make of the public money. But experience has ſhewn, that we cannot rely on this ſecurity.—The difficulty, therefore, now mentioned, is the very greateſt difficulty the nation has to ſtruggle with in the payment of its debts.

The ſinking fund was eſtabliſhed in the year 1716, or ſoon after the acceſſion of the preſent family; at a time when the public debts, though not much more than a third of what they are now, were thought to be ſo conſiderable as to be alarming and dangerous. It was intended as a SACRED DEPOSIT never to be touched; the law which eſtabliſhed it declaring, that it was to be applied to the payment of the principal and intereſt of ſuch national debts and incumbrances, as had been incurred before the 25th of *December* 1716; *and to no other uſe, intent, or purpoſe whatever.*—The faith of *parliament*, therefore,

therefore, as well as the ſecurity of the kingdom, ſeemed to require, that it ſhould be preſerved carefully and rigorouſly from alienation. But, notwithſtanding this, it has been *generally* alienated; and the produce of it employed, in helping to defray ſuch current expences as the exigences of the ſtate rendered neceſſary.

In order to juſtify this, it has been uſual to plead, that when money is wanted, it makes no difference, whether it is taken from hence, or procured by making a new loan. But in truth the difference between theſe two methods of procuring money is no leſs than *infinite*.—For, by employing the SINKING FUND in bearing current expences, rather than borrowing *new* money on new funds; the ſtate, in order to avoid giving *ſimple intereſt* for money, is made to alienate money, that *muſt* have otherwiſe been improved at *compound intereſt*; and which, in time, would have *neceſſarily* increaſed to *any* ſum.—Had a faithful uſe been made from the firſt, of only one THIRD of the produce of this fund, the greateſt part of our preſent debts would now have been diſcharged. (*a*)—Can

(*a*) The principal obſervations in this Chapter, I have given juſt as they occurred to my thoughts, without knowing that any of them had been made by other writers. Some propoſals and obſervations of a ſimilar nature, I have ſince found in an excellent pamphlet publiſhed

Can it be poſſible then to think, without the deepeſt regret, of that miſapplication of this fund, which, with the conſent of parliaments always complying, our miniſters have practiſed?

SUPPLEMENT.

THE following account of our Public Funds *in general*, and of the SINKING FUND *in particular*, having been, publiſhed in another work too incorrectly, I have thought proper to introduce it here as a *Supplement* to the preceding Chapter.

liſhed in 1726, entitled, *An Eſſay on the National Debts of this kingdom, wherein the importance of diſcharging them is conſidered, and ſome general miſtakes about the nature and efficacy of the* SINKING FUND *examined and removed. In a Letter to a Member of the Houſe of Commons. Fourth edition.*

The

The *Britiſh* Funds have been all formed into the four following claſſes or diviſions.—The AGGREGATE FUND; the SOUTH-SEA FUND; the GENERAL FUND; and the SINKING FUND.

The *Aggregate Fund* was eſtabliſhed by an act of *Geo.* I. cap. 12. in 1715. It had this name given it, becauſe it conſiſted of a great variety of taxes and ſurpluſſes of taxes which were in that year conſolidated, and given as a ſecurity for the diſcharge of the intereſt and principal of debts due to the Bank of *England*, and ſome other public debts; and alſo for the payment of 120,000*l. per ann.* to the civil liſt. Into this *fund* are brought the two-thirds and one-half ſubſidy of tonnage and poundage; half the inland duties on tea and coffee; the houſe-money granted by the 7th of *Will.* III.; the duty on hops; the duties on low wines, brandy, and *Britiſh* ſpirits; all arrears of land-taxes; all public monies not appropriated; the ſurpluſſes of the nine-penny exciſe, of the five ſevenths of the Bank nine-penny exciſe, of the revenues in the annuity acts of the 4th, 5th, and 6th of Queen *Anne*, &c. and, by an act of the 1ſt of Geo. III. all the duties conſtituting the revenue of the civil liſt.

The *South-ſea fund* was eſtabliſhed, by ſtat. 3 *Geo.* I. cap. 9. in 1716; and is ſo called, becauſe appropriated to pay the intereſt of the *South-ſea* company's capital.

It conſiſted of a duty on candles, and certain impoſts on wines, vinegar, tobacco, and *Eaſt-India* goods.

The *General Fund* was alſo eſtabliſhed, by ſtat. 3 *Geo.* I. cap. 7. in 1716, and conſiſted of a ſubſidy on goods exported; a tax on hackney-coaches and chairs; duties on ſoap, hides, ſtamps, and policies of inſurance; 700*l.* per week letter-money; a moiety of the inland duties on tea and coffee; and 39,855*l.* *per annum*, out of the hereditary exciſe on beer for the bankers annuities. All theſe taxes were appropriated to the diſcharge of the intereſt of 7,808,087*l.* (originally 10,000,000*l.*) capital ſtock of *South-ſea* annuities, together with charges of management.

All that remained of the produce of the taxes thus digeſted into theſe three *funds*, after ſatisfying the charges upon them, was in the ſame year (or 1716) carried into a fourth *fund*, to which was given the name of the *Sinking Fund*, becauſe appropriated to the purpoſe of *ſinking* the public debts. The words of the act of the 3d of *Geo.* I. which eſtabliſhed this *fund*, are, "All the "monies to ariſe from time to time, as "well of the exceſs and ſurplus of an act "made this ſeſſion for redeeming the *funds* "of the Bank of *England*; and of the ex-"ceſs or ſurplus by virtue of one other act "made likewiſe this ſeſſion for redeeming

"the

" the *funds* of the *South-ſea* company; as " alſo, of the exceſs or ſurplus of the du- " ties and revenues by this act appropriated " as aforeſaid; and the overplus monies of " the ſaid *General Fund* by this act eſta- " bliſhed, ſhall be appropriated to the diſ- " charging the principal and intereſt of " ſuch national debts as were incurred be- " fore the 25th of December, 1716, and " are declared to be national debts; and to " or for no other uſe, intent, or purpoſe " whatſoever." The tranſactions with reſpect to this fund make a very important part of the hiſtory of *Britain*; and furniſh us with a ſtriking inſtance of the depravity and folly which often ruin kingdoms.

Before its eſtabliſhment there had exiſted many ſmaller *funds* of the ſame nature; that is, ſuch duties or taxes had been provided for paying the intereſts of particular loans, as afforded ſurpluſſes by which the principal itſelf was to be gradually redeemed. This was the common practice in the reigns of King *William* and Queen *Anne*. Moſt of the public duties were given for terms of years; and at the end of thoſe terms they ceaſed of courſe, unleſs continued for farther terms by new acts of parliament: And, in general, it was provided, when any money was raiſed, that the principal ſhould be cancelled either by time, as in the caſe of the ſale of long and ſhort annuities, or by the ſurpluſſes of

the duties charged with the payment of the intereſt. This was an excellent plan; but it was by no means carried into execution. In the year 1720, moſt of the long and ſhort annuities were converted into redeemable perpetuities, at the expence of above three millions; and the ſurpluſſes of the duties charged with particular loans were often ſo broken into, by being either charged with new loans before they had cancelled the old, or ſpent on current ſervices, as to be rendered incapable of anſwering the end intended by them. In conſequence partly of this bad management, the public debts at the acceſſion of the Houſe of *Hanover* were ſo much increaſed as to be generally reckoned *inſupportable*; and their reduction was made one of the firſt objects of parliamentary attention. This gave riſe, in 1716, to the inſtitution of the *fund* of which we are giving an account, the father of which was (as has been generally ſaid) Sir *Robert Walpole*, but, in reality, Earl *Stanhope*. All the taxes, except the land-tax and ſix-pence *per* buſhel malt-tax (*a*), were then made perpetual, and diſtributed into the three *funds* which have been deſcribed, the ſurpluſſes of which, for ever afterwards, were

(*a*) Theſe taxes have been always voted by parliament from year to year; and are, on this account, diſtinguiſhed by the name of the *annual* taxes; and wholly employed, as far as they will go, in bearing the current expences of every year.

to

to be held ſacred, and to be applied *inviolably*, according to the words of the act juſt recited, to the redemption of the national debt.

A conſiderate perſon might have ſuſpected, that the ſame cauſes which had rendered former partial appropriations ineffectual, would deſtroy the efficacy of this. There ſeemed, however, to be reaſon for hoping the contrary: For,

Firſt, the future happineſs and glory of the kingdom were thought to depend (*a*) on this appropriation; and the law which eſtabliſhed it was declared to be a *fundamental* law of the realm.

Secondly, in conformity to theſe ſentiments, the words of this law were made as ſtrong as poſſible; and, in order to give additional force to it, a repetition of it, in the ſame words, was inſerted in an act of the 5th of *Geo.* I. cap. 3.—Particular notice ſhould be taken of theſe words.—They order that all the ſurpluſſes of the taxes then made perpetual, ſhall be applied to the diſcharge of the public debts, and " to no other uſe or intent

(*a*) The alarm occaſioned about this time by the public debts (which have been ſince twice doubled) and the eager expectations entertained from this fund, appear remarkably, from a fact juſt mentioned; or from the converſion (at the expence of *three millions*) of moſt of the long and ſhort annuities then ſubſiſting into *redeemable* perpetuities, in order to ſubject them to the operation of this fund.

"whatever." When, therefore, a debt had been paid off, the addition ariſing from that payment to the ſurpluſſes (or the annuity diſengaged by it) became a part of the *fund*, and, together with it, was to be employed in diſcharging farther debts. And the ſame being true of every ſucceſſive annuity diſengaged by every payment, the *fund*, if never miſapplied, muſt neceſſarily have operated in ſinking the public debt, in the ſame manner that money accumulates, when put out to bear compound intereſt. And in this way this *fund* did in fact operate for a few years. While in its infancy, it was watched with great care. The improvement and the inviolable application of it was recommended in moſt of the ſpeeches from the throne, and echoed back in the addreſſes of the Houſe of Commons. It is particularly obſervable, that ſo well did our miniſters then underſtand the nature and importance of this *fund*, that rather then encroach upon it, they frequently borrowed money in order to defray the neceſſary expences of government. From ſome publications in 1726 it appears, that ſome perſons had been led to apprehend this zeal of the miniſtry would not be permanent, becauſe it was not their intereſt to pay off the public debt, on account of the dependence and influence created by it. In anſwering this objection, the writers on the ſide of the court called

ſuch

ſuch an apprehenſion an *indecent jealouſy*, and took upon them to aſſure the public, "that in no poſſible exigence of affairs "could our miniſters ever approve of or "recommend the alienation of the *ſinking* "*fund.*" Happy would it have been for *Britain* had this proved true: But in a little time it appeared, that the apprehenſions which had been ſtiled *indecent jealouſies*, were too well grounded. Men in power came ſoon to ſee, that this *fund* was advancing too faſt in its operations, and to change their zeal for it into a reſolution to deſtroy it. This will abundantly appear from the following facts.

Charging the income of the *ſinking fund* with the payment of the intereſt of new loans, is an encroachment upon it, no leſs ſubverſive of its efficacy, than depriving it of groſs ſums; there being no difference between taking from it the annual intereſt of a ſum, and that ſum itſelf. Between the years 1727 and 1732 ſeveral incroachments of this kind had been made upon it; but, being of a leſs obvious nature, they paſſed without any particular oppoſition. The finiſhing blow was given in the year 1733. In that year, in order to keep the land-tax at one ſhilling in the pound, it was neceſſary either to borrow half a million for the current ſervice, or to take half a million from the *ſinking fund*. The laſt method was choſen; and propoſed by Sir *Robert Walpole*

to the Houſe of Commons. Long and warm debates enſued. A propoſal to alienate, in a time of profound peace, a *fund* which the law had made ſacred, and the alienation of which *no poſſible exigence of public affairs* could juſtify, only for the ſake of keeping the land-tax for one year at one ſhilling in the pound, juſtly kindled the indignation of the patriotic party. They urged the prohibition of the law, the faith of parliament, and the ſecurity of the kingdom. The propoſer of the alienation was reminded of his inconſiſtency and treachery, in endeavouring to beat down that very monument of glory which he had boaſted of having erected for himſelf; and Sir *John Barnard* warned him, that he was drawing upon himſelf the curſes of poſterity. But all arguments were vain. The miniſtry pleaded that the landed intereſt wanted eaſe; that there was no occaſion for being in a hurry to pay the national debt; and that the circumſtances of the kingdom had altered ſo much ſince the eſtabliſhment of the *ſinking fund*, that the competition then among the public creditors was, not who ſhould be *firſt*, but who ſhould be *laſt* paid. Thus argued, among others, Sir *Robert Walpole*. *His* reaſons prevailed; and the Houſe of Commons conſented.

The practice of alienating the *ſinking fund* having been thus begun, went on of courſe. In the next year, or 1734, 1,200,000 *l.* was taken

taken from it. In 1735, and 1736, it was anticipated and mortgaged.

Thus expired, after an exiſtence of a few years, the *ſinking fund*; that ſacred bleſſing (as it was once thought) and the nation's only hope. Could it have eſcaped, it would long before this time have eaſed *Britain* of all its debts, and left it ſafe and happy.

In order to obtain a juſter ſenſe of this, let us here compare what it *would* have done had it never been miſapplied, with what it *has* done.

Though the act that eſtabliſhed it was paſſed, as already ſaid, in 1716, it did not begin its operation till 1719, when three quarters of a million in old Exchequer bills were paid off with it. The intermediate time had been employed in laying the foundations of this *fund*, and providing an income for it, by a general reduction of the public debts, from an intereſt of ſix *per cent.* and other higher intereſts, to five *per cent.* What made this reduction then practicable was a rapid fall of the intereſt of money, which begun (in conſequence of an increaſe of trade producing an influx of money) a little before the acceſſion (*a*). The means uſed by government for accompliſhing this reduction were, firſt, the addition (at *Michaelmas* 1717) of the intereſt

(*a*) The *legal* intereſt of money was reduced in 1714, from ſix to five *per cent.*

of

of ſome debts bearing five *per cent.* to the principal, in order to make uſe of the produce of the taxes which ſhould have paid that intereſt, in diſcharging the bankers debt and ſome other debts bearing ſix *per cent.*

Secondly, loans at five *per cent.* obtained chiefly from the Bank and the *South-ſea* company, to pay off ſuch of the public creditors as did not chuſe to accept a lower intereſt than ſix *per cent.*

After this reduction, the three *funds*, before deſcribed, produced a ſurplus of above half a million *per annum.* In 1727 this ſurplus was increaſed to 939,103 *l.* and in 1733 (the year when the practice of alienating it begun) it had been increaſed ſo much by the redemptions made with it, and by a ſecond reduction of intereſt in 1727 from five to four *per cent.* that its medium for five years had been 1,212,000 *l.* *per annum.* Had it, from the year 1732, been allowed no increaſe beyond this ſum (except from the intereſt of debts paid by it), and been applied for the firſt twenty-five years to the payment of debts bearing four *per cent.* intereſt, and afterwards to the payment of debts bearing three *per cent.* it would (in the year 1781) have completed the redemption of more than one hundred and ſixty millions of debt, leaving the public during this whole period, in poſſeſſion of all the

the ſurpluſſes which have ariſen in the revenue beyond 1,212,000*l.* except thoſe produced by redemptions. It is not poſſible to conceive the beneficial effects with which this would have been attended, or the vigour which would have been all along given to public credit by ſuch a *fund*, and by the proſpect it would have given of the total annihilation in a few years of all the public debts, and the diſengagement of the taxes ſupporting them. No perſon who duly attends to this, and wiſhes well to *England*, can avoid execrating the policy which firſt produced, and has ſince continued, the alienation of the *ſinking fund*, and converted an expedient for ſaving the kingdom, into a ſupply for extravagance, and a ſupport of corruption and deſpotiſm. This, however, is a policy which it may be expected men in power will always uſe when they can; for few of them (except when well watched) have ever ſhewn themſelves ſuperior to the temptations of power, or virtuous enough to avoid uſing all means to extend it.

Many ſchemes of different kinds have been formed for paying the public debts; and certain it is, that nothing can be of more importance. But the nature of things doth not admit of any method of doing this ſo expeditiouſly and effectually as an unalienably *ſinking fund*; for in ſuch a *fund* (it has been ſhewn) money is improved at compound

pound intereſt, and therefore in the moſt perfect manner. The writers, therefore, who have employed themſelves in contriving ſuch ſchemes might have ſpared their labour. The beſt of all ſchemes has been long known and eſtabliſhed, and received all the weight and efficiency which could be given it by the moſt ſolemn acts of legiſlature. But no legiſlature can give ſecurity againſt itſelf. No parliament can do any thing which it may not undo.

We have now ſeen what the *ſinking fund* WOULD have done, had it ſuited the views of the *Britiſh* miniſtry in 1733 to ſuffer it to go on with its operations. Let us next compare this with what it *has* done.

In 1737 and 1738, a million of the ſtock of Bank annuities and two millions of the ſtock of *South-ſea* annuities were redeemed with it. For twelve years after 1738, it was wholly applied to the current expences of every year. In 1749, the intereſt of near fifty-eight millions of the public debts was reduced from four to three and a half *per cent.* intereſt for ſeven years, and afterwards to three *per cent.* for ever. But notwithſtanding the great addition which this *third* reduction of intereſt made to the *ſinking fund*, no more than three millions of the public debts were redeemed by it, during the interval of peace between the years 1748 and 1756.

By

By an act of the 25th of *Geo.* II. 1752, a change was made in the *ſinking fund*, which it is neceſſary to mention.

Before this act the *ſinking fund* conſiſted only of the clear ſurpluſſes of the aggregate, the general, and the *South-ſea* company's *funds*. By the war, which begun in 1740, there was an addition made to the public debts of near thirty-two millions. This occaſioned a great increaſe of taxes; and the practice was, whenever any new tax produced *leſs* than the intereſt with which it was charged, to make good the deficiency out of the *ſinking fund*, and afterwards to replace the ſum taken from it out of the ſupplies for the following year. But whenever a tax produced *more* than the charge upon it, the overplus, inſtead of being carried to the *ſinking fund*, was made a part of the ſupplies for the year. By the act juſt mentioned, all the *new* taxes, together with all the annuities to the payment of which they had been appropriated, were ordered to be carried into the *ſinking fund*, and formed into one general account. Moſt of the new taxes having proved deficient, this *fund* at firſt loſt more than it gained by the change. But the loſs was afterwards more than made up; firſt, by the ſaving which was produced by the reduction of intereſt from three and a half *per cent.* to three *per cent.* in 1757; and, ſecondly, by the addition, in the ſame year,

year, of the ſalt-duties to this *fund*, after they had completed the redemption of a million with which they had been charged in 1745.

The war which began in 1756 (*a*), added ſeventy-one millions and a half to the public debts. This produced a new increaſe of taxes, which (in conformity to the conſolidating act juſt mentioned) were brought to the general *ſinking fund* account, together with the annuities or intereſts with the payment of which they were charged. And it had been, till the laſt war, the conſtant practice to carry every new fund or tax, impoſed for paying the intereſt of a loan, into the *ſinking fund*; in conſequence of which this *fund* gained when the tax happened to produce more, but loſt when it produced leſs than the intereſt which it had been given to pay. The *ſinking fund*, therefore, which, *before* the conſolidating act, conſiſted only of the ſurpluſſes of the aggregate, general, and *South-ſea* company's *funds*, conſiſted afterwards of the clear ſurplus of all the appropriated taxes. There was only one exception; namely, the additional tax upon houſes and windows, granted in 1758, towards paying the intereſt of four millions and a half then borrowed. This tax was never made a part of the *ſinking fund*; and, having always

(*a*) See ſecond Tract on Civil Liberty, p. 147.

proved

proved deficient to the amount of about 43,000*l.* *per annum*, the deficiency was conſtantly made good by the *ſinking fund*, and afterwards replaced from the ſupplies.

Before the laſt reduction of the intereſt of the public debts, the *ſinking fund*, having ſuffered greatly from various encroachments upon it, produced little more than a million *per ann.* But after this reduction, and its increaſe by the addition of the ſalt-duties, it produced near two millions *per ann.* In 1764 it produced at *Michaelmas*, after making good deficiencies, 2,105,000*l.* nearly. For five years after 1764, its average produce, reckoned to *Chriſtmas* in every year, was 2,234,780*l.* For five years, ended in 1774, its average produce (after making good the deficiency of the fund in 1758) was 2,610,759*l.* In 1775, it produced 2,917,869*l.* In 1776, 3,166,517*l.* In 1777 it was charged with an annuity of 100,000*l.* *per ann.* to the civil liſt; and, after paying three quarters of this annuity, and half a year's intereſt of five millions borrowed in that year, it produced from October 1776 to October 1777, 2,685,669*l.* From October 1777 to October 1778, 1779, and 1780, it produced 2,442,063*l.* 2,267,399*l.*—2,403,017*l.* after paying the ſaid annuity, and alſo after making good the

the deficiency of the fund in 1758, and all the deficiencies (*a*) of the new taxes.

It appears from this detail, that after the peace in 1763 the income of the *ſinking fund* increaſed conſiderably. The cauſes of this have been partly the falling in of life-annuities, and the greater productiveneſs of the taxes occaſioned by the increaſe of luxury. But the principal cauſe was the falling in of the intereſt of about ten millions and a half of the public debts, which were diſcharged during the twelve years of peace between 1763 and 1775. This diminution of the public debts was made, not by the *ſinking fund*, but by a contribution from the *Eaſt India* company of 400,000*l. per ann.* begun in 1768, and continued for five years; by the profits of ten lotteries; by the compoſition for maintaining *French* priſoners; ſale of *French* prizes taken before the declaration of war in 1756; and other extraordinary receipts, amounting in all to above eight millions. This *fund*, therefore, did not pay off more than two millions and a half, the reſt of its produce having been employed in bearing the expences of the peace eſtabliſhment,

(*a*) Theſe deficiencies have had two cauſes. Firſt, the unproductiveneſs of ſome of the new taxes: and, ſecondly, the commencement of the intereſts of the loans before the new taxes could be collected, the payment of which intereſts, therefore, fell on the Sinking Fund.

which

which, during that period, were not much leſs than double to what they had been in any former period.

To the ſum juſt mentioned, add three millions paid off in the peace between 1748 and 1756, and three millions paid off in 1736 and 1737, and it will appear that the whole amount of the public debts paid off by the *ſinking fund*, from its firſt alienation in 1733, to the commencement of the *American* war, is only eight millions and a half.

It has been ſaid, that when money is wanted for defraying public expences, it makes no difference whether it is obtained by diverting the *ſinking fund*, or by a new loan. I have taken notice of this fallacy in page 207. Money in a *ſinking fund*, if never alienated, is improved, I have ſhewn, at *compound* intereſt; but, when procured by a loan, bears only *ſimple* intereſt. A nation, therefore, whenever it applies the income of ſuch a *fund* to current expences rather than the redemption of its debts, chuſes to loſe the benefit of compound intereſt in order to avoid paying ſimple intereſt; and the loſs in this caſe is equal to the difference between the increaſe of money at compound and ſimple intereſt. The following calculation will ſhew what this difference is.

One penny put out at our Saviour's birth to five *per cent.* compound intereſt, would,

in the year 1791, have increaſed to a greater ſum than would be contained in THREE HUNDRED MILLIONS of earths, all ſolid gold. But, if put out to ſimple intereſt, it would, in the ſame time, have amounted to no more than SEVEN SHILLINGS AND SIX-PENCE. All governments that alienate *funds* deſtined for reimburſements, chuſe to improve money in the *laſt* rather than the *firſt* of theſe ways.

N. B. The preceding account brings down the hiſtory of the Sinking Fund to the time of the laſt publication of this work. Since that time a new *Sinking Fund* has been eſtabliſhed, and a new *Conſolidating Act* has been paſſed, of which an account will be given at the end of the next volume.

STATE-

FOUR ESSAYS

ON

DIFFERENT SUBJECTS

IN THE

DOCTRINE

OF

LIFE ANNUITIES

AND

POLITICAL ARITHMETIC.

ESSAY I*.

Containing Obſervations on the Expectations of Lives; the Increaſe of Mankind; the Number of Inhabitants in LONDON; *and the Influence of great Towns, on Health and Population.*

In a LETTER *to* BENJAMIN FRANKLIN, *Eſq;* LL. D. *and* F. R. S.

DEAR SIR,

I Beg leave to ſubmit to your peruſal the following obſervations. If you think them of any importance, I ſhall be obliged to you for communicating them to the Royal Society. You will find, that the chief ſubject of them is the preſent ſtate of the city of *London*, with reſpect to healthfulneſs and number of inhabitants, as far as it can be collected from the bills of mortality. This is a ſubject which has been conſidered by others; but the proper method of calculating

* This Eſſay was read to the ROYAL SOCIETY, April 27th, 1769, and has been publiſhed in the Philoſophical Tranſactions, Vol. 59. It is here republiſhed with corrections; and with ſeveral additions, particularly the *Poſtſcript*.

from the bills has not, I think, been ſufficiently explained.

No competent judgment can be formed of the following obſervations, without a clear notion of what the writers on *Life-Annuities* and *Reverſions* have called the *Expectation of Life*. Perhaps this is not properly underſtood; and Mr. *De Moivre*'s manner of expreſſing himſelf about it is very liable to be miſtaken.

The moſt obvious ſenſe of the *expectation* of a given life is, "That particular number "of years which a life of a given age has "an equal chance of enjoying." This is the time that a perſon may reaſonably *expect* to live; for the chances *againſt* his living longer are greater than thoſe *for* it; and, therefore, he cannot entertain an *expectation* of living longer, conſiſtently with probability. This period does not coincide with what the writers on Annuities call the *expectation of life*, except on the ſuppoſition of an uniform decreaſe in the probabilities of life, as Mr. *Simpſon* has obſerved in his *Select Exerciſes*, p. 273.—It is neceſſary to add, that, even on this ſuppoſition, it does not coincide with what is called the *expectation of life*, in any caſe of joint lives. Thus, two lives of 40 have an even chance, according to Mr. *De Moivre*'s hypotheſis (*a*), of continuing together only $13\frac{1}{2}$ years. But the *expectation*

(*a*) See the Notes in page 2 and 23.

of

of two equal joint lives, being (according to the ſame hypotheſis) always a *third* of the *common complement*; it is, in this caſe, $15\frac{1}{2}$ years. It is neceſſary, therefore, to obſerve, that there is another ſenſe of this phraſe, which ought to be carefully diſtinguiſhed from that now mentioned. It may ſignify, "The *mean continuance* of any given *ſingle*, "*joint*, or *ſurviving* lives, according to any "given Table of Obſervations:" that is, the number of years which, taking them one with another, they actually enjoy, and may be conſidered as ſure of enjoying; thoſe who live or ſurvive *beyond* that period, enjoying as much *more* time in proportion to their number, as thoſe who *fall ſhort* of it enjoy *leſs*. Thus; ſuppoſing 46 perſons alive, all 40 years of age; and that, according to Mr. *De Moivre's hypotheſis*, one will die every year 'till they are all dead in 46 years; half 46, or 23, will be their *expectation of life:* That is, The number of years enjoyed by them all, will be juſt the ſame as if every one of them had lived 23 years, and then died; ſo that, ſuppoſing no intereſt of money, there would be no difference in value between annuities payable for life to every ſingle perſon in ſuch a ſet, and equal annuities payable to another equal ſet of perſons of the ſame common age, ſuppoſed to be all ſure of living juſt 23 years and no more.

In like manner; the *third* of 46 years, or 15 years and 4 months (*a*), is the *expectation* of two joint lives both 40; and this is alſo the *expectation* of the ſurvivor. That is; ſuppoſing a ſet of marriages between perſons all 40, they will, one with another, laſt juſt this time; and the ſurvivors will laſt the ſame time. And annuities payable during the continuance of ſuch marriages would, ſuppoſing no intereſt of money, be of exactly the ſame value with annuities to begin at the extinction of ſuch marriages, and to be paid, during life, to the ſurvivors.—In adding together the years which any great number of ſuch marriages and their ſurvivorſhips have laſted, the ſums would be found to be equal.

One is naturally led to underſtand the *expectation* of life in the firſt of the ſenſes now explained, when, by Mr. *Simpſon* and Mr. *De Moivre*, it is called, *the number of years which, upon an equality of chance, a perſon may expect to enjoy*; or, *the time which a perſon of a given age may juſtly expect to continue in being*; and, in the laſt ſenſe, when it is called, *the ſhare of life due to a perſon*. But, as in reality it is always uſed in the laſt of theſe ſenſes, the former language ſhould not be applied to it: And it is in this laſt ſenſe that it coincides with the *ſums* of the *preſent* probabilities, that any given ſingle or joint lives ſhall attain to the end of the

(*a*) See Note (K) at the end of next volume.

1ſt,

1ſt, 2d, 3d, &c. *moments*, from this time to the end of their poſſible exiſtence; or (in the caſe of ſurvivorſhips) with the ſum of the probabilities, that there ſhall be a ſurvivor at the end of the 1ſt, 2d, 3d, &c. moments, from the preſent time to the end of the poſſible exiſtence of ſurvivorſhip. This coincidence every one converſant in theſe ſubjects muſt ſee, upon reflecting, that both theſe ſenſes give the true preſent value of a life-annuity, ſecured by land, without intereſt of money (*a*).

This period in joint lives, I have obſerved, is never the ſame with the period which they have an equal chance of enjoying; and in ſingle lives, I have obſerved, they are the ſame only on the ſuppoſition of an uniform decreaſe of the probabilities of life. If this decreaſe, inſtead of being always uniform, is *accelerated* in the laſt ſtages of life; the former period, in ſingle lives, will be *leſs* than the latter; if *retarded*, it will be *greater*.

It is neceſſary to add, that the number expreſſing the former period, multiplied by the number of ſingle or joint lives whoſe expectation it is, added annually to a ſociety or town, gives the whole number living together, to which ſuch an annual addition would in time grow. Thus; ſince 19, or the third of 57, is the *expectation* of two

(*a*) See Note (K) at the end of next volume.

joint

joint lives whoſe common age is 29, or common *complement* 57; twenty marriages every year between perſons of this age would, in 57 years, grow to 20 times 19, or 380 marriages always exiſting together. The number of *ſurvivors* alſo ariſing from theſe marriages, and always in life together, would, in twice 57 years, increaſe to the ſame number. And, ſince the *expectation* of a ſingle life is always half its *complement*; in 57 years likewiſe, 20 ſingle perſons aged 29, added annually to a town, would increaſe to 20 times 28.5 or 570; and, when arrived at this number, the deaths every year will juſt equal the acceſſions, and no further increaſe be poſſible.

It appears from hence, that the particular proportion that becomes extinct every year, out of the whole number conſtantly exiſting together of ſingle or joint lives, muſt, wherever this number undergoes no variation, be exactly the ſame with the *expectation* of thoſe lives, at the time when their exiſtence commenced. Thus; was it found that a 19th part of all the marriages among any bodies of men, whoſe numbers do not vary, are diſſolved every year by the deaths of either the huſband or wife, it would appear that 19 was, at the time they were contracted, the *expectation* of theſe marriages. In like manner; was it found in a ſociety, limited to a fixed number of members,

members, that a 28th part dies annually out of the whole number of members, it would appear that 28 was their common expectation of life at the time they entered. So likewiſe; were it found in any town or diſtrict, where the number of births and burials are equal, that a 20th or 30th part of the inhabitants die annually, it would appear, that 20 or 30 was the *expectation* of a child juſt born in that town or diſtrict. Theſe *expectations*, therefore, for all ſingle lives, are eaſily found by a *Table of Obſervations*, ſhewing the number that die annually at all ages, out of a given number alive at thoſe ages; and the general rule for this purpoſe, is "to divide the ſum of all the living in the Table, "at the age whoſe expectation is required, "and at all greater ages, by the ſum of all "that die annually at that age, and above it; "or, which is the ſame, by the number (in "the Table) of the living at that age; and "half unity ſubtracted from the quotient will "be the required *expectation* (*a*)." Thus, in Dr. *Halley*'s Table, the ſum of all the living at 20 and upwards, is 20,724. The number living at that age is 598; and the former

(*a*) This rule, and alſo rules for finding in all caſes the expectations of joint lives and ſurvivorſhips, may be deduced with great eaſe, by having recourſe to the doctrine of fluxions. In this method, Mr. *De Moivre* ſays, he diſcovered them. See note (K), where an account will be given of theſe deductions, omitted by Mr. *De Moivre*.

number

number divided by the latter, and half unity (*a*) ſubtracted from the quotient, gives 34.15 for the *expectation* of 20. The expectation of the ſame life by Mr. *Simpſon*'s Table, formed from the bills of mortality of *London*, is 28.9 (*b*).

Theſe

(*a*) If we conceive the *recruit* neceſſary to ſupply the *waſte* of every year to be made always at the *end* of the year, the *dividend* ought to be the *medium* between the numbers living at the *beginning* and the *end* of the year. That is, it ought to be taken *leſs* than the ſum of the living in the Table at and above the given age, by *half* the number that die in the year; the effect of which *diminution* will be the ſame with the *ſubtraction* here directed.—The reaſon of this ſubtraction will be further explained, in the beginning of the 4th Eſſay.

(*b*) It appears in p. 230 and 231, that the *expectations* of *ſingle* and *joint* lives are the ſame with the values of *annuities* on theſe lives, ſuppoſing no intereſt or improvement of money.—In conſidering this ſubject, it will, probably, occur to ſome, that, allowing intereſt for money, the values of lives muſt be the ſame with the values of annuities *certain* for a number of years equal to the *expectations* of the lives. But care muſt be taken not to fall into this miſtake. The latter values are always greater than the former: And the reaſon is, that, though a number of *ſingle* or *joint* lives of given ages will, among them, enjoy a *given* number of years, yet ſome of them will enjoy a much *greater*, and ſome a much *leſs* number of years. Thus; 100 marriages among perſons, all 29, would, as I have ſaid, one with another, exiſt 19 years; and an office bound to pay annuities to ſuch marriages during their continuance, might reckon upon making 19 payments for each marriage. But then, many of theſe payments would not be made 'till the end of 30, and ſome not 'till the end of 40 years. And it is apparent, that on account of the greater value of *quick* than *late* payments, when money *bears* intereſt, 19 payments ſo made cannot be worth

Theſe obſervations bring me to the principal point which I have had all along in view. They ſuggeſt to us an eaſy method of finding the number of inhabitants in a place, from a *Table of Obſervations* or the *bills of mortality* for that place, ſuppoſing the yearly births and burials equal. "Find by "the Table, in the way juſt deſcribed, the "*expectation* of an infant juſt born; and this, "multiplied by the number of yearly births, "will be the number of inhabitants." At *Breſlaw*, according to Dr. *Halley*'s Table, though half die under 16, and therefore an

worth as much, as the ſame number of payments made regularly at the end of every year, 'till in 19 years they are all made.

This obſervation might be employed, to demonſtrate further, the error of thoſe who have maintained, that the value of a given life is the ſame with the value of an annuity certain, for as many years as the life has an equal chance of exiſting. Were this true, an annuity on a life, ſuppoſed to be expoſed to ſuch danger in a particular year as to create an equal chance whether it will not fail that year, would, at the beginning of the year, be worth *nothing*, though ſuppoſed to be ſure of continuing for ever, if it eſcaped that danger: Nor, in general, would the values of annuities on a ſet of lives be at all affected by any alterations in the rate of mortality among them, provided theſe alterations were ſuch, as did not affect the period during which they had an equal chance of exiſting. —But there can be no occaſion for taking notice of an opinion, which has been embraced only by perſons ignorant of mathematics, and plainly unacquainted with the genuine principles of calculation on this ſubject.—See a pamphlet on Life-Annuities by *Weyman Lee*, Eſq; of the *Inner Temple*.

infant

infant juſt born has an *equal chance* of living only 16 years; yet his *expectation*, found by the rule I have given, is near 28 years; and this, multiplied by 1238, the number born annually, gives 34,664, the number of inhabitants. In like manner, it appears from Mr. *Simpſon*'s Tables, that, though an infant juſt born in *London* has not an *equal chance* of living 3 years, his *expectation* is 19 years and a quarter. Let us reckon it as high as 20 years. This number, multiplied by the yearly births, would give the number of inhabitants in *London*, were the births and burials equal.—The medium of the yearly births, for ten years, from 1759 to 1768, was 15,710. And 15,710 multiplied by 20, gives 314,200; which is the number of inhabitants that there would be in *London*, according to the Bills, were the yearly burials no more than equal to the births; that is, were it to ſupport itſelf in its number of inhabitants; without any ſupply from the country. But for the period I have mentioned, the burials were, at an average, 22,956, and exceeded the chriſtenings 7,246. This is, therefore, the yearly addition of people to *London* from other parts of the kingdom, by whom it is kept up. Suppoſe them to be all, one with another, perſons who have, when they remove to *London*, an *expectation* of life equal to 30 years. That is; ſuppoſe them to be all of the age

of

of 18 or 20, a ſuppoſition certainly far beyond the truth. From hence will ariſe, according to what has been before obſerved, an addition of 30, multiplied by 7.246; that is, 217,380 inhabitants. This number, added to the former, makes 531,580; and this, I think, at moſt, would be the number of inhabitants in *London* were the bills perfect. But it is certain, that they give the number of births and burials too little. There are many burial places which are never brought into the bills. Many alſo emigrate to the navy and army and country; and theſe ought to be added to the number of deaths. What the deficiencies ariſing from hence are, cannot be determined (*a*). Suppoſe them equivalent to 6000 every year in the births, and 6000 in the burials. This would make an addition of 20 times 6000, or 120,000, to

(*a*) Two whole pariſhes are omitted in the bills, or *Marybone* and *Pancras* pariſhes. The former of theſe pariſhes is now one of the largeſt in *London*. The annual medium of burials in it for five years to 1771, was 780. In *Pancras* pariſh this medium for the ſame period was 322. From an accurate account taken in March 1772 of that part of this laſt pariſh which joins to *London*, it appeared that the number of inhabitants was then 3479, of whom 1594 were lodgers, and that the number of houſes was 476, of which about 330 had been built in ſeven years. Mr. *Wales*, in a pamphlet of which more notice will be taken preſently, gives the annual medium of *burials*, for 5 years to 1779, in *Marybone* pariſh 1145; of *births* 1008. In *Pancras*, he gives the *burials* for the ſame period, 339; the *births*, 234.

the

the laſt number; and the whole number of inhabitants would be 651,580. If the burials are deficient only two thirds of this number, or 4000; and the births the whole of it; 20 multipled by 6000, muſt be added to 314,290, on account of the defects in the births: and, ſince the exceſs of the burials above the births will then be only 5,246; 30 multiplied by 5,246 or 157,380, will be the number to be added on this account; and the ſum, or number of inhabitants, will be 591,580.—But if, on the contrary, the burials are deficient 6000, and the births only 4000; 80,000 muſt be added to 314,200, on account of the deficiencies in the births; and 30 multiplied by 9,246, or 277,380, on account of the exceſs of the burials above the births; and the whole number of inhabitants will be 671,580.

Every ſuppoſition in theſe calculations is too high. *Emigrants* from *London* are, in particular, allowed the ſame *expectation* of continuance in *London* with thoſe who are born in it, or who come to it in the firmeſt part of life, and never afterwards leave it; whereas it is not credible that the former *expectation* ſhould be ſo much as half the latter. But I have a further reaſon for thinking that this calculation gives too high numbers, which has with me irreſiſtible weight. It has been ſeen, that the number of inhabitants

bitants comes out leſs on the ſuppoſition, that the defects in the chriſtenings are greater than thoſe in the burials. Now it ſeems evident that this is really the caſe; and, as it is a fact not attended to, I will endeavour to explain diſtinctly the reaſon which proves it.

The proportion of the number of births in *London*, to the number who live to be 10 years of age, is, by the Bills, 16 to 5. Any one may find this to be true, by ſubtracting the *annual medium* of thoſe who have died under 10 for ſome years paſt (*a*), from the *annual medium* of births for the ſame number of years.—Now, though without doubt *London* is very fatal to children, yet it ſeems incredible that it ſhould be ſo fatal as this implies. The *Bills*, therefore, probably, give the number of thoſe who die under 10 too great in proportion to the number of births; and there can be no other cauſe of this, than a greater deficiency in the *births* than in the *burials*. Were the deficiencies in both equal; that is, were the *burials*, in proportion to their number, juſt as deficient as the *births* are in proportion to *their* number, the proportion of thoſe who reach 10 years of age to the number born, would be right in the *Bills*, let the deficiencies themſelves be ever ſo conſiderable. On the contrary; were the deficiencies in the *burials* greater than in the *births*, this proportion would be given too

(*a*) That is, for ſome years before the date of this letter in 1769.

great; and it is only when the former are leaſt, that this proportion can be given too little.—Thus; let the number of annual *burials* be 23,000; of *births* 15,700; and the number dying annually under 10, 10,800. Then 4,900 will reach 10, of 15,700 born annually; that is, 5 out of 16. —Were there no deficiencies in the *burials*, and were it fact that only *half* the number born die under 10; it would follow, that there was an annual deficiency equal to 4,900 ſubtracted from 10,800, or 5,900, in the *births*.—Were the *births* a third part too little, and the *burials* alſo a third too little, the true number of *births*, *burials*, and of *children dying under* 10, would be 20,933—30,666—and 14,400; and, therefore, the number that would live to 10 years of age, would be 6,533 out of 20,933, or 5 of 16 as before.—Were the *births* a third part, and the *burials* ſo much as two-fifths wrong, the number of *births*, *burials*, and children dying under 10 would be 20,933—32,200—and 15,120. And, therefore, the number that would live to 10 would be 5,813 out of 20,933, or five out of 18.—Were the *births* a third part wrong, and the *burials* but a ſixth, the foregoing numbers would be 20,933—26,833—12,600; and therefore, the number that would live to 10 would be 8,333 out of 20,933, or 5 out of 12.56: and this proportion ſeems

as

as low as is conſiſtent with probability. It is ſomewhat leſs than the proportion in Mr. *Simpſon*'s Table of *London Obſervations*; and much leſs than the proportion in the Table of *Obſervations* for *Breſlaw*. The deficiencies, therefore, in the regiſter of *births*, muſt be greater than thoſe in the regiſter of *burials* (*a*); and the leaſt number I have given, or 591,580 (*b*) is neareſt to

(*a*) One obvious reaſon of this fact is, that *none* of the *births* among *Jews*, *Quakers*, *Papiſts*, and the *three denominations of Diſſenters* are included in the Bills, whereas *many* of their *burials* are. It is further to be attended to, that the abortive and ſtill-born, amounting to about 600 annually, are included in the burials, but never in the births. If we add theſe to the chriſtenings, preſerving the burials the ſame, the proportion of the born according to the Bills, who have reached ten, for ſixteen years, from 1756 to 1771, will be very nearly one *third* inſtead of *five ſixteenths*.

(*b*) Mr. *Wales*, the ingenious maſter of the royal mathematical ſchool in *Chriſt's Hoſpital*, has lately, in a pamphlet entitled, *An Enquiry into the preſent State of the Population of England and Wales*, made ſeveral remarks on the Obſervations in this Eſſay. He objects particularly to this calculation, and expreſſes, p. 12, his ſurprize that it ſhould have eſcaped my attention, that if the births are conſiderably more deficient than the burials, the expectation of life by which the number of theſe births is multiplied will be greater, particularly at this time, when the number of births approaches ſo much nearer than it did to the number of burials.—— But Mr. *Wales* ſhould have obſerved, that in order to be certain of not making the number of inhabitants in *London* leſs than it is, I have all along in this calculation reckoned the expectation of a child at birth in *London* ſo

to the true number of inhabitants. However, ſhould any one, after all, think that it

high as 20 years; and that this is a greater expectation than ſuch a child could have, according to the Bills from 1759 to 1768, ſuppoſing the deficiencies in the chriſtenings ſo conſiderable as a third, while in the *burials* they were only a ſixth.——In page 15th, he ſays, that according to my Tables for *London*, formed on the ſuppoſition that the burials exceed the births a *fourth*, the expectation of a child juſt born in *London* is 20 years and three quarters.—Had Mr. *Wales* attended more to this ſubject, he would have found, that in reality this expectation is no more than 18; and that 20 and three quarters is the expectation, according to my Tables, not of a child juſt born in *London*, but of all the inhabitants of *London* at the time they enter it. See the 4th Eſſay towards the middle, and the Tables for *London* in the Collection of Tables.——He would alſo have found that even in the preſent improved ſtate of *London* it is not poſſible, without aſſuming ſuppoſitions which are perfectly extravagant, to frame a table from the Bills that ſhall give the expectation of a child at birth in *London* much more than 20. He *intimates*, however, that it may *now* approach even to 25¼; but *concludes*, tho' he could not ſtop to make the calculation, that it cannot be leſs than 24. He will ſee how wrong he has been in drawing this concluſion, if he will conſult the Eſſay, and the Tables to which I have juſt referred. The 16th Table, in particular, gives the probabilities of life between 8 and 16 higher than (according to Mr. *Wales*'s account) they have been found to be among the children in *Chriſt's Hoſpital* for 20 years before 1781. It gives them likewiſe too high *after* 20; and yet even this Table makes the expectation of a child juſt born in *London* only 19¼.——Mr. *Wales*, in conſequence of concluding without calculation this expectation to be 24, makes the inhabitants of *London* to be 625,131.——Had he taken it at 20, he would have taken it higher than it is, and by

it is not improbable that only 5 of 16 ſhould live in *London* to be 10 years of age; or that above *two-thirds* die under this age; the conſequence will ſtill be, that the foregoing calculation has been carried too high. For it will from hence follow, that the *expectation* of a child juſt born in *London* muſt be far ſhort of the number at which I have taken it, or of 20 years.—It is only 19¼ on the ſuppoſition that half die under 3 years of age, and that 5 of 16 live to be 29 years of age, agreeably to Mr. *Simpſon*'s Table. But if it is indeed true, that *half* die under 2 years of age, and 5 of 16 under 10, agreeably to the *Bills*, this expectation cannot be ſo much as 17; and all the numbers before given will be conſiderably reduced.

Upon the whole: I am forced to conclude from theſe obſervations, that the ſecond number I have given, or 651,580, though ſhort of the number of inhabitants commonly ſuppoſed in *London*, is, very probably, much *greater*, but cannot be *leſs*, than the true number. Indeed, it is in ge-

by proceeding on his own principles found that the inhabitants of *London* cannot be ſo many as 528,859.

I cannot conclude this Note without adding, that though it appears from hence, that Mr. *Wales* has been much too haſty in ſome of his remarks, yet I think myſelf greatly obliged to him for them. It will come in my way to take notice of more of them in the courſe of this work.

neral evident, that in caſes of this kind numbers are very much over-rated. The ingenious Dr. *Brakenridge*, 14 years ago, when the Bills were lower than they are now, from the number of houſes, and allowing ſix to a houſe, made the number of inhabitants 751,800. But he has taken the number of houſes much greater than it really is; and ſix to a houſe is probably too large an allowance (*a*).

Another

(*a*) Vid. Phil. Tranſactions, Vol. xlviii. p. 788. In a paper ſubſequent to this, read to the Royal Society in March 1758, Dr. *Brakenridge* tells us, that in a late ſurvey it appeared, that in all *Middleſex*, *London*, *Weſtminſter*, and *Southwark*, there were 87,614 houſes, of which 19,324 were cottages, and 4810 empty. And he acknowledges, that this, if right, proves *London* to be much leſs populous than he had made it. See Phil. Tranſ. Vol. L. p. 471.—Mr. *Maitland* gives two accounts of the number of houſes within the Bills. One carefully taken from the books of all the pariſhes and precincts belonging to *London*; and another taken from a particular ſurvey in 1737, made by himſelf with incredible pains. The firſt account makes the number of houſes 85,805. The ſecond account makes it 95,968. And the reaſon of the difference he obſerves, is, that many landlords of ſmall places paying all taxes, they are in the pariſh books reckoned as ſo many ſingle houſes, though each of them contain ſeveral houſes. See Mr. *Maitland*'s Hiſtory of *London*, 2d Book at the end.—It will be obſerved preſently, that the number of inhabitants in *London* in 1737, was conſiderably greater than it is now.

From a Table which I have given at the end of this Eſſay, containing the reſults of actual ſurveys of the number

Another method which Dr. *Brakenridge* took to determine the number of inhabitants in *London* was from the annual number of burials, adding 2000 to the Bills for omiſſions, and ſuppoſing a 30th part to die every year. In order to prove this to be a moderate ſuppoſition, he obſerves that, according to Dr. *Halley*'s Obſervations, a 34th part die every year at *Breſlaw*. But this obſer-

number of inhabitants, houſes, and families in many different places, it will appear that *ſix* to a houſe is probably too large an allowance for LONDON; and that certainly *five* to a houſe is an allowance ſufficiently large for ENGLAND in general. And this will prove that Dr. *Brakenridge* has over-rated the number of people in ENGLAND as well as in LONDON. In a letter to *George Lewis Scott*, Eſq. publiſhed in 1756 in the Phil. Tranſ. Vol. xlix. p. 877, he ſays, that he had been certainly informed that the number of houſes rated to the window-tax was 690,000. The number of cottages not rated, he adds, could not exceed 200,000; and from theſe data, by allowing ſix to a houſe, he makes the number of inhabitants in ENGLAND to be 5,340,000.—Dr. *Brakenridge* was much miſtaken with reſpect to the cottages, Their number in 1761 was (according to the returns of the ſurveyors of the houſe duties) 276,149; and the whole number of houſes in *England* and *Wales* was in the ſame year 980,692.—In 1777, according to the ſame returns, the cottages were 251,261, and the whole number of houſes 952,734. Let, however, the number of houſes then in *England* and *Wales* be reckoned a MILLION; and, allowing *five* to a houſe, the number of people muſt have been FIVE MILLIONS.—The inhabitants of IRELAND may, I ſuppoſe, be ſtated at TWO MILLIONS.—The inhabitants of SCOTLAND conſiſted, in 1754.

obſervation was made too inadvertently. The number of annual burials there, according to Dr. *Halley*'s account, was 1174, and the number of inhabitants, as deduced by him from his Table, was 34,000; and therefore a 29th part died every year. Beſides; any one may find, that in reality the Table is conſtructed on the ſuppoſition, that the whole number born, or 1238, die every year; from whence it will follow that a 28th part died every year (*a*). Dr. *Brakenridge*, therefore, had he attended to this, would have ſtated a 24th part as the pro-

of between 16,000 and 17,000 *Papiſts*, and between 1,240,000 and 1,280,000 Proteſtants, according to an eſtimate that was made, I am informed, with labour and expence by the Rev. Dr. *Webſter*.—It follows, therefore, that the whole number of people in *Britain* and *Ireland* may be about EIGHT MILLIONS AND A HALF, or NINE MILLIONS. In the Supplement in the next volume I ſhall have occaſion to ſay more on this ſubject, and to take notice of the arguments offered by Mr. *Wales* and Mr. *Howlett*, to prove that our population is increaſing.

(*a*) Care ſhould be taken, in conſidering Dr. *Halley*'s Table, not to take the firſt number in it, or 1000, for ſo many juſt born. 1238, he tells us, was the annual medium of births, and 1000 is the number he ſuppoſes all living at one year and under. It was inattention to this that led Dr. *Brakenridge* to his miſtake.

It will be ſhewn in the 4th Eſſay, that the number of the living under 20, is given too high in this table; and from hence it will follow, that more than a 28th part of the inhabitants die at *Breſlaw* annually.

portion

portion that dies in *London* every year, and this would have taken off 150,000 from the number he has given. But even this muſt be leſs than the juſt proportion. For let three-fourths of all who either die in *London* or migrate from it, be ſuch as have been born in *London*; and let the reſt be perſons who have removed to *London* from the country, or from foreign nations. The *expectation* of the former, it has been ſhewn, cannot be 20 years; and 30 years have been allowed to the latter. One with another, then, they will have an *expectation* of $22\frac{1}{2}$ years. That is; one of $22\frac{1}{2}$ will die every year (*a*). And, conſequently, ſuppoſing the annual

(*a*) The mean number of inhabitants in *Rome*, of all ages and conditions, for ten years ending in 1771, was 158,957. The annual medium of births for the ſame time was 4851; and of burials 7367. One in $21\frac{1}{2}$, therefore, died annually. See Phil. Tranſ. Vol. lxv. p. 445. In 1752, the accurate and diligent Mr. *Struyk* took particular pains to determine the number of inhabitants in *Amſterdam*; and the reſult of his enquiry was, that very probably it did not amount to 200,000. The annual medium of burials for ſix years, from 1747 to 1752, was 8247; and for five years, from 1772 to 1776, it was 8447. *One* in 24, therefore, died annually.—At *Amſterdam*, there is a great number of Jews, and their burials are not included in the Bills. There muſt, I ſuppoſe, be other deficiencies, and an allowance for theſe would, I doubt not, increaſe the proportion of inhabitants who die annually to one in 21 or 22.—At *Dublin*, in the year 1695, the number of inhabitants was found, by an exact ſurvey, to be 40,508. (See Philoſ.

annual recruit from the country to be 7000 (*a*), the number of *births* 3 times 7000

Philoſ. Tranſactions, No. 261). I find no account of the annual burials juſt at that time; but from 1661 to 1681, the medium had been 1613; and from 1715 to 1728 it was 2123. There can, therefore, be no material error in ſuppoſing that, in 1695, it was 1800; and this makes 1 in 22 to die annually. See Dr. *Short's Comparative Hiſtory*, p 15, and *New Obſervations*, p. 228.—The annual medium of burials for five years ending in 1775, in *Mancheſter* and *Salford*, was 973. The number of inhabitants in 1773 was 27,246. About a 28th part, therefore, died annually. But it ſhould be conſidered here, that *Mancheſter* increaſes faſt by acceſſions from the country, and that the effect of ſuch an increaſe muſt be to raiſe the proportion of *inhabitants* to the *deaths* and alſo the proportion of the *births* and *weddings* to the *burials*, higher than they would otherwiſe be.—The annual medium of burials at *Stockholm* in *Sweden*, from 1758 to 1763, was 3802. The number of inhabitants in 1763 was 72,979. One in 19 therefore died annually. See a memoir by M. *Vargentin*, in the 15th Vol. of the *Collection Academique*, printed at *Paris* 1772.

Mr. *Maitland*, in his Hiſtory of *London*, Vol. II. page 744, by a laborious, but too unſatisfactory inveſtigation, makes 1 in 24½ to die in *London* annually; and on the ſuppoſitions, that this is the true proportion dying annually, *at all times*, in *London*, and that the deficiencies in the burials (including the burials in *Marybone* and *Pancras* pariſhes) amounted to 3038 annually; he determines, that the number of inhabitants within the bills was 725,903, in the year 1737.

The number of burials not brought to account in the Bills is, probably, now much greater than either Dr. *Brakenridge* or Mr. *Maitland* ſuppoſe it. I have reckoned it ſo high as 6000, in order to be more ſure of not falling below the truth.

It will appear in the laſt Eſſay, with an evidence little ſhort of demonſtration, that, at leaſt, 1 in 20¾ die annually

7000 or 21,000, and the *burials* and *migrations* 28,000 (which are all very high ſuppoſitions),

nually in *London*, and that, conſequently, the number of inhabitants, if the burials are 26,000, cannot exceed 539,500.

(*a*) Mr. *Wales*, though he ſeems to acknowledge that *formerly* the number of annual recruits from the country to *London* was much greater than it is here ſuppoſed, yet reckons that when he wrote on this ſubject in 1782, it might be fairly ſtated at no more than 1779. See Mr. *Wales*'s Enquiry, p. 16. It may be proper to conſider here how improbable it is that ſuch a change as this ſhould have taken place at a time when the communication between *London* and the country has been made ſo eaſy as it is, and when alſo a diſpoſition to migrate to *London* ſeems to be more prevalent than ever.—But it is unneceſſary to inſiſt on this, for in the 4th Eſſay it will be proved by deciſive evidence, that theſe recruits could not even then be ſo little as double the number at which Mr. *Wales* has ſtated them. It is true indeed, that though the burials have been falling, the chriſtenings have been riſing, for ſeveral years. But this does not neceſſarily imply, that the emigrants from the country are leſs numerous than they were. It may, on the contrary, be owing to a greater afflux of people to *London* in the prolific ſtages of life occaſioning an increaſe of the chriſtenings, without at preſent occaſioning ſuch an increaſe of the burials as is ſufficient to balance the cauſes that diminiſh them. The Lying-in Hoſpitals lately eſtabliſhed in *London* increaſe the chriſtenings, by drawing many into them to lye-in who reſide out of the limits of the Bills; and the burials are diminiſhed by the cuſtom of ſending infants to be nurſed in the country, by the new burying grounds which have been lately opened, and particularly by an Act of Parliament which we owe to the humanity of Mr. *Hanway*, paſſed in 1767, and requiring all pariſh infants to be ſent in three weeks into the country to be nurſed there for ſix years.—The improved ſtate of *London* with reſpect

poſitions), the number of inhabitants will be, 22½ multitipled by 28,000, or 630,300 (*a*).

I will juſt mention here one other inſtance of exaggeration on the preſent ſubject.

Mr. *Corbyn Morris*, in his uſeful *Obſervations on the paſt growth and preſent ſtate of the city of London*, publiſhed in 1651, ſuppoſes that no more than a 60th part of the inhabitants of *London*, who are above 20, die every year, and from hence he concludes that the number of inhabitants was near a million. In this ſuppoſition there was an error of at

ſpect to healthineſs might be alſo here mentioned; but this has been greatly over-rated. The values of lives in *London after* the age of 20, are much the ſame that they were 50 years ago; and there is no evidence to prove, that they are much greater *before* 20. This will be ſhewn at the end of the 4th Eſſay, and in the Obſervations on Table 15th in the next volume.—According to Mr. *Howlett*'s account, in page 91, of his Examination of my Eſſay on the Population of *England* and *Wales*, above 2000 deaths of children under two years of age have been taken out of the Bills by the Pariſh Act juſt mentioned. This probably goes much beyond the truth. Should the true number be only a *thouſand*, it will follow that the ſtate of infants in *London* is but little mended. For on this ſuppoſition a thouſand muſt be added to the number given in the Bills as dying under two years of age, which will make it near half the number born as it was 20 years ago. But the addition of 2000 would make the mortality of infants (ſuppoſing pariſh infants not ſent into the country) greater now in *London* than it ever was.

(*a*) If with Mr. *Wales* the annual recruit is taken at no more than 1779, the inhabitants on the high ſuppoſitions here made that the burials are 28,000, the expectations at birth 20, and at migration 30, will be only 577,790.

leaſt

leaſt one half. According to Dr. *Halley's* Table, it has been ſhewn, that a 34th part of all at 20 and upwards, die every year at *Breſlaw*. In *London*, a 29th part, according to Mr. *Simpſon's* Table, and alſo according to all other Tables of *London* Obſervations. Had, therefore, Mr. *Morris* ſtated a 30th part of all above 20 dying annually in *London*, he would have gone beyond the truth, and his concluſion would have been 400,000 leſs than it is.

Dr. *Brakenridge* obſerved, that the number of inhabitants, at the time he culculated, was 127,000 leſs than it had been. The Bills have lately advanced a little, but ſtill they are much below what they were from 1717 to 1743. The medium of the annual *births*, for 20 years, from 1716 to 1736, was 18,000, and of *burials* 26,529; and, by calculating from hence on all the ſame ſuppoſitions with thoſe which made 651,580 to be the preſent number of inhabitants in *London*, it will be found that the number then was 735,840, or 84,260 greater than the number in the preſent year 1769 (*a*). *London, therefore*, for the laſt 30 years, has been decreaſing; and though now it is increaſing again, yet there

(*a*) In the Eſſay on the Population of *England* and *Wales*, I have mentioned ſeveral facts which ſeem to ſhew, that even ſo long ago as the *Revolution*, *London* was more populous than it is at preſent. The chief of theſe facts are the following:

Firſt;

there is reaſon to think that the additions lately made to the number of buildings round it, are owing, chiefly to the increaſe of

Firſt; The returns in 1777 of the ſurveyors of the houſe and window duties made the number of houſes then in *Southwark*, *Weſtminſter*, *London*, and all *Middleſex*, including cottages and uninhabited houſes, to be 90,578.—Sir *William Petty*, in 1687, ſays, that the number of *houſes* (which he expreſsly diſtinguiſhes from *families*) in *London* appeared by the regiſter to be 105,315. See his Political Arithmetic, p. 74. His words in p. 79 are, "by certificate from the hearth-office, I find the "houſes within the Bills of Mortality to be 105,315."—Dr. *Devenant*'s account agrees with this, who, from the ſame hearth-office, gives 111,215 as the number of houſes in *London* (excluſive of *Southwark*) *Weſtminſter*, and all *Middleſex*, on Lady-day 1690. See Dr. *Davenant*'s Works, Vol. I. p. 38. The annual average of regiſtered burials alſo for five years before 1690 was conſiderably greater than it is at preſent.

This ſeems as direct evidence as can well be given in a point of this kind. In order to give more weight to the fact laſt mentioned, I have, in the Eſſay juſt referred to, obſerved that there are twelve pariſhes now included in the Bills, which were omitted formerly. But Mr. *Wales* has very properly corrected me in this inſtance by obſerving, that theſe pariſhes at the time they were added to the Bills were *new* pariſhes formed out of old pariſhes, which had been always included in the Bills. There is, therefore, no ſuch regard due to this omiſſion as I imagined.—It may be farther obſerved with reſpect to the exceſs of the burials at the Revolution, that the deficiencies in the regiſter of burials are greater now than they were then; and there are two cauſes that may poſſibly have produced this effect. Firſt, the opening of ſome burial places among the *Methodiſts*, where many are now buried who uſed to be buried in churches. And, Secondly, the interment out of the Bills of the greater part of the pariſh-children who die, in conſequence of the Act of Parliament mentioned in the note, p. 251.—

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of luxury, and the inhabitants requiring more room to live upon (*a*).

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There are, however, other cauſes which have leſſened theſe deficiencies; and, particularly, the decreaſe of the three donominations of Diſſenters in *London*. My own recollection, as well as a great deal of other evidence, leave me no room to doubt of this. Mr. *Howlett*, however, in the pamphlet already quoted, aſſerts the contrary; and gives a liſt of burials among Diſſenters, which makes their number more than three times greater than it was when Mr. *Maitland* publiſhed his Hiſtory of *London*. But this is all a miſtake. The principal burying places in his liſt happen to be places lately opened, to which, partly from a regard to cheapneſs, not Diſſenters only, but people of all ſorts are brought to be buried. This is particularly the caſe with *Coughland*'s ground, *Holywell Mount*, and *Britain*'s ground, *Whitechapel*.—The chief burying place of Diſſenters has always been *Tindall*'s ground in *Bunhill Fields*; but even this is by no means confined to Diſſenters, and the number of burials in it has been for a courſe of years decreaſing; and inſtead of being as Mr. *Howlett* gives it, 1400 annually, is not a *third* of this number.—In 1779 the exact number was 434, according to an account which has been extracted for me from the Regiſter.

(*a*) The medium of annual burials in the 97 pariſhes within the walls was,

From 1655 to 1664,	—	3264
From 1680 to 1690,	—	3139
From 1730 to 1740,	—	2316
From 1758 to 1768,	—	1620
From 1771 to 1780,	—	1491
From 1781 to 1784	—	1354

This account proves, that though, ſince 1655, *London* has doubled its inhabitants, yet *within the walls*, they have decreaſed; and ſo rapidly ſince 1690, as to be now reduced to leſs than half.—The like may be obſerved of the 17 pariſhes immediately without the walls.

Since

It ſhould be remembered, that the number of inhabitants in *London* is now ſo much leſs as I have made it, than in was 40 years ago, on the ſuppoſition, that the proportion of the omiſſions in the *births* to thoſe in the *burials*, was the ſame then that it is now. But it appears that this is not the fact.—From 1728, (the year when the ages of the dead were firſt given in the *Bills*) to 1742, near five-ſixths of thoſe who were born died under 10, according to the Bills. From 1742 to 1752 three quarters: and ever ſince 1752, this proportion has ſtood nearly as it is now, or at ſomewhat more than two-thirds. The omiſſions in the *births*, therefore, compared with thoſe in the *burials*, were greater formerly; and this muſt render the difference between the number of inhabitants now and formerly ſomewhat leſs conſiderable than it may ſeem to be from the face of the Bills. One reaſon, why the proportion of the amounts of the *births* and *burials* in the Bills, comes now nearer than

Since 1730, theſe pariſhes have been decreaſing ſo faſt, that the *annual burials* in them have ſunk from 8672 to near 4000, which is lower than they were before the year 1660. In *Weſtminſter*, on the contrary, and the 23 out-pariſhes in *Middleſex* and *Surrey*, the *annual burials* have ſince 1660 advanced from about 4000 to 16,000, the medium for ſome years before 1769.—These facts prove, that the inhabitants of *London* are now much leſs crowded together than they were. It appears, in particular, that *within the walls* the inhabitants take as much room to live upon as double their number did formerly.

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it did, to the true proportion, may, perhaps, be, that the number of Diſſenters is leſſened (*a*).

I will add, that it is probable that *London* is now become leſs fatal to children than it was; and that this is a further circumſtance which muſt reduce the difference I have mentioned; and which is likewiſe neceſſary to be joined to the greater deficiencies in the births, in order to account for the very ſmall proportion of children who ſurvived 10 years of age, during the two firſt of the periods I have ſpecified.—Since 1752, *London* has been thrown more open. The cuſtom of keeping country-houſes, and of ſending children to be nurſed in the country, has prevailed more. But, particularly, the deſtructive uſe of ſpirituous liquors among the poor has been checked (*b*).

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(*a*) See the end of the Note in p. 254.

(*b*) The enquiry in the preceding pages into the number of inhabitants in *London* was firſt publiſhed above twenty-one years ago. Fourteen years ago (or in 1777) the ſurveyors of the houſe and window duties were ordered to make returns to the tax office of the number of houſes of *all ſorts* in *London*, *Southwark*, *Weſtminſter*, and the county of *Middleſex*. The number returned was 90,570. This ſeems to leave no room for much diſpute. Allowing ſix to a houſe, the number of inhabitants *within the Bills*, with the addition of the whole county, will be 543,420 See the Note in p. 260 and 252; and a more particular account in my Eſſay *on the Population of England from the Revolution to the preſent time.*

Mr. *Wales*, in the pamphlet quoted in the Notes, p. 243 and p. 251, without taking any notice of theſe re-

I have ſhewn that in *London*, even in its preſent ſtate, and according to the moſt moderate

turns, calculates the number of houſes and inhabitants in *London* in the following manner.—Mr. *Maitland*, in 1737 (when the regiſtered burials for 20 years had been near 8000 *per ann.* more than they are at preſent) found the number of houſes in *London* to be 95,968. To this number Mr. *Wales* adds 4032, in order to make up 100,000; and by allowing 6½ to each houſe, finds the number of inhabitants to be 650,000.

Leaving the reader to judge as he pleaſes of this calculation, I ſhall reckon myſelf more out of danger of being wrong in following the documents I have juſt mentioned, and in ſtating from them the inhabitants of *London within the Bills*, with the addition of *Pancras* and *Maryhone* pariſhes, at *half a million*.—The annual medium of burials for the five years ending in 1780 was, according to the Bills, 20,743. Add 6000 for omiſſions, and the number of burials will be 26,743, or a 19th part nearly of the inhabitants, which is the proportion dying annually at *Stockholm*. See the Note, p. 250.

If the omiſſions are only 3038, agreeably to the reſult of Mr. *Maitland*'s enquiries, one in 21 will die annually.—Mr. *Howlett*, in his examination already quoted, (publiſhed about eight years ago) p. 91, makes the deficiencies in the burials to be much greater than either of theſe eſtimates. He reckons that a deficiency of 2100 burials has been occaſioned by the Act of Parliament, requiring pariſh infants to be nurſed for ſix years in the country, which implies that ſo many now die annually in the country who ought to be included in the Bills. But this is not his meaning; for he ſays, that of 2800 infants which come annually upon pariſhes, and are required to be removed in three weeks into the country, only 250 die there in *ſix years*; whereas 450 die in the *three weeks* before their removal. The deficiency, therefore, in the Bills ariſing from hence, can be only 250. But this carries us to the contrary extreme, and makes the probabilities of the duration of life among infants,

derate computation, half the number born die under *three* years of age. In *Vienna* and *Stock-*

faults, committed by parishes to the care of foster-mothers, to be much greater than were ever known among infants in the best situations.—Mr. *Howlett*'s meaning appears to be, that 2100 deaths are *prevented* annually by this Act of Parliament. The observation just made shews, that it is impossible this should be true; but supposing it true, it will be obvious, that a *prevention* of deaths ought not to be reckoned among *deficiencies*; for on the same ground the deaths prevented by cleansing and opening the streets, and other salutary regulations, might be so reckoned.—This Act of Parliament has undoubtedly prevented a great number of deaths. Before it was passed, almost all parish infants died in the first six years. Let us reckon that *now* of 2800 brought annually into workhouses, only a thousand die in this time, after being removed in three weeks into the country to be nursed. This would be a change unspeakably for the better; and it would imply that the probabilities of the duration of life among them is higher than is common among children in *London*. On this supposition the deficiency under consideration will be a *thousand*; and it will appear that 1100 ought to be taken from Mr. *Howlett*'s total of deficiencies. But much greater deductions ought to be made on other accounts.—He gives 2000 as a deficiency occasioned by carrying out so many to be buried in the neighbouring villages, without making any allowance for the burials brought in. He gives also the burials in the *East-India* ships serving abroad; the burials in the *hospitals*, *Northampton-chapel*, *Bunhill*, as all burials of persons residing within the Bills; and thus makes the deficiencies amount to 11,273, and the total of annual burials to 31,941. He farther calculates that the kingdom in general, and *London* in particular, is improved a *tenth* in healthiness; and on this account he adds a tenth to the total just mentioned, and in *this* way makes the number of inhabitants in *London* to be about 800,000.—Such are Mr. *Howlett*'s calculations.—In his list of deficiencies he sets down 1400 for the annual burials in *Bunhill*. From

Stockholm, under *two*. In *Manchester*, under *five*. In *Norwich*, under *five*. In *Northampton*, under (*a*) *ten*.—But it appears from *Graunt*'s (*b*) accurate account of the births, weddings, and burials in three country parishes for 90 years; (and also, from Dr. *Short*'s collection of observations in his *Comparative History*, and his Treatise, entitled, *New Observations on Town and Country Bills of Mortality*) that in country villages and parishes, the major part live to

the note in p. 261, it appears that this number is near 1000 greater than the truth. The annual burials at *Northampton-chapel*, *Clerkenwell*, in 1782, he makes to be 2080. The information I received from thence was, that, taking one week with another, they might be then reckoned at 30 in a week, or 1560 in a year. This, probably, Mr. *Howlett* mistook for 40 in a week, and thus has been led to make them 2080 in a year. They were, however, increasing, and every year diminishing more and more the burials in the churches, the lowness of the fees gaining for this burying ground, and the other burying grounds mentioned in the note p. 261, a particular preference among the lower ranks of people.

(*a*) See the Tables at the beginning of the next volume.—The whole number buried in the parish church of *Manchester* for six years, from 1773 to 1778, was 4126, of whom 2174 were children under five. But it must be considered, that in this town the births then exceeded the burials, and that consequently the Bills give the proportion dying in childhood too high.

(*b*) See *Natural and Political Observations on the Bills of Mortality*, by Capt. *John Graunt*, F. R. S.—Also Mr. *Derham*'s *Phisico-Theology*, p. 174, where it appears, that in the parish of *Aynho* in *Northamptonshire*, though the *births* had been for 118 years to the marriages as 6 to 1; yet the *burials* had been to the marriages only as 3¼ to 1.

mature

mature age, and even to marry. In the parish of *Holy-Cross* (*a*), near *Salop*, it appears from a curious register, which has been kept by the Rev. Mr. *Gorsuch*, the vicar, that, of 655 who died there at all ages for 20 years to 1770 near

(*a*) This parish contains in it a village which is a part of the suburbs of *Shrewsbury*. It consists of 1400 acres of arable and pasture land; besides 300 acres taken up by houses and gardens. It is six miles in circumference; half of which lies along the banks of the river *Severn*.—I mention these particulars to shew, that it may be reckoned a *country* parish; though, perhaps, not perfectly so, on account of its nearness to *Shrewsbury*.—The christenings in it exceed the burials in the proportion of 15 to 13; and the number of inhabitants (mostly labouring people) had for 20 years before 1771 kept nearly to 1050, without any considerable increase.—The register of this parish from 1750 to 1760, has been published in the LIId volume of the *Philosophical Transactions*, Part I. Art. 25. And a continuation of it from 1760 to 1770, in the LXIst volume, p. 57. It is kept with particular care and accuracy by Mr. *Gorsuch*; and furnishes very useful *data* for determining the value of country lives.—It deserves to be mentioned particularly, that no *foreigners* or *strangers*, who happen to die in this parish, or who may be brought into it to be buried, are entered into the register: Nor are any of the fixed inhabitants omitted, though carried out to be buried.

In Nov. 1781. Mr. *Gorsuch* was so kind as to favour me with a continuation of his Observations to 1780, which makes them complete for 30 years. An abstract of them, and a Table of the decrements of life deduced from them, which I reckon one of the most correct that has been ever published, will be found in the Collection of Tables in the next Volume. The conclusions mentioned above are confirmed by the addition of these last Observations.

one half lived to 30 years of age: And, by forming a Table of Obſervations from this regiſter, in the manner which will be deſcribed in the laſt Eſſay, I find that a child juſt born in this pariſh has an expectation of 33 years; and that, in general, under the age of 50, the *expectations* of lives there exceed thoſe in *London*, in the proportion of about 4 to 3.—In the pariſh of *Ackworth*, *Yorkſhire*, it appears, from an exact account kept by Dr. *Lee*, of the ages at which all died there for 20 years to 1767, that half the inhabitants live to the age of 46—In the province of *Vaud*, *Switzerland*, conſiſting of 112,951 (*a*) inhabitants, half live to 41.—So great is the difference between the duration of human life in *towns* and in the *country*.—Further evidence for the truth of this obſervation may be deduced from the account given by Dr. *Thomas Heberden*, and publiſhed in the Philoſophical Tranſactions (Vol. LVII. p. 461), *of the increaſe and mortality of the inhabitants of the iſland of Madeira*. In this iſland, it ſeems, the weddings have been to the births, for 8 years, from 1759 to 1766, as 10 to 48.8; and to the burials, as 10 to 27.5, or 9 to 24.75. Double theſe proportions, therefore, or the proportion of 20 to 46.8, and of 18 to 24.75, are the proportions of the

(*a*) See the Supplement in the next Volume.

num-

number marrying annually, to the number born and the number dying. Let one marriage in three be a 2d or (*a*) 3d marriage on the ſide of either the man or the woman; or, in other words, let one in ſix of all that marry be *widows* and *widowers*; and 9 marriages will imply 15 perſons who have grown up to maturity, and lived to marry once or oftener; and the proportion of the number marrying annually the firſt time, to the number dying annually, will be 15 to 24.75, or 3 to 5. It may ſeem to follow from hence, that in this iſland three-fifths of thoſe who die have been married; and, conſequently, that only two-fifths of the inhabitants die in childhood and celibacy; and this would be a juſt concluſion were there no increaſe, or had the births and burials been equal. But it muſt be remembered, that the general effect of an increaſe while it is going on in a country, is to render the proportion of perſons marrying annually, to the annual deaths, *greater*, and to the annual births *leſs*, than the true proportion marrying out of any given number born. This proportion generally lies between the other two proportions, but always

(*a*) This proportion is taken nearly from fact.—In all *Pomerania*, during 9 years, from 1748 to 1756, the number of perſons who married was 56,956; and of theſe, 10,586 were *widows* and *widowers*. *Suſmilch*'s Works, Vol. I. Tables, p. 98.

neareſt to the firſt (*a*); and, in the preſent caſe, it cannot be ſo little as one half. Agreeably

(*a*) In a country where there is no increaſe or decreaſe of the inhabitants, and where alſo life, in its firſt periods, is ſo ſtable, and marriage ſo much encouraged, that half of all who are born live to be married, the *annual* births and burials muſt be equal, and alſo *quadruple* the number of weddings, after allowing for 2d and 3d marriages. Suppoſe in theſe circumſtances (every thing elſe remaining the ſame) the *probabilities of life*, during its firſt ſtages, to be improved. In this caſe, more than *half* the born will live to be married, and an increaſe will take place. The births will exceed the burials, and both fall below *quadruple* the weddings; or, which is the ſame, below *double* the number annually married.—Suppoſe next (the *probabilities of life* and the *encouragement to marriage* remaining the ſame) the *prolificknefs* only of the marriages to be improved. In this caſe it is plain, that an increaſe alſo will take place; but the *annual* births and burials, inſtead of being leſs, will now both riſe above *quadruple* the weddings; and therefore the proportion of the born to that part of the born who marry (being by ſuppoſition two to one) will be leſs than the proportion of either the *annual* births or the *annual* burials, to the number marrying *annually*. Suppoſe again (the *encouragement to marriage* remaining the ſame) that the *probabilities of life* and the *prolificknefs of marriages* are both improved. In this caſe, a more rapid increaſe will take place, or a greater exceſs of the births above the burials; but at the ſame time they will keep nearer to *quadruple* the weddings, than if the latter cauſe only had operated, and produced the ſame increaſe.—I ſhould be too minute and tedious, were I to explain theſe obſervations at large. It follows from them, that, in every country or ſituation where, for a courſe of years, the *burials* have been either *equal to* or *leſs* than the *births*, and both under *quadruple* the marriages; and alſo that, wherever the burials are *leſs* than quadruple the annual marriages, and at the ſame time

greeably to this, it appears alſo from Dr. *Heberden*'s account, that the *expectation* of a child juſt born in *Madeira* is about 39 years; or more than double the expectation of a child juſt born in *London*. For the number of inhabitants was found, by a ſurvey made in the beginning of the year 1767, to be 64,614. The annual medium of *burials* had been, for eight years, 1293; of *births*

time the births *greater*, there the major part of all that are born live to marry.

I have ſhewn how the allowance is to be made for 2d and 3d marriages. Very wrong concluſions will be drawn if this allowance is not made. But it is, in part, compenſated by the natural children which are included in the births, and which raiſe the proportion of the births to the weddings higher than it ought to be, and therefore bring it nearer to the true proportion of the number born *annually*, to thoſe who marry annually, after deducting thoſe who marry a 2d or 3d time.

In drawing concluſions from the proportion of *annual* births and burials, in different ſituations, ſome writers on the increaſe of mankind, have not given due attention to the difference in theſe proportions, ariſing from the different circumſtances of increaſe or decreaſe among a people. One inſtance of this I have now mentioned; and one further inſtance of it is neceſſary to be mentioned. The proportion of *annual* births to weddings has been conſidered as giving the true number of children derived from each marriage, taking all marriages one with another. But this is true only when, for many years, the births and burials have kept nearly equal. Where there is an exceſs of the births occaſioning an increaſe, the proportion of *annual* births to weddings muſt be leſs than the proportion of children derived from each marriage; and the contrary muſt take place where there is a decreaſe.

2201. The number of inhabitants, divided by the annual medium of *burials*, gives 49.89; or the *expectation* nearly of a child juſt born, ſuppoſing the *births* had been 1293, and conſequently equal to the *burials*, the number of inhabitants remaining the ſame. And the ſame number, divided by the annual medium of *births*, gives 29.35; or the *expectation* of a child juſt born ſuppoſing the burials 2201, the number of births and of inhabitants remaining the ſame. And the true *expectation* of life muſt be ſomewhere near the mean between 49.89 and 29.35.

Again: A 50th part of the inhabitants of *Madeira*, it appears, die annually. In *London*, I have ſhewn, that above twice this proportion dies annually. In ſmaller towns a ſmaller proportion dies (*a*); and the births alſo

(*a*) In *London*, this proportion is, at the higheſt, 1 in $20\frac{3}{4}$.—In *Norwich*, 1 in $24\frac{1}{2}$.—In *Northampton*, 1 in $26\frac{2}{5}$. See the laſt Eſſay. In the pariſh of *Newbury*, *Berks*, conſiſting of 3732 perſons, all *town* inhabitants, the annual medium of deaths for 19 years, or from 1747 to 1765, was 136. In this town, therefore, 1 in $27\frac{1}{2}$ die annually. The contiguous pariſh of *Speen* conſiſted, in 1757, of 1200 inhabitants, about 520 of whom were inhabitants of that part of the town of *Newbury* which is in this pariſh, and the reſt were *country* inhabitants. For 34 years, or from 1724 to 1757, *thirty-nine* died here *annually*; or 1 in 31.—In both theſe pariſhes the births and burials are nearly equal.—I believe theſe facts may be depended on; and they ſeem to ſhew us very diſtinctly

alſo come nearer to the burials.—In general; there ſeems reaſon to think that in towns (allowing for particular advantages of ſituation, trade, police, cleanlineſs, and openneſs, which ſome towns may have,) the exceſs of the burials above the births, and the proportion of inhabitants dying annually, are more or leſs as the towns are greater or ſmaller. In *London* itſelf, about 160 years ago, when it was ſcarcely a fourth of its preſent bulk, the births were much nearer to the burials, than they are now. But in country pariſhes and villages, the births almoſt always exceed the burials; and I believe it never happens, except in very particular ſituations, that more than a 40th (*a*) part

ſtinctly the gradations in the degrees of human mortality from *great* towns to *moderate* towns, and from *moderate* towns to *ſmall* towns, and to pariſhes conſiſting partly of town and partly of country inhabitants. The next note will ſhew what the degree of human mortality is in places purely country.

(*a*) According to *Graunt*'s account of a pariſh in *Hampſhire*, not reckoned, he ſays, remarkably healthful, a 50th part of the inhabitants had died annually for 90 years, *Natural and Political Obſervations, &c.* Chap. xii.—In the pariſh of *Ackworth*, *Yorkſhire*, one of 47 die annually. See the regiſter of this pariſh at the end of the firſt additional Eſſay in the next volume. In the province of *Vaud*, *Switzerland*, one in 45 die annually. See the firſt part of the *Supplement* in the next volume. In 1098 country pariſhes, mentioned by *Suſmilch*, the annual average of deaths, for ſix years ending in 1749, was 5255. The number of inhabitants was 225,357. One,

part of the inhabitants die annually. In the four provinces of *New-England* there is a very rapid increaſe of the inhabitants; but, notwithſtanding this, at *Boſton*, the capital, the inhabitants would decreaſe, were there no ſupply from the country: For, if the account I have ſeen is juſt, from 1731 to 1762, the burials all along exceeded the

One, therefore, in 43 died annually.—In 106 other pariſhes, mentioned by him, this proportion was 1 in 50.

In the dukedom of *Wurtemberg*, the inhabitants, Mr. *Suſmilch* ſays, are numbered every year; and from the average of five years, ending in 1754, it appeared that, taking the towns and country together, 1 in 32 died annually.—In another province, which he mentions, conſiſting of 635,998 inhabitants, 1 in 33 died annually. From theſe facts he concludes, that taking a whole country in *groſs*, including all cities and villages, mankind enjoy among them about 32 or 33 years each of exiſtence. This, very probably, is below the truth; from whence it will follow, that a child born in a country pariſh or village, has, at leaſt, an expectation of 36 or 37 years; ſuppoſing the proportion of *country* to *town* inhabitants to be as $3\frac{1}{2}$ to 1; which, I think, this ingenious writer's obſervations prove to be nearly the caſe in *Pomerania*, *Brandenburgh*, and ſome other kingdoms.

In all *Sweden*, conſiſting in 1763 of 2,446,394 inhabitants, the annual medium of deaths for 9 years, ending in 1763, was 69,125; and therefore one in 35 and two-fifths died annually. The medium of births was 90,245; of marriages 21,220. See the firſt additional Eſſay in the next volume.—In the kingdom of *Naples*, conſiſting of 4,311,503 inhabitants in 1777, the medium of deaths for 5 years was 115,412; and therefore one in 37 and a third died annually. The births were 166,808. *See the Eſſay on the Population of* England, &c. page 15.

births

births (*a*). So remarkably do towns, in consequence of their unfavourableness to health, and the luxury which generally prevails in them, check the increase of countries.

Healthfulness and prolificness are, probably, causes of increase seldom separated. In conformity to this observation, it appears from comparing the births and weddings, in countries and towns where registers of them have been kept, that in the former, marriages, one with another, seldom produce less than four children each; generally between four and five, and sometimes above five (*b*). In all *Sweden* the births and weddings

(*a*) See a particular account of the births and burials in this town from 1731 to 1752 in the *Gentleman's Magazine* for 1753, p. 413.

(*b*) Any one may see what evidence there is for this, by consulting Dr. *Short*'s two books already quoted, and the *Abridgment of the Philosophical Transactions*, Vol. VII. Part iv. p. 46, and *Graunt*'s account, already quoted, of the births, weddings, and burials in three country parishes for 90 years; compared with similar accounts in towns. In considering these accounts, it should not be forgotten that allowances must be made for the different circumstances of increase or decrease in a place, agreeably to the observations at the end of the note in page 264.

In April 1779 the inhabitants of the parish of *Biddulph*, in *Staffordshire*, were numbered, and found to be 495 males, and 540 females, making 207 families. The annual average of births for 20 years preceding 1780 had been 21.4 males, and 17.5 females, of burials 10.85 males, and 10.3 females; of marriages 6.15.—The same averages

dings are to one another as $4\frac{1}{4}$ to 1.—In all *France* as $4\frac{2}{3}$ to 1. But in towns this proportion is *generally* between 3 and 4 to 1.

I have ſometimes heard the great number of old people in *London* mentioned, to prove its favourableneſs to health and long life. But no obſervation can be more erroneous. There ought, in reality, to be more old people in *London*, in proportion to the number of inhabitants, than in any ſmaller towns; becauſe at leaſt one quarter of its inhabitants are perſons who come into it from the country, in the moſt robuſt part of life, and with a much greater probability

averages for 60 years had been 16.9 males *born* annually and 14.7 females; 9.4 males *buried* annually, and 9.93 females; marriages 5.46.—Taking, therefore, the higheſt of theſe averages, it appears that in this pariſh a 46th part of the *males* die annually, but only a 52d part of the *females*; that the annual births are nearly a 26th part of the inhabitants; and that every marriage, ſuppoſing no allowance for illegitimate births, produces ſix children. This account I owe to an information communicated by the Rev. Mr. *Wilſon*, the miniſter of this pariſh, to Dr. *Haygarth* at *Cheſter*.

The pariſh of *Swinderby*, in *Lincolnſhire*, conſiſted in June 1771 (as I have learnt from Dr. *Diſney*, then the worthy miniſter of that pariſh) of 52 families and 224 ſouls, 95 of whom were heads of families, 87 children, 32 ſervants, and 10 inmates.—The *births*, *marriages*, and *burials* for 30 years before 1771 had been 199, 47 and 154. The proportion of marriages to births therefore, was as 1 to $4\frac{1}{4}$.—A number equal to a 34th of the inhabitants had been born annually, and a 44th part died annually.—The inhabitants of *Okeford*, in *Devonſhire*, were in 1770, 422.

The

bility of living to old age, than if they had come into it in the weakneſs of infancy. But, notwithſtanding this advantage, there are much fewer perſons who live to great ages in *London*, than in moſt other places where obſervations have been made.—At *Breſlaw* it appears, by Dr. *Halley*'s Table, that 41 of 1238 born, or a 30th part, live to be 80 years of age. In the pariſh of *All-ſaints*, in *Northampton*, an account has been kept ever ſince 1733 of the ages at which the inhabitants die; and I find that a 22d part die there turned of 80. At *Norwich* a like account has been kept; and

The average of births for 20 years to 1769 had been 12, and of burials 7½. A 35th part, therefore, was born annually, and a 56th part died.

From the returns of the Intendants of the different provinces of FRANCE, it appeared, that the annual medium of births in that kingdom for ten years to 1780, had been 940,935; of deaths 818,491; and of marriages 213,774.—See Mr. NECKER on the Adminiſtration of the finances of *France*, Vol. I. chap. 9.—The births and marriages were, therefore, in the proportion mentioned in the text. From the laſt note but one it appears that a 35th part of the inhabitants of a country may be reckoned to die annually. Multiply, therefore, 818,491 by 35, and the kingdom of *France* will appear to have conſiſted in 1780 of 28 millions and a half of inhabitants. Nor is there any reaſon to think this to be greater than the true number; for the deaths, as well as the births and marriages, are probably given too ſmall, it being ſcarcely poſſible to avoid omiſſions in ſuch returns. It appears further from the great exceſs of births, that the population of *France* was then increaſing.—See more on this ſubject in the *Appendix* to my *Diſcourſe on the Love of our Country*, p. 1.

it

it appears, that a 27th part of the inhabitants die turned of the ſame age.—According to Mr. *Kerſſeboom*'s Table of Obſervations, publiſhed at the end of Mr. *De Moivre*'s Treatiſe on the Doctrine of Chances, a 14*th* part die turned of 80. And this is the very proportion that died turned of 80 in the pariſh of *Ackworth*, for the 20 years mentioned page 262. In the pariſh of *Holy-Croſs*, already mentioned, p. 261, 1 in 11½, or 2 in 22 of the inhabitants live to 80 (*a*).—But in *London*, for 30 years, ending at the year 1768, only 25 of every 1000, who had died, or a 40th part, had lived to this age (*b*); which may be eaſily diſcovered, by dividing the ſum of all who have died during theſe years at all ages, by the ſum of all who had died above 80 (*c*).

Among

(*a*) This, however, will appear itſelf inconſiderable, if the following account is true: "In 1761 the burials "in the diſtrict of *Chriſtianna*, in *Norway*, amounted "to 6,927, and the chriſtenings to 11,024. Among "thoſe who died, 394, or 1 in 18, had lived to the "age of 90; 63 to the age of 100, and ſeven to the "age of 101.—In the dioceſe of *Bergen*, the perſons "who died amounted to 2,580, of whom 18 lived "to the age of 100; one woman to the age of 104, and "another woman to the age of 108."

See the *Annual Regiſter* for 1761, p. 191.

(*b*) For five years to 1780 only one in 46 lived to 80.

(*c*) In the pariſh church of *Mancheſter*, of 4126 buried during ſix years ending in 1778, a hundred and twenty

Among the peculiar evils to which great towns are ſubject, I might further mention the PLAGUE. Before the year 1666, this dreadful calamity laid *London* almoſt waſte once in every 15 or 20 years; and there is no reaſon to think, that it was not generally bred

twenty-nine, or a 32d part, had lived to 80 or more. This proportion would be conſiderably greater were there no increaſe of *Mancheſter*, and no exceſs of the births above the burials.—The ſame is true of *Warrington*, in *Lancaſhire*, where of 2430 buried in eight years ending in 1780, ſixty-ſeven, or a 36th part, had lived to 80 or upwards; and alſo of the pariſh of ECCLES in the ſame county, where of 1123 buried in four years, from 1776 to 1779, fifty-one, or a 22d part, had lived to 80.—In CHESTER, where the births and burials are nearly equal, of 1969 females who died in the courſe of 9 years, from 1772 to 1780, 149 or a 13th part, had lived to 80; but of *males*, only 72 out of 1764, or a 25th part. See the Tables in the next Volume. In all SWEDEN, where the births exceed the burials in the proportion of nearly 13 to 10, 710 *females* of 10,000 born (or a 14th part) and 555 *males*, of 10,000 (or an 18th part) live to 80: But in STOCKHOLM only one in a 100 of the *females* born there, and one in 300 of the *males*, live to this age. See the Tables in the next Volume.

Theſe facts give a frightful view of the fatality of great towns to human life. A farther account, with anſwers to ſome objections, may be found in the next volume, in the firſt additional Eſſay on the *difference between the duration of human life in great towns and in country pariſhes*.

I have ſaid above, that a 40th part of all who die in *London* live to 80. But it ſhould be conſidered, that a great proportion of thoſe who die in *London* came into

bred within itſelf. A moſt happy alteration has taken place; which, perhaps, in part is owing to the greater advantages of cleanlineſs and openneſs which *London* has enjoyed ſince it was rebuilt; and which lately have been very wiſely improved.

The facts I have now taken notice of are ſo important that, I think they deſerve more attention than has been hitherto beſtowed upon them. Every one knows that the ſtrength of a ſtate conſiſts in the number of people. The encouragement of population, therefore, ought to be one of the firſt objects of policy in every ſtate; and ſome of the worſt enemies of population are the luxury, the licentiouſneſs, and debility produced and propagated by great towns.

it in the firmeſt parts of life, and that conſequently nothing can be from hence determined with reſpect to the proportion of the *natives* of *London* who live to 80. This muſt be a much ſmaller proportion. The corrected Table of Obſervations for *London* (or Table 15th in the next Volume) makes it as 25 to 1518, or as 1 to 60. But even this corrected Table certainly gives the probabilities of living in *London*, at moſt ages, too high; and were there ſuch accurate *data* for forming a table for *London* as have been furniſhed by the Obſervations at *Stockholm*, the rate of mortality in the two cities would not perhaps appear to be very different. More will be ſaid on this ſubject in the introduction to the Tables in the next volume.

I have

I have obſerved that *London* is now (*a*) increaſing. But it appears, that, in truth, this is an event more to be dreaded than deſired. The more *London* increaſes, the more the reſt of the kingdom muſt be deſerted; the fewer hands muſt be left for agriculture; and, conſequently, the leſs muſt be the plenty, and the higher the price of all the means of ſubſiſtence. *Moderate* towns, being ſeats of refinement, emulation, and arts, may be public advantages. But *great* towns, long before they grow to half the bulk of *London*, become checks on population of too hurtful a nature, nurſeries of debauchery and voluptuouſneſs; and, in many reſpects, greater evils than can be compenſated by any advantages (*b*).

Dr.

(*a*) If we may truſt the Bills, *London* has decreaſed ſince this was written. The annual medium of burials for five years ending in 1770, 1777, and 1780, was 22,688—21,087—and 20,743. The medium for three years to 1780, was 20,445. But this decreaſe has probably been owing to the cauſes mentioned in the notes, p. 251 and 260.

(*b*) The mean annual *births*, *weddings*, and *burials* in the following towns, for ſome years before 1772, have been nearly,

	Births.	Weddings.	Burials.
At *Paris*, — —	19,100	4,400	19,400
Vienna, from 1757 to 1769	5,800		6,600
Amſterdam, from 1761 to 1770	4,600	2,400	7,922
Copenhagen, — —	2,700	886	3,300

Dr. *Heberden* obſerves that, in *Madeira*, the inhabitants double their own number in 84 years. But this (as you, Sir, well know) is a very ſlow increaſe, compared with that which takes place among our colonies in AMERICA. In the back ſettlements, where the inhabitants apply themſelves entirely to agriculture, and luxury is not known, they double their own number in 15 years; and all through the northern colonies, in 25 years (*a*). This is an inſtance of increaſe ſo rapid, as to have ſcarcely any parallel. The births in theſe countries muſt exceed the burials much more than in *Madeira*; and a greater proportion of the born muſt reach maturity.—In 1738, the number of inhabitants in

	Births.	Weddings.	Burials.
At *Berlin*, for 5 years, ending at 1759	3,855	980	5,054
Stockholm, for nine years, ending in 1763	2,535		3,781

It deſerves notice, that before 1770, all that died in the hoſpitals at *Vienna* were omitted in the Bills.—Of the *Paris* Bills a more particular account will be given in the Poſtſcript to this Eſſay.—The annual medium of burials at *Amſterdam* for 10 years to 1710, was 7,288. For 10 years to 1780, it was 8,710; but three of theſe laſt years were reckoned very ſickly years.

(*a*) See a Diſcourſe on *Chriſtian Union*, by Dr. *Styles*, *Boſton*, 1761, p. 103, 109, &c.—See alſo, *The Intereſt of Great Britain conſidered with regard to her Colonies, together with Obſervations concerning the Increaſe of Mankind, peopling of Countries*, &c. p. 35. 2d edit. *London*, 1761.

New

New Jerſey was taken by order of the government, and found to be 47,369. Seven years afterwards, the number of inhabitants was again taken; and found to be increaſed, by procreation only, above 14,000; and very near one *half* of the inhabitants were found to be under (*a*) 16 years of age. In 22 years, therefore, they muſt have doubled their own number, and the births muſt have exceeded the burials 2000 annually. As the increaſe here is much quicker than in *Madeira*, we may be ſure that a ſmaller proportion of the inhabitants muſt die annually. Let us, however, ſuppoſe it the ſame, or a 50th part. This will make the annual burials to have been, during theſe ſeven years, 1000; and annual births 3000; or an 18th part of the inhabitants.—Similar obſervations may be made on the much quicker increaſe in *Rhode Iſland*, as related in the preface to the *Collection of the London Bills of Mortality*; and alſo in the valuable pamphlet laſt quoted, on *the Intereſt of Great Britain with regard to her Colonies*, p. 36.—What a prodigious difference muſt there be, between the vigour and the happineſs of human life in ſuch ſituations, and in ſuch a place as *London?*—The original number of perſons who, in 1643,

(*a*) According to Dr. *Halley*'s Table, the number of the living under 16, is but a *third* of all the living at all ages.

had ſettled in *New-England*, was 21,200. Ever ſince, it is reckoned, that more have left them than have gone to them (*a*). In the year 1760, they were increaſed to half a million. They had, therefore, all along doubled their own number in 25 years. It is not probable that they will continue to increaſe at the ſame rate; but ſhould this happen they will, 70 years hence, in *New-England* alone, be four millions; and in all the colonies (*b*), above twice the number of inhabitants in *Great Britain* (*c*).—But I am wandering

(*a*) See Dr. *Style*'s pamphlet, juſt quoted, p. 110, &c.

(*b*) In the original letter to Dr. *Franklin*, containing theſe obſervations, and communicated by him to the Royal Society (in April 1769), the following words were here added.—"Formerly an increaſing number of "FRIENDS, but now likely to be converted, by an un-"juſt and fatal policy, into an increaſing number of "ENEMIES."—This reflection was occaſioned by the diſcontents which were then prevalent in the colonies, and which had been produced firſt by the Stamp Act, and after the repeal of that act, by the duties laid in *America* on tea, paper, glaſs, &c.—When read to the Royal Society, it was ſoftened by the omiſſion of the words "unjuſt and fatal policy;" but, notwithſtanding this, it gave offence; and was ſuppreſſed in all the former publications of theſe Obſervations.—I need not ſay how dreadfully the apprehenſions expreſſed by it were afterwards verified.

(*c*) The rate of increaſe, ſuppoſing the procreative powers the ſame, depends on two cauſes: The "encouragement to marriage;" and the "*expectation* of "a child juſt born." When one of theſe is given, the increaſe

dering from my purpoſe in this letter. The point I had chiefly in view was, the preſent ſtate

increaſe will be always in proportion to the other.—— That is; as much *greater* or *leſs* as the *ratio* is of the numbers who reach maturity, and of thoſe who marry, to the number born, ſo much *quicker* or *ſlower* will be the increaſe.—Let us ſuppoſe the operation of theſe cauſes ſuch, as to produce an annual exceſs of the *births* above the *burials*, equal to a 36th part of the whole number of inhabitants. It may ſeem to follow from hence, that the inhabitants would double their own number in 36 years; and thus ſome have calculated. But the truth is, that they would double their own number in much leſs time. Every addition to the number of inhabitants from the births, produces a proportionably greater number of births, and a greater exceſs of theſe above the burials; and if we ſuppoſe the exceſs to increaſe annually at the ſame rate with the inhabitants, or ſo as to preſerve the *ratio* of it to the number of inhabitants always the ſame, and call this *ratio* $\frac{1}{r}$, the period of doubling will be the *quotient* produced by dividing the logarithm of 2 by the *difference* between the logarithms of $r + 1$ and r; as might be eaſily demonſtrated. In the preſent caſe, r being 36, and $r + 1$ being 37, the period of doubling comes out 25 years. If r is taken equal to 22, the period of doubling will be 15 years.—But it is certain that this ratio may, in many ſituations, be greater than $\frac{1}{22}$; and, inſtead of remaining the ſame, or becoming leſs, it may *increaſe*, the conquence of which will be, that the period of doubling will be ſhorter than this rule gives it.—According to Dr. *Halley*'s Table, the number of perſons between 20 and 42 years of age is a third part of the whole number living at all ages. The prolific part, therefore, of a country may very well be a 4th of the whole number of inhabitants; and ſuppoſing four of theſe, or every other marriage between perſons all under 42, to produce *one*

ſtate of *London* as to healthfulneſs, number of inhabitants, and its influence on population. The obſervations I have made may, perhaps, help to ſhew, how the moſt is to be made of the lights afforded by the *London* Bills; and ſerve as a ſpecimen of the proper method of calculating from them. It is indeed extremely to be wiſhed, that they were leſs imperfect than they are, and extended further. More pariſhes round *London* might be taken into them; and, by an eaſy improvement in the pariſh regiſters now kept, they might be extended through all the pa-

birth every year, the annual number of births will be a 16th part of the whole number of people. And, therefore, ſuppoſing the burials to be a 48th part, the annual exceſs of the births above the burials will be a 24th part, and the period of doubling 17 years.

I muſt not conclude this note without adding a remark to remove an objection which may occur to ſome in reading Dr. *Heberden*'s account of *Madeira*, to which I have referred. In that account 5945 is given as the number of children under ſeven in the iſland, at the beginning of the year 1767. The medium of annual births, for eight years, had been 2201; of burials 1293. In ſix years, therefore, 13,206 muſt have been born; and if, at the end of ſix years, no more than 5945 of theſe were alive, 1210 muſt have died every year. That is; almoſt all the burials in the iſland, for ſix years, muſt have been burials of children under ſeven years of age. This is plainly incredible; and, therefore, it ſeems certain, that the number of children under ſeven years of age muſt, through ſome miſtake, be given, in that account, 3000 or 4000 too little.

riſhes

riſhes and towns in the kingdom. The advantages ariſing from hence would be very conſiderable. It would give the preciſe law according to which human life waſtes in its different ſtages; and thus ſupply the neceſſary *data* for computing accurately the values of all *life-annuities* and *reverſions*. It would, likewiſe, ſhew the different degrees of healthfulneſs of different ſituations, mark the progreſs of population from year to year, keep always in view the number of people in the kingdom, and, in many other reſpects, furniſh inſtruction of the greateſt importance to the ſtate. Mr. *De Moivre*, at the end of his book on the Doctrine of Chances, has recommended a general regulation of this kind; and obſerved, particularly, that at leaſt it is to be wiſhed, that an account was taken, at proper intervals, of all the living in the kingdom, with their ages and occupations; which would, in ſome degree, anſwer moſt of the purpoſes I have mentioned.—But, dear Sir, I am ſenſible it is high time to finiſh theſe remarks. I have been carried in them far beyond the limits I at firſt intended. I always think with pleaſure and gratitude of your friendſhip. The world owes to you many important diſcoveries; and your name muſt live as long as there is any knowledge of philoſophy among mankind. That you may ever enjoy all that,

can

can make you moſt happy, is the ſincere wiſh of,

SIR,

Your much obliged,

and very humble Servant,

Newington-Green,
April 3, 1769.

RICHARD PRICE.

POST-

POSTSCRIPT.

AT *Edinburgh*, bills of mortality, of the ſame kind with thoſe of *London*, have been kept for many years. I have, ſince the foregoing letter was written, examined theſe Bills, and formed a Table of Obſervations from them, as I found them for a period of 20 years, beginning in 1739, and ending in 1758.—As this is a town of moderate bulk, and ſeems to have a particular advantage of ſituation; I expected to find the probabilities of life in it, nearly the ſame with thoſe at *Breſlaw*, *Northampton*, and *Norwich*; but I have been ſurprized to obſerve, that this is not the caſe. During the period I have mentioned, only one in 42 of all who died at *Edinburgh*, reached 80 years of age.—In general; it appears, that the probabilities of life in this town are much the ſame, through all the ſtages of life, with thoſe in *London*, the chief difference being, that after 30, they are rather lower at *Edinburgh*.—It is not difficult to account for this.—It affords, I think, a ſtriking proof of the pernicious effects ariſing from uncleanlineſs, and crowding together on one ſpot too many inhabitants. At *Edinburgh*, Mr. *Maitland* ſays, "the buildings, elſewhere called *houſes*, are denominated *lands*, and the *apartments*, in other places

" places named *ſtories*, here called *houſes*, are " ſo many freeholds inhabited by different " families; whereby the houſes are ſo ex- " ceſſively crowded with people, that the " inhabitants of this city may be juſtly pre- " ſumed to be more numerous than thoſe of " ſome towns of *triple* its dimenſions." See *Maitland's Hiſtory of Edinburgh*, p. 140.

In the year 1748, the whole number of *apartments* or *families* in the city and liberties of *Edinburgh*, was 9064. This Mr. *Maitland* mentions as the reſult of particular examination, and undoubtedly right. *Ib.* p. 217, 218.—In 1743, an accurate account was taken, by the deſire of this writer, of the number of *families* and *inhabitants* in the pariſh of St. *Cuthbert*. *Ib.* p. 171. The number of *families* was 2370, and of *inhabitants* at all ages, 9731. The proportion, therefore, of *inhabitants* to *families*, was $4\frac{1}{10}$ to 1; and, ſuppoſing this the true proportion for the whole town, the number of inhabitants was $4\frac{1}{10}$ multiplied by 9064, or 37,162. The yearly medium of deaths in the town and liberties for eight years, from 1741 to 1748, was 1783. *Ib.* p. 220 and 222. And, conſequently, *one* in $20\frac{4}{5}$ died annually.

Mr. *Maitland*, though poſſeſſed of the *data* from which theſe concluſions neceſſarily followed, has made the number of inhabitants 50,120, in conſequence of a diſpoſition to exaggerate in theſe matters, and of aſſuming, without

without any reaſon, a 28th part of the inhabitants as dying annually.

In page 220, he expreſſes much ſurprize at finding, that the number of males in this town was leſs than the number of females, in the proportion of 3 to 4. But this is by no means peculiar to *Edinburgh*.

All I have been ſaying muſt be underſtood of the ſtate of *Edinburgh*, before the year 1758. The Bills, ſince this year, have been ſo irregular, and ſo different from the ſame Bills for the preceding years, and from all other Bills, that I cannot give them any credit. Either ſome particular incorrectneſs has crept into the method of keeping them; or there has been ſome change in the ſtate of the town which renders them of no uſe.

From the note in p. 275, it appears, that the chriſtenings and burials at PARIS, come very near to equality. This once led me to ſuſpect, that there muſt be ſome particular ſingularity in the ſtate of *Paris*, which rendered it much leſs prejudicial to health and population than great towns commonly are. But better information has lately obliged me to entertain very different ſentiments.—The difference between the births and burials at *Paris*, is much greater than the Bills ſhew. "Children here are baptized the inſtant "they are born; and, in a day or two af-"terwards, it is the cuſtom to ſend them to "the

" the adjacent villages to be nurſed. A " great number, therefore, of the infants born " at *Paris* die in the country, and theſe " appear only in the regiſter of chriſten- " ings." See a book entitled the *Police of France*, page 127. And *Buffon*'s Natural Hiſtory, Tom. II. at the end.—" All the " children alſo received into the *Foundling-* " *Hoſpital*, are immediately ſent to be nurſ- " ed in the country, at a diſtance from *Paris*, " where they remain 5 or 6 years; at the end " of which time they are brought again to " *Paris*, the boys to be placed in the ſuburbs " of St. *Antoine*, and the girls at *Salpetriere*, " to be further maintained 'till they arrive at " the age of twelve years." *Police of France*, p. 81.—The following paſſage in the ſame writer, containing a further account of this Hoſpital, is important; and therefore, though long, I cannot help tranſcribing it.—" Let " us ſuppoſe, that out of 4000 children an- " nually carried into the country, two thirds " may die, during the five years they are " deſtined to remain at nurſe; ſo that only " 1333 would conſtantly be the annual " number ſent back to *Paris*; who, being " kept at the two Hoſpitals St. *Antoine* and " *Salpetriere* juſt mentioned, till they are 12, " and ſucceeded by a like number each year, " the total number compoſed of all brought " in the ſucceſſive years, would make the " conſtant reſting ſtock to amount to 9331.

" But

" But of theſe we will ſuppoſe a 5th part
" to die every year. Yet even then the
" conſtant reſting ſtock of children ought to
" be 7465. How greatly then muſt we be
" ſurprized to find, by the authentic account
" taken from their own books, only 640
" boys in the college of St. *Antoine*, and not
" more than 600 girls at the *Salpetriere*; ſo
" that the reſting ſtock of returned foundlings
" appears to be no more than 1240, which
" being deducted from 7465, will leave 6225.
" What then becomes of theſe?—Are they
" reclaimed by their parents?—Or do they
" periſh for want of care?—In anſwer to
" which queſtion it was explained to me;
" that as many of the lower claſs of people
" were induced to marry, in order to be ex-
" cuſed from ſerving in the militia; ſo when
" theſe have children, which they are un-
" able to maintain, they uſually ſend them to
" this hoſpital; which, therefore, muſt be
" looked upon, as not only a charity for the
" care of expoſed and deſerted children whoſe
" parents are unknown, but alſo as a public
" *nurſery* for the ſuſtenance of the children
" of poor people, who, tho' regiſtered at the
" office, are often reclaimed from their coun-
" try nurſes by their parents. This accounts,
" in ſome meaſure, for the ſmall ſtock of
" children brought back to the hoſpital at
" *Paris*.—The further difference is ſuſpected

" to

" to be owing to the inſufficient nouriſhment " they receive; as this particular charity, as " well as the General Hoſpital, adopts that " prepoſterous method of taking in an un- " limited number, while there is only a li- " mited income for their ſubſiſtence." *Ib.* page 83.

Theſe facts prove, that, at the ſame time that the regiſter of *chriſtenings* at *Paris* muſt be full, the regiſter of *burials* muſt be very deficient. Let the deficiencies be reckoned at 3700; and, conſequently, the annual burials at 23,100. The annual average of weddings, given in p. 275, is 4400; and, therefore, the number of perſons who marry annually muſt be 8800. Deduct a 6th part (*a*) for *widows* and *widowers*, and 7134 will be the number of *virgins* and *batchelors* marrying annually.—The difference between the chriſtenings and burials is 4000; which, therefore, is the number of annual recruits from the country. Theſe, in general, muſt be perſons in mature life. Suppoſe 3000 of them to marry after ſettling at *Paris*. Then, 7134 leſſened by 3000, or 4134 will be the number of perſons born at *Paris* who grow up to marry; and 14,966, or near *four-fifths* of all who are born at *Paris*, will be the number dying annually in childhood and celibacy. Nor is this at all improbable, for

(*a*) Vid. Note, p. 263.

it

it appears from the moſt authentic documents that *three-fifths* of all that are born at *Stockholm* die under five years of age. It has been obſerved in p. 264, &c. that in country pariſhes above half the inhabitants live to marry.

The ſuppoſitions on which I have made this computation for *Paris*, ſeem moderate; but if any one thinks otherwiſe, he may make the ſame calculation on any other ſuppoſitions.

The births at *Paris* are above four times the weddings; and it may ſeem, therefore, that here, as well as in the moſt healthy country ſituations, every wedding produces above four children. I have obſerved nothing like this in any other great town. Many children born in the country are, I ſuppoſe (*a*), brought to the Foundling-Hoſpital, and there chriſtened. This Hoſpital may likewiſe occaſion a more than common number of illegitimate births. And, beſides, ſome who leave the country to ſettle at *Paris*, may come thither already married. Theſe are circumſtances that will ſwell the regiſter of births, without having any effect on the weddings. I do not, however, know that

(*a*) "If the parents of a child brought to this Hoſpi-"tal are known, the regiſter of its baptiſm muſt be "produced. If the parents are unknown, the child "muſt be baptiſed after being received." *Police of France*, page 82.

any of them take place at *Paris*; and, perhaps, it muſt be granted, that it is diſtinguiſhed in this reſpect from moſt other towns. Nor can I wonder at this, if it be indeed true, not only, that all married men in *France* are excuſed ſerving in the militia from whence draughts are made for the army, but alſo, that a *fifth* of all the children born at *Paris* are ſent to the *Foundling-Hoſpital* (*a*). Theſe are encouragements to mar-

(*a*) See the Police of *France*, p. 83.—This writer adds, that a *third* of all that die at *Paris* die in Hoſpitals. "In the *Hôtel Dieu* (a great Hoſpital, ſituated in "the middle of the city) we may, he ſays, behold a "horrid ſcene of miſery; for, the beds being too few "for the number admitted, it is common to ſee 4, or "6, or even 8 in a bed together, lying 4 at one end and "4 at the other, ill of various diſtempers in ſeveral de-"grees; ſome bad, others worſe; ſome dying, others "dead.—Above a *fifth* of all admitted to this Hoſpital "die; the annual numbers admitted being 21,823. The "medium of deaths for three years from 1751 to 1753, "4650.—The medium of deaths for the ſame years in "*all* the Hoſpitals was 6181." *Ib.* p. 85.—In our two great city Hoſpitals, St. *Thomas*'s and St. *Bartholomew*'s, about 600 die annually; or one in 13 of all admitted as in-patients.——An account of the *Hotel Dieu* in *Paris*, much the ſame with that now given, may be found in the *Memoirs of the Year Two Thouſand Five Hundred* lately publiſhed, and tranſlated from the *French* by *W. Hooper*, M. D. "A citizen or ſtranger (this writer "ſays) who falls ſick, and is ſent thither, is impriſoned "in a noiſome bed, between a corpſe and a perſon ex-"piring in agonies, to breathe the noxious vapours from "the dead and the dying, and convert a ſimple indiſpo-"ſition into a cruel diſeaſe.—Six thouſand wretches are "crowded

marriage that no other city enjoys. It has been ſeen that the *Foundling-Hoſpital*, though attended with this effect, is, probably, in the higheſt degree pernicious.

At the end of the 2d vol. of Monſieur *De Buffon's Natural Hiſtory*, there are Tables formed from the Obſervations of M. *Du Pre de S. Maur*, of the *French Academy*, containing an account of the ages at which 13,189 perſons died in three pariſhes at *Paris*; and alſo, of the ages at which 10,805 perſons died in 12 country pariſhes and villages near *Paris*.—According to theſe Tables, many *more* die in the beginning of life, and much *fewer* in the latter part of life, in the country than in *Paris*. But the circumſtances of *Paris*, and the country round it, are ſuch, that no argument can be drawn from hence in favour of *Paris*. Many of the children dying in the country, are children ſent thither from *Paris* to be nurſed; and, on the other hand, *many*,

" crowded together into this Hoſpital, where the air has " no free circulation; and the arm of the river which " flows by, receives all its filth, and is drank, abound- " ing with the ſeeds of corruption, by half the city." The *London* Hoſpitals, it appears, have greatly the advantage; but indeed, with reſpect to hoſpitals in general, *as now conſtructed and regulated*, I cannot help fearing that they cauſe more diſtempers than they cure, and deſtroy more lives than they ſave. See *Thoughts on Hoſpitals*, by Mr. *Aikin*, phyſician, at YARMOUTH, together with a Letter to the Author, by Dr. *Percival*.

perhaps *moſt*, of thoſe who die in old age at *Paris*, are perſons who have removed thither from the country, ſome to *Hoſpitals*, and ſome to places and ſettlements. It is evident, therefore, that theſe Tables give a repreſentation of the probabilities of life at *Paris*, which, when compared with thoſe in the adjacent country (*a*), is juſt the reverſe of the truth. Were the children born at *Paris*, who die in the country, to be tranſferred to the town regiſter; and, on the contrary, the adults born in the country, who die at *Paris*, to be transferred to the country regiſter, there is no reaſon to doubt, but that the probabilities of life at *Paris*, would be found as low, in compariſon with thoſe in the country, as the probabilities of life in *London* are; or, perhaps, much lower.—This obſervation is applicable, in ſome degree, to moſt other great towns; and, in general, on account of the migrations from the country to towns, navies and armies, we may be ſatisfied, that we err on the ſide of *defect*, whenever we judge of the probabilities of life in the *country*, from the numbers dying in the ſeveral ſtages of life; and, on the ſide of *exceſs*, whenever,

(*a*) It is for this reaſon that theſe Tables, when combined, exhibit juſtly the *mean* probabilities of life for town and country taken together; and that the Table of the *decrements* of life deduced from them by *M. Buffon* and Mr. *Du Pre*, agrees nearly with Dr. *Halley*'s Table.

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in the ſame way, we judge of the probabilities of life in *towns*. And this, it is obvious, has a tendency to confirm all that has been ſaid in the preceding Eſſay, concerning the pernicious effects of great towns on human life.

There are ſeveral *ordonnances* and *arrets* of council which fix the boundaries of *Paris*, and prohibit all new buildings beyond thoſe boundaries.—The reaſons of this regulation, as ſet forth in one of theſe *arrets*, are remarkable; and it will not be improper to recite them.—"By the exceſſive aggrandizing of the city, it is ſaid, the air would be rendered unwholeſome, and the cleaning the ſtreets more difficult."—"Augmenting the number of inhabitants would augment the price of proviſions, labour, and manufactures."—"That ground would be covered with buildings which ought to be cultivated in raiſing the neceſſary ſubſiſtence for the inhabitants; and thereby hazard a ſcarcity."—"The people in the neighbouring towns and villages would be tempted to come and fix their reſidence in the capital, and deſert the country."—"And, laſtly; the difficulty of governing ſo great a number of people, would occaſion a diſorder in the *Police*, and give an opportunity to rogues to commit robberies and murders (*a*)."

(*a*) Vid. Police of *France*, p. 130.

No one can think overgrown cities greater evils than I do. But, yet, I can by no means approve of this policy. The effect of it muſt be, crowding together too many people within the preſcribed boundaries, and rendering a town more the ſeat of uncleanlineſs, infection and diſeaſe.—The number of houſes in *Paris* is reckoned about 28,000 (*a*), but the number of inhabitants (ſuppoſing a 20th part to die annually, and the true number of burials to be 23,000) muſt be 460,000; or about 16 times the number of houſes.

It is happy for LONDON, that there have been no laws to reſtrain its increaſe. In conſequence of being allowed to extend itſelf on all ſides into the country, the inhabitants now take near twice the room to live upon that they did; and it muſt be rendered leſs the means of ſhortening human life.

(*a*) Vid. Police of *France*, p. 130.

I find, in a Book entitled, *Recherches ſur la Population des Generalites d'Auvergne, de Lyon, de Rouen*, &c. by M. MESSANCE, and printed at *Paris* in 1766, the number of houſes at *Paris* is given 23,565, from a capitation tax in 1755; and the number of families 71,114. There muſt, I ſuppoſe, be ſome deficiencies in this account; but M. *Meſſance*, by allowing moſt extravagantly (See the Table at the end of this Poſtſcript) 8 to a family, infers from it that the number of inhabitants at *Paris* is 568,912.—On very unſatisfactory grounds alſo he makes the inhabitants of FRANCE to be near 24 millions. *Suſmilch* calls them 16 millions. But the returns mentioned in the note, p. 271, determine them to be a much larger number, and leave little room for controverſy on this ſubject.

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In page 276, I have given the annual *medium* of births, weddings and burials at BERLIN, from 1755 to 1759.—In 1747, an account was taken with the utmoſt care, by the order of the King of PRUSSIA, of the number of inhabitants in this town; and, it was found to be 107,224.—In order to be more certain, a *ſecond* account was taken the ſame year; and the number found the ſame within 200.—In 1755, the inhabitants were increaſed to 126,661. Their number, therefore, in 1758, could ſcarcely be leſs than 134,000; and muſt have been to the annual burials nearly as 26½ to 1.—This proportion is higher than could be expected in a town ſo conſiderable; and alſo ſo much crowded, as to have, at an average, 16 inhabitants in every houſe. But an obſervation already made, muſt be here remembered. —BERLIN, for many years, had been increaſing very faſt, by a conflux of people from the ſurrounding country and provinces. About the year 1700, the medium of annual burials was no more than 1000. In 50 years, therefore, it has more than quadrupled itſelf.—In a city increaſing with ſuch rapidity, the *ratio* of inhabitants to the annual deaths, muſt be greatly above the juſt ſtandard.—Were there now, ſuch acceſſions to LONDON of deſerters from the country, in the beginning of mature life, as would cauſe the number of inhabitants to increaſe at the rate

of 10,000 every year, it would in 50 years be doubled; and the proportion of inhabitants to deaths would riſe gradually, 'till it came to be about one-third greater. BERLIN, we have ſeen, has, in fact, increaſed at *double* this rate; and, therefore, the number of inhabitants dying annually in it is in reality very high.

The ingenious *Suſmilch*, to whoſe works I owe my information concerning BERLIN, makes the proportion of people who die annually in *great* towns, to be from $\frac{1}{24}$ to $\frac{1}{28}$; in *moderate* towns, from $\frac{1}{28}$ to $\frac{1}{31}$; and in the country from $\frac{1}{40}$ to $\frac{1}{50}$.—The obſervations and facts in this Eſſay, joined to thoſe which will be found in the 4th Eſſay, and the *Supplement* in the next volume, prove, I think, that theſe proportions may be more truly ſtated as follows.—*Great* towns, from $\frac{1}{19}$ or $\frac{1}{20}$ to $\frac{1}{23}$ or $\frac{1}{24}$. *Moderate* towns, from $\frac{1}{23}$ to $\frac{1}{28}$. The *country*, from $\frac{1}{35}$ or $\frac{1}{40}$, to $\frac{1}{35}$ or $\frac{1}{60}$.—This, however, muſt be underſtood with exceptions. There may be *moderate* towns ſo ill ſituated, or whoſe inhabitants may be ſo crowded together, as to render the proportion of deaths in them greater than in the largeſt towns: And, of this, EDINBURGH, if it is not now, was 30 years ago an example.—There may be alſo *great* towns in which, from a ſudden increaſe, this proportion may be leſs than in ſmall towns: And of this I have juſt given an example in BERLIN.—On the con-

contrary; there may be *moderate* towns ſo advantageouſly circumſtanced as to be equally healthy with many *country pariſhes*; and of this, *Cheſter* ſeems to be a very ſingular inſtance. See the Introduction to the Tables in the next volume.—And there are ſome country pariſhes ſo ill ſituated as to be no leſs unhealthy than great towns; of which a marſhy pariſh in *Switzerland*, deſcribed in a letter to Dr. *Horſley* at the end of the next volume, is an inſtance.

In Nottingham, according to a ſurvey in Sept. 1779, excluſive of 294 in hoſpitals and workhouſes	Families,	3,267 / 3,556	Inhabitants,	17,417	To a houſe, / To a family,	5⅓. / 4⅘.
Norwich, according to a ſurvey in 1752	Families	7,139		36,169	To a houſe,	5.
Shrewſbury, by a ſurvey in 1750		3,078		13,328		4⅓.
Northampton, by a ſurvey in 1746		1,083		5,136		4¾.
The pariſh of Ackworth, Yorkſhire, in 1767		184		728		4.
Newbury, Berkſhire, in 1768		930		3,732		4.
Speen, adjoining to Newbury, in 1768		303		1,200		4.
Aldwinckle, Northamptonſhire, in 1772		96		402		4⅕.
The pariſh of Holy Croſs, near Shrewſbury, in 1760		242		1,050		4⅓.
Altringham, Cheſhire, in 1772		248		1,029		4⅛.
The Pariſh of St. Michael's, Cheſter, in 1772		127		618		4⅘.
The town and pariſh of Bala, North-Wales, in 1774		401		1,723		4 3/5.
Fifty-nine Dutch villages mentioned by Struyk		12,005		45,888		3⅘.
Birmingham, in 1770		6,025		30,804		5⅛.
Liverpool*, in 1773, including 400 in the Poor-houſe, but excluſive of about 4000 abſent at ſea		6,340		34,407		5⅖.

* Houſes in Liverpool in 1753 — 3700
Annual average of *deaths* for five years to 1771 — 1391
of *births* — 1398
See an *Eſſay towards the Hiſtory of Liverpool*, by Dr. Enfield, p. 23—34.

In Biddulph, in Staffordſhire, according to an accurate enumeration in April, 1779, by the Rev. Mr. Wilſon	— Families	1,035	—Inhabitants,	207 — To a houſe,	5.
Swinderby, in Lincolnſhire, according to an enumeration by the Rev. Mr. Diſney — — — —	— Families	52	——	224 ——	4 1/3.
Holbee, Hunſlett, and ſix other villages near *Leeds*. See Mr. *Wales*'s Inquiry, p. 41 —— ——	— Families	2,631	——	11,468 ——	4 1/3.
Mancheſter and Salford, in 1773 — —	— Houſes	4,338	——	27,246 ——	6 1/4.
Leeds, in 1775 —— —— —	— Families	4,096	——	17,121 — To a family	4 1/3.
The Diſtrict of Vaud in Switzerland	Families	25,778	——	112,951 ——	4 1/2.
Cheſter in 1774 —— ——	— Families	3,428	——	14,713 ——	4 7/10.
Rome, in 1770 —— ——	— Families	37,449	——	158,442 ——	4 1/4.
Calne, Wiltſhire —— ——	— Families	776	——	3,467 ——	4 1/2.
Liverpool —— —— ——	— Families	8,002	——	34,407 ——	4 3/10.
Mancheſter —— —— ——	— Families	6,416	——	27,246 ——	4 2/3.
Bolton in Lancaſhire, in 1773, including Little Bolton —— —	— Houſes	1,178	——	5,339 — To a houſe,	4 1/2.
Bury in Lancaſhire, in 1772 ——	— Houſes	463	——	2,090 ——	4 1/2.
The pariſh of Bala in North-Wales, in 1774 —— —— —	— Houſes	401	——	1,723 ——	4 3/10.
Chippenham, Wilts, in 1773 ——	— Houſes	483	——	2,407 ——	5.
Brenhill, near Calne, in Wiltſhire	— Houſes	218	——	1,206 ——	5 1/2.
The Iſland of Sicily (ſee end of 2d vol. of Brydone's Travels) ——	— Houſes	268,120	——	1,123,163 ——	4 1/3.

In Fourteen market towns mentioned by Dr. Short, Comparative Hiſtory, page 58	— Families	20,371	— Inhabitants,	97,611	— To a family $4\frac{3}{4}$.
Sixty-five country pariſhes, ibid.	— Families	17,208		76,284	$4\frac{2}{3}$.
The Pariſh of *Skelton*, Yorkſhire, in 1777	— Houſes	139		506	— To a houſe, $4\frac{1}{3}$.
The town and pariſh of Wycombe, Bucks	— Families	500		2,461	— To a family $4\frac{9}{10}$.
Worſley, Barton, Pendleton, Pendlebury, and Clifton, Lancaſhire, in 1778	— Families	1,685		9,117	$5\frac{2}{3}$.
Pariſh of St. Cuthbert, Edinburgh, in 1743 (ſee Maitland's Hiſtory of Edinburgh, page 171)	— Families	2,370		9,731	$4\frac{1}{10}$.
In a number of ſmall towns and pariſhes in the Generalities of Auvergne, Lyon, and Rouen, in France (ſee *Recherches ſur la Population, par* M. Meſſance, pages 8, 26, and 62)	— Families	24,931		99,332	$4\frac{1}{10}$.
Pariſh of Mancheſter, excluſive of the town, in 1774	— Families	2,525		13,786	$5\frac{1}{2}$.
	— Houſes	2,412		13,786	— To a houſe, $5\frac{7}{10}$.
Pariſh in the city of London (ſee Phil. Tranſ. vol. 48, part 2d, page 796)					— 6 to a houſe.

In Warrington and its vicinity, by a survey in April 1781	— Houſes	2,145	—Inhabitants,	9,770	— To a houſe, $4\frac{4}{5}$.
Maidſtone, by a ſurvey in 1781 —	Houſes	1,106		5,650	$5\frac{1}{10}$.
	Families	1,276		5,650	— To a family $4\frac{2}{5}$.
Town and pariſh of Aſhton under-Line, near Mancheſter, by a ſurvey in 1775. See Phil. Tranſ. vol. 66, p. 164	— Houſes	1,494		7,956	— To a houſe, $5\frac{1}{3}$.
	— Families	1,570		7,956	— To a family $5\frac{1}{17}$.
Tattenhall and Waverton, two pariſhes near Cheſter, by a ſurvey in 1774.—Ib. p. 165	— Houſes	261		1,413	— To a houſe, $5\frac{2}{5}$.
	— Families	292		1,413	— To a family $4\frac{4}{5}$.
Swindon, Wilts, by a ſurvey in 1781	— Houſes	190		1,064	— To a houſe, $5\frac{3}{5}$.
Pariſh of St. Michael's, in Cheſter, by a ſurvey in 1772	— Families	151		618	— To a family $4\frac{1}{10}$.
	— Houſes	127			— To a houſe, $4\frac{5}{8}$.
Bridgend, in Glamorganſhire, in March 1783	— Houſes	205		824	4.

ESSAY II.

On Mr. De Moivre's *Rules for calculating the Values of* Joint *Lives; with a* Poſtſcript, *containing a Specimen of the moſt expeditious Method of calculating the Values of Single and Joint Lives, according to any Table of Obſervations.*

THE calculation of the values of *ſingle* and *joint* lives, from given Tables of Obſervation, being tedious and troubleſome; Mr. *De Moivre* has had recourſe to two *hypotheſes*, which give eaſy rules for this purpoſe; and which, he thought, correſponded with ſufficient exactneſs to Obſervations.—The firſt of theſe *hypotheſes* is, that the probabilities of life decreaſe, as we advance from childhood to old age, in an *arithmetical progreſſion*; or in ſuch a manner, that the *difference* is always the ſame, between the number of perſons living at the beginning of any one year, and the number living at the beginning of the next following year.—The other *hypotheſis* is, that the probabilities of life

life decreaſe in a *geometrical* progreſſion; or, in ſuch manner, that the *proportion* is always the ſame, between the number of perſons living at the beginning of any one year, and the number living at the beginning of the next following year.—All the Tables of Obſervation ſhew, that the real law, according to which human life waſtes, comes much nearer to the former *hypotheſis*, than the latter.—In Tables V, VI, and VII, in the next volume, it is ſo near the former *hypotheſis*, that the difference between them in the middle ſtages of life is ſcarcely worth regarding. According to this *hypotheſis*, therefore (accommodated to the *Breſlaw* Table, in the manner mentioned in the note, page 2.) Mr. *De Moivre* calculated the values of *ſingle lives*; and the rules founded upon it for this purpoſe are ſo eaſy, that an operation which would otherwiſe take up much time, may be performed almoſt immediately.

By proceeding on the ſame principles, the values of *joint* lives might have been calculated; but the rules for this purpoſe derived from theſe principles, are far from being equally eaſy in practice. Here, therefore, Mr. *De Moivre* quitted his *firſt* hypotheſis; and finding, that the *ſecond* hypotheſis afforded, in the caſe of *joint* lives, rules that were as eaſy as the rules given by the other hypotheſis were in the caſe of *ſingle* lives, he choſe to adopt this *hypotheſis*; believing

at

at the ſame time, that the values of *joint* lives, obtained by rules derived from it, would not deviate much from the truth. But in this he was greatly miſtaken. The values of *two joint* lives obtained by theſe rules are ſo wrong, that in finding the preſent value, in a *ſingle payment*, of one life after another, they generally give reſults which are near a *quarter* of the true value too great; and about *two-fifths* too great, when the value is ſought in *annual payments* during the joint lives. Theſe are errors ſo conſiderable, that I think it of particular importance that the public ſhould be informed of them, in order to prevent the inconveniencies and perplexities they may occaſion.

Mr. *Simpſon* (in the Appendix to his Treatiſe on the *Doctrine of Annuities and Reverſions*) has obſerved, that Mr. *De Movre*'s rules for finding the values of joint lives are wrong. But I don't know, that it has been ever attended to, that they are *ſo* wrong as I have found them. Mr. *Simpſon*'s remarks point out chiefly the errors in theſe rules, when the values of *three* or more joint lives are calculated by them; but, 'till I was forced to a particular examination of this ſubject by ſome difficulties into which I found myſelf brought by following Mr. *De Moivre* too implicitly, I did not at all ſuſpect, that any ſuch errors as I have mentioned, could ariſe from theſe rules, when

the values of only *two* joint lives are calculated by them. Mr. *De Moivre*, in conſequence of other remarks contained in Mr. *Simpſon's Appendix*, altered, in the 4th edition of his Treatiſe, ſome of his rules. It is ſurprizing he did not ſee reaſon at the ſame time to alter theſe.

That there may be no doubt about the truth of theſe obſervations, I will juſt mention a few examples of the difference between the values of a given reverſionary annuity, according to the rules to which I have objected, and the values, according to the exact method of deducing them from Mr. *De Moivre's firſt hypotheſis*.

Let the propoſed annuity be 30*l.*, to be enjoyed for what ſhall happen to remain of the life of a perſon now 40 years of age, after the life of another perſon of the ſame age. The value of the joint lives (intereſt being at 4 *per cent.*) is, by the 2d hypotheſis, or problem 2d of Mr. *De Moivre*'s Treatiſe on Life-Annuities, 8.964; which ſubſtracted from 13.196, (the value, by the firſt hypotheſis, of a ſingle life at 40) gives 4.23; which remainder, multiplied by 30, gives *l.* 126.9, or the value of the reverſion in a ſingle preſent payment. And 126.9, divided by the foregoing value of the joint lives, is *l.* 14.16; or, the value of the reverſion in annual payments, during the joint lives.—But the *true* values are *l.* 101.1

in a *ſingle* payment, by Queſt. I. chap. I:; and *l.* 10.3, in *annual* payments, by Queſt. IV.—The former values, therefore, are a *quarter* of the true value too great in the *ſingle* payment; and near *two-fifths* too great in the *annual* payments.

The *true* value of the ſame annuity for a life at 66, after another life of the ſame age, is, (reckoning intereſt as before, at 4 *per cent.*) 68*l.* in a *ſingle* payment; and *l.* 13.5 in *annual* payments.—But theſe values, according to the Problem juſt quoted, are 91 *l.* and 21 *l.* one of which is near a *third*, and the other above *half* the true value too great.

In *unequal* lives theſe errors may be no leſs conſiderable.—Thus; if the value of the propoſed annuity be required for a life at 70, after a life at 30 years of age; it will, by the ſame Problem, be *l.* 26.5, in a *ſingle* payment; and *l.* 5.1, in *annual* payments during the joint lives. But the *true* values are 17 *l.* and *l.* 3.05.

Where 3 or more lives are concerned the errors will be ſtill greater.

The true values of the joint lives, mentioned in theſe Examples, have been calculated by a rule in page 16, of Mr. *Simpſon's* Treatiſe on the Doctrine of *Annuities* and *Reverſions*, and explained in note (L) at the end of the next volume.—To ſave, however, a great deal of trouble hereafter, I have thought proper

proper to calculate Table II. at the end of the next volume, which gives the exact values according to Mr. *De Moivre's first* hypotheſis, of two joint lives, for every five years of human life, from 10 to 70.

This *hypotheſis*, I have obſerved, does not differ much from the Tables of Obſervation for *Breſlaw*, *Northampton* and *Norwich*. Between the ages of 30 and 40, it gives the values of *ſingle* lives almoſt the ſame with the *Breſlaw* Table. Under 30, it gives them ſomewhat *leſs*; and above 40, ſomewhat *greater*. But it ought to be remembered, that wherever it does this, it gives, at the ſame ages, the values of the *joint* lives alſo too little or too great; and that, conſequently, the reſults from it, in calculating the values of *Reverſions*, and of the *longeſt* of given lives, come ſo much nearer to exactneſs.

The rules to which I have objected are the only ones given by Mr. *De Moivre*, in all the editions of his Treatiſe on Life-Annuities. But it ſeems, this great mathematician became at laſt ſenſible, that they were too incorrect; and therefore, at the end of the laſt edition of his Treatiſe on the *Doctrine of Chances*, page 320, (a work which gets into comparatively few hands) he had given other rules which come nearer the truth. But even theſe rules produce errors ſo great in

many caſes, (particularly when combined with the errors of the hypotheſis) that it will be beſt never to uſe them.

Poſtſcript *for the* Fourth Edition.

SINCE the former editions of this work I have found reaſon to be diſſatisfied with Mr. *De Moivre's firſt* as well as his *ſecond* hypotheſis. There is no ſituation in which, in the firſt and laſt periods of life, it correſponds to fact; and in ſome ſituations, particularly great towns and country pariſhes, it does not correſpond ſufficiently to fact in *any* periods of life. An inſpection of the Tables of Obſervation in the next volume will prove this. However uſeful, therefore, this hypotheſis may be in many caſes, it would be beſt not to be under any neceſſity of having recourſe to it; and for this reaſon, and alſo to render this work as complete as I am capable of making it, I have, while this edition has been in the preſs, and with the help of ſome friends, calculated the tables in the next volume of the values of ſingle and joint (*a*) lives from the *Northampton* regiſter of mortality. This

(*a*) The value of the reverſionary annuity, mentioned in p. 306, is by theſe Tables 14.83 in annual payments, inſtead of 13.5, as there given from the hypotheſis.

regiſter

regiſter has been choſen for this purpoſe, becauſe it gives the mean values of lives between the higheſt and loweſt, and is on this account, and alſo in conſequence of the corrections I have made in it, better fitted for general uſe than any other.——I have, however, retained the Tables of theſe values according to Mr. *De Moivre*'s Hypotheſis, publiſhed in the former editions of this work, becauſe all the examples in the preceding part of this work have been taken from them, and there are ſome caſes in which they may ſtill prove of uſe.

The computation of the values of joint lives correctly from a given table of Obſervations, is a buſineſs ſo tedious and tireſome, that it has ſcarcely been ever executed, except by Mr. *Simpſon* from the *London* Obſervations; and as theſe give the values of lives among a body of people taken in the groſs in one of the worſt of all ſituations, they are by no means fit for common uſe.——I have, therefore, employed a good deal of attention to find out the moſt eaſy and expeditious method of making theſe calculations; and I ſhall here give the following *Specimen* of a method (deduced from that deſcribed by Mr. *Morgan* in his Treatiſe on Life-Annuities and Aſſurances, chap 2d, ſect. 2d, p. 56) which, at the ſame time that it renders miſtakes impoſſible, will ex-

pedite this work as much as the nature of it will allow, and render the computation of the values of any number of *joint* lives not more difficult or tedious than the computation of the values of an equal number of *ſingle* lives.

Let the Table of Obſervations be that for *Northampton*, or Table 6th, in the next volume; and let the rate of intereſt be 4 *per cent.*

Write down on a paper to be always kept in ſight the Logarithms of all the numbers in the column of the living without the Indices.

EXAMPLE FIRST.

Living at age 0 —	11650 —	Log^{ms}.	.066325
1 year	8650 —		.937016
&c.	&c.		&c.
age 81 —	406 —		.608526
82 —	346 —		.539076
83 —	289 —		.460897
84 —	234 —		.369215
&c.	&c.		&c.
age 91 —	34 —		.531478
92 —	24 —		.380211
93 —	16 —		.204119
94 —	9 —		.954242
95 —	4 —		.602059
96 —	1 —		.000000

Find

Find the Logarithm of 1 *l.* increaſed by its intereſt for a year, and alſo the Logarithm of the value of 1 *l.* payable at the end of a number of years equal to the difference between the greateſt and leaſt ages in the Table of Obſervations leſſened by the difference of age between the joint lives whoſe values are to be calculated.

EXAMPLE SECOND.

Intereſt being at 4 *per cent.* 1 *l.* increaſed by its intereſt for a year is 1.04; and the Logarithm of 1.04 is .0170333.

In the *Northampton* Table of Obſervations the greateſt age is 96, and the leaſt age 0. The difference, therefore, is 96; and ſuppoſing the given difference of age between the two joint lives to be 10 years, the value of 1 *l.* payable at the end of a number of years equal to the difference between the greateſt and leaſt ages in the table leſſened by the difference of age between the joint lives, will be the value of 1 *l.* payable at the end of 86 years. Table 1ſt, in the next volume, ſhews this value to be .0342872 (reckoning intereſt at 4 *per cent.*) the Logarithm of which number (ſtriking out the Index) is .535133.

N. B. The beſt way of finding this Logarithm is by multiplying the Logarithm of 1 *l.* with its intereſt for a year by the dif-

ference between the greateſt and leaſt ages in the Table leſſened by the difference of age between the joint lives, and ſubtracting the product from unity. The remainder will be the Logarithm ſought. Thus, in the preſent example .0170333 multiplied by 86, gives (without the Index) .464864, which ſubtracted from unity leaves .535136.—— Had the given difference of age between the two joint lives been 15 years, and the youngeſt age in the Table of Obſervations 3, and the oldeſt 94, the Logarithm .0170333, inſtead of being multiplied by 86, muſt have been multiplied by 76, and the product (without the Index) ſubtracted from unity would have been .705469.

Having made theſe preparations, the calculations muſt begin with the oldeſt joint lives, and proceed upwards according to the following ſpecimen.

SPECIMEN

SPECIMEN of an eafy and expeditious Method of calculating the Values of two Joint Lives.

Intereft at 4 *per Cent.*——Difference of Age 10 Years.——*Northampton* Table of Obfervations.

Values. — ——	Ages $\frac{95}{85}$	.18739	Ages $\frac{94}{84}$	.403347	Ages $\frac{93}{83}$	.61495
Ages $\frac{92}{82}$	.86446	Ages $\frac{91}{81}$	1.07846			

.017033 (*a*) added continually to .535136 See *fecond* Example — — — —	A	.535136	.552169	.569202	.586235	.603268
Log^{ms}. of the numbers living at 96, 95, 94, &c. years. See *firft* Example —	B	.000000	.602059	.954242	.204119	.380211
Log^{ms}. of the numbers living at 86, 85, 84, &c. years. See *firft* Example —	C	.161368	.269512	.369215	.460897	.539076
B + C —— ——	D	.161368	.871572	.323457	.665016	.919287
Log^{ms}. of the numbers (with unity added) of 1ft, 2d, 3d, &c. Log^{ms}. in H —	E	- - - - -	.074597	.147162	.208051	.270553
D + E —— ——	F	.161368	.946168	.470619	.873067	.189840
.017033 (*a*) added to 2d, 3d, 4th, &c. Logarithms in D ——	G	.838604	.340490	.682049	.936320	.157037
F — G —— ——	H	.272764	.605678	.788570	.936747	.032803
A + G —— —— ——	I	.423740	.892659	.251251	.522555	.760305
H + I —— —— ——	K	.696504	.498337	.039821	.459302	.793108
Numbers of the Log^{ms}. in I —	L	.265303	.781016	.178341	.333087	.575846
Numbers of the Log^{ms}. in K ——	M	49717	.315019	.109603	.287941	.621025
Proof or L + M —— ——		315020	1096035	.287944	.621028	

(*a*) This muft be .0128372—0149403—.017033—.021189—.0253059, as the rates of intereft are 3, 3½, 4, 5, or 6 *per cent.*; and the firft Logarithm in A for thefe feveral rates of intereft, on which all that follow in that line depend, is to be deduced from them by the rule in the *fecond* Example.

OBSERVATIONS.

IN the addition of the Logarithms in this Specimen, the decimal parts only are to be retained.

In ſubtracting them, it is of no conſequence whether a Logarithm is greater or leſs than that from which it is to be ſubtracted.

In every column, the numbers in the lines B, C, D, E, F, G, H, give the *value*. The other numbers give the *proof*.

The *firſt* Logarithms in the lines B and C are always the Logarithms of the numbers of the living at the oldeſt ages in the Table of Obſervations, which have the given difference of age; and the following Logarithms are the Logarithms of the numbers living at the next ages, each one year younger than the preceding.

The values of the two joint lives are the numbers of the Logarithms in H; and the proof of theſe values conſiſts in the equality of the ſum of the numbers in L and M in one column, to the number in M in the following column. And it ſhould be particularly obſerved that this proof anſwers ſufficiently if, in conſequence of placing the numbers in L and M over one another in *any* order, a ſum can be made out whoſe firſt ſix figures are equal within 5 or 6 units to the ſucceeding number

ber in M. If the proof does not anſwer within this limit, the calculations have been too incorrect (*a*), and it will be neceſſary to examine the numbers not verified by the laſt proof; namely, the numbers of the two Logarithms in I and H in the *preceding* column; the Logarithms in B and C and D in the *ſubſequent* column; and the Logarithms in A, E, F, G, H, I and K in the column where the proof is found to be deficient.

EXAMPLE.

The addition, in the foregoing ſpecimen, of the firſt numbers in L and M to one another, when the *firſt* figure in M is placed under the *ſecond* in L, makes 315020, which is within one unit the ſame with 315019, the ſecond number in the line M; and this proves the calculations ſo far to be ſufficiently correct.——In like manner; the addition of the *third* numbers in L and M gives the *fourth* in M within three units. But had the addition of the numbers in L and M given the ſubſequent number in M only within *ſix* units; that is, had it given the number 287947 or 287935, an incorrectneſs of too much conſequence muſt have inſinuated it-

(*a*) This ſuppoſes that the values of the joint lives are to be found to three places of decimals. The agreement of *five* figures within 4 or 5 units will be ſufficient, if the values are required only to two places of decimals.

ſelf,

ſelf, and it would have been proper to examine the numbers and Logarithms juſt mentioned, in order to detect it.

In calculating the *laſt* value; that is, the value when the youngeſt of the lives is the youngeſt in the Table (or a life juſt born, according to moſt Tables) this proof will change into a new proof, verifying all the preceding values. For the Logarithm of 1*l.* with its intereſt for a year (that is, .017033 when the intereſt is 4 *per cent.*) ſubtracted from the Logarithm in F, will leave a Logarithm, the number of which will be the ſum of the numbers in L and M in the preceding column.

This Specimen will be accommodated to the calculation of the values of *ſingle* lives (*a*), by ſtriking out the Logarithms in C, and making thoſe in D the ſame with thoſe in B; and alſo making the firſt Logarithm in A, from which all that follow are deduced, the Logarithm of 1*l.* payable at the end of a number of years, equal to the oldeſt age in the Table when it begins at birth, or to the difference between the oldeſt and youngeſt when it begins at any age after birth.

(*a*) In this caſe, and alſo in calculating the values of *equal* joint lives, the firſt Logarithm in F (when the number living at the oldeſt age in the Table is 1) will be nothing; but notwithſtanding this, the firſt Logarithm in G muſt be ſubtracted from it juſt as if it was unity, in order to obtain the firſt Logarithm in H.

This

This Specimen will be accommodated to the calculation of the values of any *three* joint lives, if to the Logarithms in B and C are added (in order to obtain D) the Logarithms of the numbers living at any other ages, the difference between which and the ages in B and C is given; and if, likewiſe, the ſeries of Logarithms in A is deduced from the Logarithm of the value of *1l.* payable at the end of a number of years equal (if the ages begin at 0) to the oldeſt in the Table; or (if they do not begin at 0) to the differeuce of age between the oldeſt and youngeſt in the Table; leſſened (in both caſes) by the differences of age between the oldeſt and youngeſt of the three joint lives whoſe values are ſought.——Thus. Supposing the given differences between the ages of the joint lives whoſe values are to be calculated to be 5 and 10 years, and the intereſt 4 *per cent.* and the Table of Obſervations to terminate (as the *Northampton* Table does) at 96 years of age, and to begin at 0. The ſeries of Logarithms to be added to thoſe in B and C in order to obtain D, will be the Logarithms of the living at 91 in the firſt column, at 90 in the ſecond, at 89 in the third, &c. And the Logarithms in A will be the ſame with thoſe in the Specimen. But had the differences of age been 10 and 15 years, the Logarithms to be added to thoſe in B and C would have been

been the Logarithms of the living at 81, 80, 79, &c. and the firſt Logarithm in A would have been the product of 81 into .017033 ſubtracted from unity or .620302, and the following Logarithms in A would have been .017033 added continually to this Logarithm.

It is hence evident that in this method computations of the values of any given *two* or *three* or *four* joint lives are nearly as eaſy as computations of the values of *ſingle* lives; and may, after ſome practice, be performed almoſt as expeditiouſly as the numbers can be written.

An error in a book of Logarithms may, if not ſuſpected, produce infinite perplexity; and therefore, when, after repeating any calculation, the ſource of an error cannot be diſcovered, it will be right to examine the Tables from whence the Logarithms have been taken. In general, the order in which the numbers follow one another will immediately diſcover an error of the preſs; but if not, a different book of Logarithms ſhould be conſulted; and if poſſible, Mr. GARDINER'S, which is ſo correct as to be almoſt invaluable.

It may be proper to obſerve once more, that it is very eaſy to take from SHERWIN'S or GARDINER'S Tables the numbers of Logarithms, and the Logarithms of numbers to *ſix* figures; and that if this is done, the re-

reſulting values will be always accurate to at leaſt the *third* place of decimals. But, if ſuch a degree of accuracy is thought needleſs, it will be ſufficient to take them to *five* figures.

The Theorems on which the method of calculation here explained (excluſive of the proof) are grounded, will be given in note N at the end of the next volume. But a more diſtinct inveſtigation of theſe Theorems, and alſo an explanation of the principles on which the *proof* is founded, has been given by Mr. *Morgan* in his Treatiſe on Annuities and Aſſurances, Chap. IId. Sect. 2d.

The rules for finding, from the values of two or three joint lives, the values of the *longeſt* of any *two* or *three* lives; and alſo a very eaſy rule for obtaining *nearly* the value of any *three* joint lives from the values of *two* joint lives, will be given in the next volume at the end of the Table, ſhewing the values of two joint lives according to the *Northampton* Obſervations.

ESSAY

ESSAY III.

Of the Method of calculating the Values of Reverſions depending on Survivorſhips.

ALL Queſtions relating to the values of lives and reverſions, are at preſent of particular importance in this kingdom. Much buſineſs is continually tranſacted in this way; and any conſiderable errors in the methods of ſolving ſuch queſtions, muſt in time produce very bad conſequences.—The deſign of the following obſervations is to point out a particular error, into which there is danger of falling, in finding the values of ſuch reverſions as depend on ſurvivorſhips. In doing this, I ſhall, in order to be as plain as poſſible, take the following caſe. "A, "aged 40, expects to come to the poſſeſſion "of an eſtate, ſhould he ſurvive B, aged "likewiſe 40. In theſe circumſtances he "offers, in order to raiſe a preſent ſum, to "give ſecurity for 40*l. per annum*, out of "the eſtate at his death, provided he ſhould

"get

" get into poſſeſſion; that is, provided he " ſhould ſurvive B. What is the ſum that " ought now to be advanced to him, in " conſideration of ſuch ſecurity, reckoning " compound intereſt at 4 *per cent.*?"

Mr. *De Moivre*'s directions in his Treatiſe on Annuities, Problems 17th and 20th, lead us to ſeek the required ſum in this caſe, by the following proceſs.

Find firſt, the preſent ſum A ſhould receive, for the reverſion of 40*l.* *per annum* for ever after his death; ſuppoſing it *not* dependent on his ſurviving B. The preſent value of ſuch a reverſion is " the (*a*) value of the " life ſubtracted from the *perpetuity*, and " the *remainder* multipled by the annual " rent."—The value of the life is, by Mr. *De Moivre*'s Hypotheſis, 13,196. This ſubtracted from 25, the *perpetuity*, leaves 11,80; which, multiplied by 40, gives *l.*472; the value of the ſuppoſed eſtate, after the life of A. But, as Mr. *De Moivre* obſerves, the lender having a chance to loſe his money, a compenſation ought to be made to him for the riſk he runs, which is founded on the poſſibility, that a man of 40 years of age may not ſurvive another perſon of the ſame age. This chance is an *equal* chance; and, therefore, half the preceding ſum, or

(*a*) By *Scholium*, p. 34, and Problem 26th, p. 293 of Mr. *Simpſon*'s Select Exerciſes.

236*l.* is the money which ſhould be advanced now on the expectation mentioned.

This ſolution carries a plauſible appearance; and moſt perſons will, probably, be ready to pronounce it right; nor will this be at all wonderful, as ſo great a maſter of theſe ſubjects as Mr. *De Moivre*, appears to have been miſled by it.—Nothing more is neceſſary to prove it to be fallacious, than proceeding in the ſame way to ſolve the following ſimilar Queſtion.

"A, aged 40, offers to give ſecurity for "40*l. per annum*, to be entered upon at his "death, provided it ſhould happen *before* the "death of B, aged likewiſe 40. What ſum "ſhould now be advanced to him for ſuch "a reverſion, intereſt being reckoned at 4 "*per cent.*?"

In ſolving this problem, agreeably to the method juſt deſcribed, we are to find the value of 40*l. per annum*, to be entered upon *certainly* at the death of A; and then to multiply this value by the chance that A ſhall *not* ſurvive B, or by $\frac{1}{2}$; and in this way the anſwer comes out the ſame with that already given.

Now it may be eaſily ſeen, that this muſt be wrong. The value of a reverſion, to be received when a perſon of a given age dies, cannot be the ſame, whether the condition of obtaining it is, that he ſhall die *before*, or that he ſhall die *after* another perſon. That

is,

is, whether it is provided, that a purchaſer, if he ſucceeds, ſhall get into poſſeſſion *ſooner* or *later*. The reverſion in the latter caſe muſt, without doubt, be of leſs value than in the former.

The firſt Queſtion here propoſed, reſolves itſelf into the following general Queſtion.

" What is the preſent value of a given re-
" verſionary eſtate, to be entered upon after
" the failure of two lives, provided one *in*
" *particular* of them ſhould be the *longeſt*
" *life?*"

Now, the preſent value of an eſtate to be enjoyed for ever, after the failure of the *longeſt* of two lives, is " the value of the *longeſt*
" of the two lives, ſubtracted from the *per-*
" *petuity*; and the remainder multiplied by
" the annual rent of the eſtate."—The value of the *longeſt* of two lives is (as is well known) the value of the two *joint* lives, ſubtracted from the ſum of the (*a*) values of the two *ſingle* lives. In the preſent caſe, therefore, it is 9.82, (the value of two joint lives at the age of 40 by Table IId at the end of the notes in the next volume) ſubtracted from twice 13.196; (the value of a *ſingle* life at the ſame age) that is, 16.57 year's purchaſe. And this ſubtracted

(*a*) See Mr. *De Moivre* on Annuities, Problem IV. or Mr. *Simpſon's Doctrine of Annuities and Reverſions*, Problem II.

from 25, (the perpetuity) gives 8.43; which, multiplied by 40, gives *l.* 337.2, the value of the given eſtate were it *certainly* to be enjoyed, after the extinction of the longeſt of two lives both 40; that is, whether *one* or *other* of them failed laſt. But that A's life in particular ſhould fail laſt, is an even chance. The true value of the reverſion, therefore, is half the laſt value, or *l.* 168.6.

In like manner. The ſecond Queſtion is the ſame with the Queſtion, "What is the "preſent value of 40*l. per ann.* for ever, to "be entered upon after the extinction of two "*joint* lives both 40; that is, whenever *ei-* "*ther* of them ſhall fail; provided the firſt "that fails ſhould happen to be A's life in "particular?"—And the anſwer is found by ſubtracting the preſent value of the *two joint* lives from the *perpetuity*, and multiplying the remainder by $\frac{1}{2}$, that is, by the chance that A in particular ſhall die firſt: And this will give the required value, *l.* 303.4 (*a*).

In ſhort. It appears in *both* theſe caſes, that, according to the firſt method of ſolution, we are to ſubtract from the *perpetuity* the value of *one* of the ſingle lives, when, in the *former* caſe, the value of the *longeſt* of the two lives, and, in the *latter* caſe, the value

(*a*) I have, tho' ſcarcely neceſſary, given a demonſtration of theſe Solutions in note M. at the end of the next volume.

of

of their *joint continuance*, ought, in reality, to be ſubtracted. I need not ſay what prodious errors may often ariſe from hence; and how unfit ſuch a method of ſolution is for practice.

Mr. *Simpſon*, in p. 322, of his Select Exerciſes, ſpeaks on this ſubject in the following manner.—" I have been very particular " on theſe kinds of Problems; and the more " ſo, as there has been no method before " publiſhed, that I know of, by which they " can be rightly determined. 'Tis true, the " manner of proceeding, by firſt finding the " probability of Survivorſhip (which me- " thod is uſed in my former work, and " which a celebrated author has largely in- " ſiſted on in three ſucceſſive editions), may " be applied to good advantage, when the " given ages are nearly equal; but then it is " certain, that this is not a genuine way of " going to work, and that the concluſions " hence derived are at beſt but near approxi- " mations."

This excellent mathematician has here expreſſed himſelf much too favourably of the method of ſolution on which I have remarked.—In both the caſes I have ſpecified, the ages are equal; and yet, in one of them the error is a good deal above a *third* of the true value, and in the other a *fifth*: And, it is obvious, that in caſes where three equal lives are taken, the errors will be much greater.

—Mr. *Simpſon*'s Obſervations in this paſſage are true only, when applied to a *different* method uſed by himſelf, in the 28th and following Problems of his Treatiſe on the *Doctrine* of *Annuities* and *Reverſions*. This method is exact when the lives are equal; but it gives reſults which are too far from the truth, when there is any conſiderable inequality between the lives.

It is with reluctance I have made ſome of theſe remarks. Mr. *De Moivre* has made very important improvements in this branch of ſcience; and the higheſt reſpect is due to his name and authority. This, however, only renders theſe remarks more neceſſary.

In the firſt Chapter (Queſtion 11th, 12th, 15th, and 16th) (*a*). I have given a minute account of the method of finding, in all caſes, the values of the reverſions which have been the ſubject of this Eſſay.—But Mr. *Morgan* has, in his Treatiſe on Life Aſſurances, carried this enquiry much farther.

(*a*) In ſolving queſtions 11th and 12th, here referred to, I have made uſe of Mr. *Simpſon*'s rules in his Select Exerciſes, p. 297 and p. 298. Mr. *Morgan*, in his Treatiſe on Life Aſſurances, adopted the method of ſolution from which Mr. *Simpſon* deduced theſe rules; but he has lately diſcovered that it is an approximation which in ſome caſes gives reſults that are much too diſtant from the truth. See more on this ſubject in the Preface to this Edition; and alſo, at the end of the next volume, where rules (inveſtigated by Mr. *Morgan*) may be found which give in all caſes the correct values of ſums payable on any ſurvivorſhips between any two or three lives.

ESSAY

ESSAY IV.

Obſervations on the proper Method of conſtructing Tables for determining the Rate of human Mortality, the Number of Inhabitants, and the Values of Lives in any Town or Diſtrict, from Bills of Mortality in which are given, the Numbers dying annually at all Ages.

IN every place that juſt ſupports itſelf in the number of its inhabitants, without any recruits from other places; or where, for a courſe of years, there has been no increaſe or decreaſe, the number of perſons dying every year at any particular age, and above it, muſt be equal to the number of the living at that age.—The number, for example, dying every year, at all ages, from the *beginning* to the utmoſt *extremity* of life, muſt, in ſuch a ſituation, be juſt equal to the whole number *born* every year. And for the ſame reaſon, the number dying every year at *one* year of age and upwards; at *two* years of age and upwards; at *three* and upwards, and ſo

on; muſt be equal to the numbers that reach to thoſe ages every year; or, which is the ſame, to the numbers of the living at thoſe ages. It is obvious, that unleſs this happens, the number of inhabitants cannot remain the ſame. If the former number is greater than the latter, the inhabitants muſt *decreaſe*; if leſs, they muſt increaſe.—From this obſervation it follows, that in a town or country where there is no increaſe or decreaſe, bills of mortality which give the ages at which all die, will ſhew the exact number of inhabitants; and alſo the exact law, according to which human life waſtes in that town or country.

In order to find the number of inhabitants; the mean numbers dying annually, at every particular age and upwards, muſt be taken as given by the bills, and placed under one another in the order of the ſecond column of the 6th, 7th, &c. Tables at the beginning of the next volume. Theſe numbers will, it has appeared, be the numbers of the living at 0, 1, 2, 3, &c. years of age; and, conſequently, the ſum, diminiſhed by half the number living at age 0, or by half the number born annually (*a*), will be the whole

(*a*) This ſubtraction is neceſſary for the following reaſon.—In a Table formed in the manner here directed, it is ſuppoſed, that the numbers in the ſecond column are all living together at the beginning of every year. Thus; the number in the *ſecond* column oppoſite to 0 in the *firſt*

whole number of inhabitants.—In ſuch a ſeries of numbers, the exceſs of each number above that which immediately follows it, will be the number dying every year, out of the particular number alive at the beginning of the year; and theſe exceſſes ſet down regularly, as in the third column of the Table to which I have referred, will ſhew the different rates at which human life waſtes through all its different periods, and the different probabilities of life at all particular ages.

It muſt be remembered, that what has been now ſaid goes on the ſuppoſition that the place, whoſe bills of mortality are given, ſupports itſelf, by procreation only, in the number of its inhabitants. In towns this very ſeldom happens, on account of the

firſt column, the Table ſuppoſes to be all juſt born together on the firſt day of the year. The number, likewiſe, oppoſite to 1, it ſuppoſes to attain to one year of age juſt at the ſame time that the former number is born. And the like is true of every number in the ſecond column.—During the courſe of the year, as many will die at all ages as were born at the beginning of the year; and, conſequently, there will be an exceſs of the number alive at the beginning of the year, above the number alive at the end of the year, equal to the whole number of the annual births; and the true number conſtantly alive together, is the arithmetical mean between theſe two numbers; or, agreeably to the rule I have given, the *ſum* of the numbers in the ſecond column of the Table, leſſened by *half* the number of annual births. See Eſſay I. page 235, &c.

luxury

luxury and debauchery which generally prevail in them. They are, therefore, commonly kept up by a conſtant acceſſion of ſtrangers or *ſettlers*, who remove to them from country pariſhes and villages. In theſe circumſtances, in order to find the true number of inhabitants, and probabilities of the duration of life, from bills of mortality containing an account of the ages at which all die; it is neceſſary that the proportion of the annual births to the annual ſettlers ſhould be known; and alſo the period of life at which the latter remove.—Both theſe particulars may be diſcovered in the following method.

If for a courſe of years there has been no ſenſible increaſe or decreaſe in a place, the number of annual ſettlers will be equal to the exceſs of the annual burials above the annual births. If there is an *increaſe*, it will be *greater* than this exceſs. If there is a *decreaſe*, it will be *leſs*.

The period of life at which theſe ſettlers remove, will appear in the Bills by an increaſe in the number of deaths at that period and beyond it. Thus; in the *London* Bills, the number of deaths, between 20 and 30, is generally above double, and between 30 and 40, near triple the number of deaths between 10 and 20: And the true account of this is, that from the age of 18 or 20 to 35 or 40, there is a confluence of people every year to *London* from the country, which occaſions

caſions a great increaſe in the number of inhabitants at theſe ages; and, conſequently, raiſes the deaths for all ages *above* 20 conſiderably above their due proportion, when compared with the number of deaths *before* 20.—This is obſervable in all the bills of mortality for towns with which I am acquainted, not excepting even the *Breſlaw* Bills. Dr. *Halley* takes notice, that theſe Bills give the number of deaths between 10 and 20, too ſmall. This he conſidered as an irregularity, owing to chance; and therefore, in forming his Table of Obſervations, he took the liberty ſo far to correct it, as to render the proportion of thoſe who die to the living in this diviſion of life, nearly the ſame with the proportion which, he ſays, he had been *informed* (*a*) die annually of the young lads in *Chriſt-Church Hoſpital*. But the truth is, that this irregularity in the Bills was derived from the cauſe I have juſt aſſigned.—During the five years for which the *Breſlaw* Bills are given by Dr. *Halley*, the births did, indeed, a little exceed the burials; but, it appears, that this was the effect of ſome peculiar cauſes that happened to operate juſt at that time; for, during a complete century from 1633 to 1734, the annual

(*a*) See *Lowthorp's* Abridgment of the Philoſophical Tranſactions, vol. III. p. 670.

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medium of *births* was 1089 (*a*), and of burials 1256 (*b*). This town, therefore, muſt have been all along kept up by a number of yearly recruits from other places, equal to about *a ſeventh* part of the yearly births.

What has been now obſerved concerning the period of life at which people remove from the country to ſettle in towns, would appear ſufficiently probable, were there no ſuch evidence for it as I have mentioned; for it might be well reckoned, that theſe people in general, muſt be ſingle perſons in the beginning of mature life, who not having yet obtained ſettlements in the places where they were born, migrate to towns in queſt of employments.

Having premiſed theſe Obſervations, I ſhall next endeavour to explain diſtinctly, the effect which theſe acceſſions to towns muſt have, on Tables of Obſervation formed from their bills of mortality. This is a

(*a*) See Dr. *Short*'s Comparative Hiſtory, p. 63.

(*b*) It appears from the account in the *Philoſophical Tranſactions* (Abridgment, vol. VII. No. 380, p. 46, &c.) that from 1717 to 1725, the annual medium of births at *Breſlaw* was 1252, of burials 1507; and alſo, that much the greateſt part of the births died under 10 years of age.—From a Table in *Suſmilch*'s works, Vol. I. p. 38, it appears, that, in reality, the greater part of all that die in this town are children under five years of age,

ſubject

ſubject proper to be inſiſted on, becauſe miſtakes have been committed about it; and becauſe alſo, the diſcuſſion of it is neceſſary to ſhew, how near to truth the values of lives come as deduced from ſuch Tables.

The following general rule may be given on this ſubject.

If a place has, for a courſe of years, been maintained in a ſtate nearly ſtationary, as to number of inhabitants, by ſupplies or recruits coming in every year, to prevent the decreaſe that would ariſe from the exceſs of the burials above the births; a Table formed on the prin- "ciple, that the number dying annually, after "every particular age, is equal to the num- "ber living at that age," will give the number of inhabitants and the probabilities of life, too *great* for all ages preceding that at which the ſupplies ceaſe; and after this, it will give them *right*.—If the acceſſions are ſo great as to cauſe an *increaſe* in the place, ſuch a Table will give the number of inhabitants and the probabilities of life, too *little*, after the age at which the acceſſions ceaſe (*a*); and too great, if there is a decreaſe.

(*a*) Agreeably to theſe Obſervations; if a place increaſes, not in conſequence of acceſſions from other places, but of a conſtant exceſs of the births above the deaths; a Table, conſtructed on the principle I have mentioned, will give the probabilities of life too low through the whole extent of life; becauſe, in ſuch circumſtances, the number of *deaths* in the *firſt* ſtages of life

creaſe. *Before* that age it will in *both* caſes give them too great; but moſt conſiderably ſo in the former caſe, or when there is an increaſe.

For example. Let us ſuppoſe, that 244 of thoſe born in a town, attain annually to 20 years of age; and that 250 more, all likewiſe 20 years of age, come into it annually from other places; in conſequence of which, it has for a courſe of years; been juſt maintained in the number of its inhabitants, without any ſenſible increaſe or decreaſe. In theſe circumſtances the number of the living in the town of the age of 20, will be always 244 *natives* and 250 *ſettlers*, or 494 in all; and, ſince theſe are ſuppoſed all to die in the town, and no more recruits are ſuppoſed to come in; 494 will be likewiſe the number dying annually at 20 and upwards.—In the ſame manner; it will appear on theſe ſuppoſitions, that the number of the living, at every age, ſubſequent to 20, will be equal to the number dying annually at that age and above it; and conſequently, that the number of inhabitants and the decrements of life, for every ſuch age, will be given exactly by the Table I

life muſt be too great, in compariſon of the number of deaths in the *latter* ſtages; and more or leſs ſo, as the *increaſe* is more or leſs rapid. The contrary, in all reſpects, takes place where there is a decreaſe, ariſing from the exceſs of the *deaths* above the *births*.

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have ſuppoſed. But for all ages *before* 20, they will be given much too great. For let 280 of all born in the town, reach 10. In this caſe, 280 will be the true number of the living in the town, at the age of 10; and the recruits not coming in 'till 20, the number given by the Bills, as dying between 10 and 20, will be the true number dying annually of the living in this diviſion of life. Let this number be 36; and it will follow, that the Table ought to make the numbers of the living at the ages between 10 and 20, a ſeries of decreaſing means between 280 and (280 diminiſhed by 36, or) 244. But in forming the Table on the principle I have mentioned, 250 (the number above 20 dying annually in the town who were not born in it) will be added to each number in this ſeries; and, therefore, the Table will give the numbers of the living, and the probabilities of life in this diviſion of life, almoſt twice as great as they really are.—This obſervation, it is manifeſt, may be applied to *all* the ages under 20.

It is neceſſary to add, that ſuch a Table will give the number of inhabitants, and the probabilities of life, equally wrong before 20, whether the recruits all come in at 20, agreeably to the ſuppoſition juſt made, or only *begin* then to come in. In this laſt caſe, the Table will give the number of inhabitants, and probabilities of life, too great

great throughout the whole extent of life, if the recruits come in at all ages above 20. But if they ceaſe at any particular age, it will give them right only from that age; and before, it will err all along on the ſide of exceſs; but leſs conſiderably between 20 and that age, than before 20——For example. If, of the 250 I have ſuppoſed to come in at 20, only 150 then come in, and the reſt at 30; the numbers of the living will be given 100 too high, at every age between 20 and 30; but as juſt ſhewn, they will be given 250 too high at every age before 20.—In general, therefore, the number of the living *at* any particular age, muſt be given by the ſuppoſed Table, as many too great as there are annual ſettlers *after* that age: And, if theſe ſupplies come in at all ages indiſcriminately, during any certain interval of life; the number of inhabitants and the probabilities of life will be continually growing leſs and leſs wrong, the nearer any age is to the end of that interval. —Theſe Obſervations prove, that Tables of Obſervation formed in the common way, from bills of mortality for places, where there is an exceſs of the burials above the births, muſt be erroneous, for a great part of the duration of life, in proportion to the degree of that exceſs. They ſhew likewiſe, at what parts of life the errors in ſuch Tables are moſt conſiderable, and how

how they may be in a great meaſure corrected.

All this I ſhall exemplify and illuſtrate in the particular caſe of *London*.

The number of deaths, between the ages of 10 and 20, is always ſo ſmall in the *London* Bills, that it ſeems certain few recruits come to *London* under 20; or at leaſt not ſo many as before this age are ſent out for education to ſchools and univerſities. After 20, great numbers come in 'till 30, and ſome perhaps 'till 40 or 50.—The *London* Tables of Obſervation, therefore, being formed on the principle I have mentioned, cannot give the probabilities of life right 'till 40. Between 30 and 40 they muſt be a little too high; but more ſo between 20 and 30; and moſt of all ſo before 20.—It follows alſo, that theſe Tables muſt give the number of inhabitants in *London* much too great.

Table XIII. in the next volume, is a Table formed in the manner I have explained, from the *London* Bills for 10 years, from 1759 to 1768; and adapted to a 1000 born as a *radix*. The ſum of the numbers in the ſecond column diminiſhed by half the number born, is 25,757. According to this Table then, for every 1000 deaths in *London*, there are 25 times and $\frac{3}{4}$ that number of inhabitants; or, in other words, the expectation of a child juſt born is $25\frac{3}{4}$; and

the inhabitants are to the annual burials, as $25\frac{3}{4}$ to 1.—But it has appeared, that the numbers in the ſecond column being given on the ſuppoſition, that all who die in *London* were born there, muſt be too great; and we have from hence a DEMONSTRATION, that the probabilities of the duration of life are given in the common Tables of *London* Obſervations, too high, for, at leaſt, the firſt 30 years of life; and alſo, that the number of inhabitants in *London* muſt be *leſs* than $25\frac{3}{4}$, multiplied by the annual *burials*.—The common Tables, therefore, of *London* Obſervations, undoubtedly want to be corrected (*a*); and the way of doing this, and, in general, the right method of forming genuine Tables of Obſervation for towns, may be learnt from the following rule.

"From the ſum of all that die annually, "*after* any given age, ſubtract the number "of annual ſettlers *after* that age; and the "remainder will be the number of the liv-"ing *at* the given age."

This rule can want no explication or proof, after what has been already ſaid.

(*a*) The ingenious and accurate Mr. *Simpſon* ſaw that it was neceſſary to correct the *London* Tables, and he has done it with great judgment; but, I think, too imperfectly, and without going upon any fixt principles, or ſhewing particularly, how Tables of Obſervation ought to be formed, and how far in different circumſtances, and at different ages, they are to be depended on.

If, therefore, the number of annual ſettlers in a town at every age could be aſcertained; a perfect Table of Obſervations might be formed for that town, from Bills of mortality containing an account of the ages at which all die in it. But no more can be learnt in this inſtance from any Bills, than the whole number of annual *ſettlers*, and the general diviſion of life in which they enter. This, however, may be ſufficient to enable us to form Tables that ſhall be tolerably exact.—For inſtance. Suppoſe the *annual deaths* in a town which has not increaſed or decreaſed, to have been for many years, in the proportion of 4 to 3 to the *annual births*. It will hence follow, that $\frac{1}{4}$ of the perſons who die in ſuch a town are *ſupplies*, or *emigrants* from other places; and not *natives*: And the ſudden increaſe in the deaths after 20, will alſo ſhew, agreeably to what was before obſerved, that they enter after this age. In forming therefore a Table for ſuch a town, a *quarter* of all that die at all ages throughout the whole extent of life, muſt be deducted from the ſum of all that die after every given age before 20; and the remainder will be the true number living at that given age. And if, at 20, and every age above it, this deduction is omitted, or the number of the living at every ſuch age is taken the ſame with the ſum of all that die after it, the reſult will

be (ſuppoſing *moſt* of the ſupplies to come in before 30, and *all* before 40) a Table exact 'till 20; too high between 20 and 30; but nearly right for ſome years before 40; and after 40 exact again.—Such a Table, it is evident, will be the ſame with the Table laſt deſcribed at all ages above 20; and different from it only under 20.—It is evident alſo that, on account of its giving the probabilities of the duration of life too great for ſome years, after 20, the number of inhabitants deduced from it may be depended on as greater than the truth; and more or leſs ſo as the annual recruits enter in general later or ſooner after 20.

Let us now conſider, what the reſult of theſe remarks will be, when applied particularly to the *London* Bills.

It muſt be here firſt obſerved, that, at leaſt one *quarter* of all that die in *London* are *emigrants* from the country, and not *natives*.—The medium of annual burials for 10 years, from 1759 to 1768, was 22,956; of births 15,710. The exceſs is 7246; or near a *third* of the burials.—The ſame exceſs, during ten years, before 1750, was 10,500; or near *half* the burials. *London* was then *decreaſing*. For 12 or 15 years before 1769 it was *increaſing*. This exceſs, therefore, agreeably to the foregoing obſervations, was then *greater* than the number

ber of annual recruits; and it is now *leſs*. I have choſen, however, to ſuppoſe the number of annual recruits to be now (*a*) no more than a *quarter* of the annual burials, in order to allow for more omiſſions in the births than the burials; and alſo, in order to be more ſure of obtaining reſults that ſhall not exceed the truth.

Of every thouſand then who die in *London*, only 750 are *natives*, and 250 are *ſettlers* who come to it after 18 or 20 years of age: And, conſequently, in order to obtain from the Bills a more correct Table than the 13th in the next volume, 250 muſt be ſubtracted from every one of the numbers in the ſecond column 'till 20; and the numbers in the third column muſt be kept the ſame, the Bills always giving theſe right.—After 20, the Table is to be continued unaltered; and the reſult will be, a Table which will give the numbers of the living at all ages in *London* much nearer the truth, but ſtill too high.—Such is the 14th Table in the next volume.—The ſum of all the numbers in the ſecond column of this Table, diminiſhed by 500, is 20,750. For every 1000 deaths, therefore, in *London*, there are, according to this Table, 20,750 living perſons

(*a*) It muſt not be forgotten that this refers to the time of the firſt publication of this work; that is, the year 1769.

in it; or for every ſingle death, $20\frac{3}{4}$ inhabitants. It was before ſhewn, that the number of inhabitants in *London* could not be ſo great as 25 times $\frac{3}{4}$ the deaths. It now appears, (ſince the numbers in the ſecond column of this Table are too high) that the number of inhabitants in *London* cannot be ſo great as even 20 times $\frac{3}{4}$ the deaths. And this is a concluſion which, I believe, every one who will beſtow due attention on what has been ſaid, will find himſelf forced to receive. It will not be amiſs, however, to confirm it by the following fact, the knowledge of which I owe to the particular enquiry and kind information of Mr. *Harris*, the ingenious maſter of the Royal Mathematical School in Chriſt-Church Hoſpital.--The average number of lads in this ſchool has, for 30 years ending in 1768, been 831. They are admitted at all ages betwen ſeven and eleven; and few ſtay beyond 16. They are, therefore, in general, lads between the ages of eight and 16. They have better accommodations than it can be ſuppoſed children commonly have; and about 300 of them have the particular advantage of being educated in the country. In ſuch circumſtances it may be well reckoned that the proportion of children dying annually, muſt be leſs than the general proportion of children dying annually at the ſame ages

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in *London* (*a*).—The fact is, that, for the laſt 30 years, $11\frac{1}{5}$ have died annually; or *one* in $70\frac{1}{2}$.

According to Table XIV, of all who complete their 8th year in *London*, and who are living at that age and at every intermediate age till 16, one in 74 die annually. It follows, therefore, that, according to this Table, ſuppoſing the lads in *Chriſt-Church* School all admitted exactly at

(*a*) Mr. *Wales*, the preſent maſter of this ſchool, has, in his Enquiry, &c. p. 33, confirmed this account; adding, that the number of children in it for *twenty* years ending in 1780, had been 851, and the average of annual deaths $10\frac{1}{4}$, or one in 83; but that the number of children for *ten* years (ending in 1780) had been 894, and the average of annual deaths only $8\frac{9}{10}$, or about one in $100\frac{1}{4}$.—From hence Mr. *Wales* infers a great improvement in the ſtate of *London* with reſpect to healthineſs. But this fact is by no means a ſufficient foundation for ſuch a concluſion. In numbers ſo inconſiderable, an average of ten years cannot be depended on. Were it, however, the true average, the reaſons above given have a tendency to prove, notwithſtanding the centrical ſituation of this ſchool, that it muſt be too low for *London* in general. If ſo many as *three-fourths* of all that die in *London* are natives, the proportion dying annually between 8 and 16 muſt be as high as *one in ſeventy-five*; and poſſibly this is even now leſs than the true proportion. But it would be unreaſonable to take it leſs than the firſt proportion mentioned by Mr. *Wales*, or one in 83. The conſequence however of ſtating it at even one in 90, and alſo one in a hundred, will be ſhewn in the next note; from which it will appear to be impoſſible, without exceeding all the bounds of credibility, to make the expectation of a child juſt born in *London* much more than twenty years.—See 1ſt Eſſay, notes p. 243, and p. 251.

eight years of age, and none diſcharged before they have completed 16 years of age; or before they have reſided eight years (ſuppoſitions much too favourable), only a 74th part ought to die annually. That Table, therefore, gives the decrements of life in *London* at theſe ages too little, and the numbers of the living too great: And if this is true of theſe ages, it muſt be true of all other ages *under* 20; and it follows demonſtrably, in conformity to what was before ſhewn, that more people ſettle in *London* after 20, than the *quarter* I have ſuppoſed; and that from 20 to 35 or 40, the numbers of the living are given too great in proportion to the decrements of life.

In this Table the numbers in the ſecond column are doubled at 20, agreeably to what really happens in *London*; and the ſum of the numbers in this column diminiſhed by half the whole number of deaths, gives the *expectation* of life, not of a child juſt born, as in other Tables, but of all the inhabitants of *London* at the time they enter it, whether that be at birth, or at 20 years of age. The *expectations*, therefore, and the *values* of *London* lives under 20, cannot be calculated from this Table. But it may be very eaſily fitted for this purpoſe by firſt finding the number of births which, according to the given decrements of life, will leave 494 alive at 20; and then adapting the intermediate numbers

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in ſuch a manner to this radix, as to preſerve all along the number of the living, in the ſame proportion to the numbers of the dead. This is done in the 15th Table in the next volume; and this Table may, I fancy, be recommended as better adapted to the preſent ſtate of *London* than any other Table (*a*). The values of lives, however, deduced from

(*a*) Had I, inſtead of ſubtracting 250 from Table 13th before the age of 20 (agreeably to the directions in p. 341), ſubtracted only 200 (or ſuppoſed that only a *fifth* part of all that die annually in *London* are emigrants to it after 20), the reſulting Table would have made the number dying between 8 and 16, one in 90; and the expectation at *entrance* into *London*, would have been 22, and at *birth*, 19$\frac{4}{5}$.—Had 166 only been ſubtracted, or a 6th part of all that die in *London* ſuppoſed to be emigrants from the country, the reſult would have been a Table which would have made one in a 100 die between 8 and 16, and the expectations juſt mentioned 23 and 21$\frac{1}{4}$. Nor will any difference worth regarding ariſe, if Table 15th in the next volume, inſtead of being formed after 19 from the Bills for ten years ended at 1768, had been formed from the Bills for ten years ending in 1780. Table 16th is ſuch a Table; and the obſervations annexed to it will ſhew how wrong the ideas are which ſome have lately entertained of the improved ſtate of *London*. Some alteration for the better there muſt be; but the correſpondence between the Tables of Obſervation for whatever period they are formed from the Bills, demonſtrates that it is not conſiderable. The great evils which produce the unhealthfulneſs of towns are the cloſeneſs and foulneſs of the air, and the irregular modes of living. If the former of theſe has been diminiſhed in *London*, the latter may have increaſed. But the truth may be, that the diminution of the former of theſe evils has not much extended itſelf to the lower ranks of people in *London*, who form the body of the inhabitants.

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it, are in general nearly the ſame with thoſe deduced by Mr. *Simpſon*, from the *London* Bills as they ſtood 40 years ago. The main difference is, that after 52 and in old age, this Table gives them ſomewhat lower than Mr. *Simpſon's* Table.

It has ſufficiently appeared, what judgment we are to form of the values of lives thus deduced. During the greateſt part of the interval of life, in which the annual recruits that keep up *London* come to it, theſe values err certainly on the ſide of *exceſs*: And it is alſo *probable*, that they exceed the truth in all the laſt ſtages of life (*a*).

The

(*a*) When the former editions of this Treatiſe were publiſhed, it appeared to me probable, that, in conſequence of retirements from *London* in the advanced periods of life, the Bills gave the probabilities of living in *London* after ſeventy years of age too *low* rather than too *high*. But I am now convinced of the contrary. Thoſe who withdraw from *London* in advanced life are only a part of the inhabitants in the higher claſſes, *themſelves* a ſmall part of the whole body of inhabitants; and they withdraw, if at all, before ſeventy years of age, and therefore, the loſs of them in the Bills can have no effect on the *proportions* of the numbers that die at all ages after ſeventy.—It has alſo occurred to me, that though the probabilities of living *before* the age of ſeventy, as given by the Bills, have continued remarkably the ſame from 1728 (when the ages were firſt included in the Bills) to 1780, (as will be ſhewn in the Obſervations on the *London* Tables in the next volume), yet *after* the age of 70 there has been a gradual diminution in them; ſo that now, of all who die at *all* ages, only one in 44 dies at a greater age than 80; whereas at the period juſt men-

The number of inhabitants in *London* may alſo be learnt from what has been offered, more

mentioned, one in 32 died above this age; and of all who die above 70, only 31 in a hundred now die above 80, and 4 in a 100 above 90; whereas, at the ſame period, 43 in a hundred uſed to die above 80, and 11 in a hundred above 90.

But what has principally determined my judgment in this inſtance is a compariſon of the probabilities of living in STOCKHOLM, as deduced from the STOCKHOLM Bills, with the correct probabilities as determined by an actual account taken at three different times of the number of the inhabitants living there at all ages.—This compariſon ſhews that Bills of mortality for great towns give the probabilities of living too high at all ages; and particularly at the *end* as well as the *beginning* of life; for the proportion of inhabitants between 70 and 80 dying annually at STOCKHOLM was, according to the *ſurvey*, 10 out of 63; and between 80 and 90, ten out of 28; and above 90, ten out of 25; whereas, according to the *Bills*, theſe proportions are ten out of 100, 55, and 24 reſpectively.

The *London*, *Vienna*, and *Berlin* Bills give the probabilities of living between 70 and 80, and between 80 and 90, nearly the ſame with theſe, as may be learnt from the Tables of Obſervations for theſe towns in the next volume; and as at *Stockholm*, they are certainly too high; the reaſonable concluſion is, that they are ſo likewiſe in the other towns: The truth, perhaps, may be, that more perſons (invited by the conveniencies in towns) remove into them in old age, than withdraw from them.

No one, probably, will think that the change which I have mentioned in the *London* Bills can be owing to a growing unfavourableneſs of *London* to the health of old people. The following obſervations will ſufficiently account for this fact.

LONDON, after the loſs of a quarter of its inhabitants by the plague in 1665, and the devaſtation of the fire

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more nearly than by any method which has been hitherto taken. It cannot, it has been ſhewn, exceed 20 times $\frac{3}{4}$ the number of annual deaths. Could, therefore, the annual deaths be aſcertained, we ſhould know the number of inhabitants within pretty narrow limits. But the omiſſions in the Bills are ſuch, that it is not poſſible to aſcertain, with exactneſs, the annual deaths. Dr. *Brakenridge* ſuppoſed theſe omiſſions to amount to 2000 annually. The reſult of a very minute enquiry by Mr. *Maitland* is, that in the year 1729, they amounted to 3038. But they are probably now more conſiderable than they ever were (*a*). Let them be 6000; and the

in 1666, recovered ſo faſt as in three or four years to become more populous than it had ever been; and it continued to increaſe till the *Revolution* in 1688; after which period, and during the reigns of King *William* and Queen *Anne*, it ſeems, if we may judge from the Bills, to have ſtagnated and declined. There muſt, therefore, for ſome years after 1666, have been a very extraordinary influx of people to it; and they muſt have been, for the moſt part, people in the beginning of mature life, who would not all die off in leſs than 60 or 70 years, and, therefore, would, about the year 1728, render *London* fuller of inhabitants turned of 80 and 90, than it could be at any other period.

(*a*) Vid. Preface to a Collection of the Bills of Mortality from 1657 to 1758, p. 4, &c.—Since the above was written the burials, as given in the Bills, have fallen from 22,688 (the annual average for five years to 1770) to 20,743, the ſame average to 1780. Adding 6000 to this

the number of inhabitants (ſuppoſing the burials 29,000) will be 601,750 at moſt.

All the preceding Obſervations are, it is plain, applicable to Bills of mortality for towns in general; and point out the way of deducing from them genuine Tables of Obſervations, which ſhall give the true probabilities and values of lives, and the true number of inhabitants, in the towns whoſe Bills are given.—I ſhall beg leave to confirm and illuſtrate this, in the particular caſe of the town of NORTHAMPTON.

In this town, containing four pariſhes, namely, *All-Saints*, *St. Sepulchre's*, *St. Giles*, and *St. Peter's*, an account has been kept ever ſince the year 1741, of the number of males and females that have been chriſtened and buried (Diſſenters included) in the whole town. And in the pariſh of *All-ſaints*, containing the greateſt part of the town, an account has been kept ever ſince 1735, of the *ages* at which all have died there.

In 1746, an account was taken of the number of *houſes*, and of *inhabitants* in the town. The number of *houſes* was found to be 1083;

this laſt number, and multiplying the total by $20\frac{3}{4}$, will make the number of inhabitants in *London* in 1780 554,917. But even this computation is too high, as appears from the note in p. 257.

Since 1780, the cauſes mentioned in the note p. 251 and at the concluſion of the Poſtſcript in the next volume, have ſunk the regiſtered burials in *London* to 19,494, which was the average for three years to 1790.

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and the number of *inhabitants* 5136.—In the pariſhes of *All-ſaints* and *St. Giles*, the number of *male* and *female heads* of families, *ſervants*, *lodgers*, and *children*, were particularly diſtinguiſhed.—The *heads of families* were, 707 males ; and 846 females.——*Children*, males 624; females 759.—*Servants*, males 203; females 280.—*Lodgers*, males 137; females 287.—In *St. Peter's*, males 99; females 129.—In *St. Sepulchre's*, *adults* 638; *children* 427. In the laſt pariſh the ſexes were not diſtinguiſhed.

The *Chriſtenings* and *Burials* in the *whole town* for 40 years, from 1741 to 1780, have been as follows:

Chriſtened	Males 3218 Fem. 3108	6326	Annual medium 158
Buried	Males 3757 Fem. 3823	7580	Annual medium 189½

In the pariſh of *All-Saints*, from 1735 to 1780, or 46 years,

Chriſtened	Males 2152 Fem. 2068	4220	Annual medium 91¾
Buried	Males 2377 Fem. 2312	4689	Annual medium 102

Of theſe died,

Under 2 years of age	—	1529
Between 2 and 5	—	362
Between 5 and 10	—	201
Between 10 and 20	—	189

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Between 20 and	30	—	373
Between 30 and	40	—	329
Between 40 and	50	—	365
Between 50 and	60	—	384
Between 60 and	70	—	378
Between 70 and	80	—	358
Between 80 and	90	—	199
Between 90 and	100	—	22
		Total	4689

A Table formed from theſe *data* in the manner of Table XIII in the next volume; or, on the ſuppoſition, that all who *die* in *Northampton* were *born* there, would give the expectation of a child juſt born 28.83 years; and conſequently the proportion of the inhabitants to the annual deaths, as 28.83 to 1. It has been ſhewn, that this proportion, in a place where the burials exceed the births, muſt be *greater* than the *true* proportion of the number of inhabitants to the annual deaths: And this appears to be the real caſe. For the Bills ſhew, that, from 1741 to 1750, or for 10 years, about the time when the number of inhabitants was 5136, the annual medium of burials was 197.5; which, multiplied by 28.83, gives 5693; or a 9th part more than the *true* number.

A Table formed in the manner of Table XIV in the next volume, would give the proportion

portion of inhabitants to the annual deaths, as 26.41 to 1; and this makes the inhabitants 5216, or very nearly the true number.

The VIth Table, in the next volume, is formed in the ſame manner with Table XV, for *London*: And this is the genuine Table of Obſervations for *Northampton* (*a*), from which may be calculated the true probabilities and values of lives in that town.

(*a*) In the fourth edition of this Treatiſe the following corrections were made in this Table. Firſt. The Table printed in the three firſt editions having been formed from the *Northampton* Bills for 36 years, this Table was rendered a little more correct in conſequence of being formed from the ſame Bills for 46 years.—Secondly. The Bills give the numbers dying annually between 20 and 30 greater than between 30 and 40; but this being a circumſtance which does not exiſt in any other regiſter of mortality, and undoubtedly owing to ſome accidental and local cauſes, the decrements were made equal between 22 and 40; preſerving, however, the total of deaths between 20 and 40 the ſame that the Bills give them.—Thirdly. The Bills giving only the totals of deaths under two years of age, and between 2 and 5, the proportions of deaths for every particular year between 2 and 5, and for every *quarter* of a year after birth till one year of age, were made the ſame nearly that the Cheſter regiſter makes them. See the Introduction to the collection of Tables in the next volume.

In conſequence of theſe alterations, and alſo of increaſing the *radix* from 1165 to 11650, in order to adjuſt the decrements with greater regularity and preciſion, this Table, in my opinion, gives the *mean* probabilities and values of lives at every age with more accuracy than any other Table now extan

At NORWICH, Bills of Mortality, of the ſame kind with thoſe in *London* and *Northampton*, have been kept for many years. I have been favoured with a copy of theſe Bills for 30 years, from 1740 to 1769. The annual medium of *chriſtenings*, during this period, has been 1057 (*a*), of *burials* 1206. And from hence, together with the account of the numbers dying in the ſeveral decads of life, after 10, I have formed Table VII, which ſhews the true probabilities of the duration of life in this town.

The following particulars ſeem to deſerve notice here.

Firſt. Had theſe Tables been formed from the NORTHAMPTON and NORWICH Bills, for no longer time than any 10 years taken together, of the periods I have men-

(*a*) In this regiſter all that die before baptiſm, and alſo all that are born and die among *Quakers*, *Jews*, &c. are omitted. There are alſo ſome other omiſſions; and the true annual medium of births and burials muſt be greater than they are given in the bills. But this will have no effect on a Table of Obſervations, ſuppoſing the *proportions* of the births to the burials, and of the numbers dying in the different ſtages of life, given right. —It is proper I ſhould mention further here, that theſe Bills give only the whole number of children dying under 10, without ſpecifying the numbers dying under two years of age, between 2 and 5, and between 5 and 10, as in other Bills. I have, therefore, in forming the Table for NORWICH, ſuppoſed the proportions of theſe numbers the ſame that I have given them for NORTHAMPTON.

tioned; they would not have given the values of lives materially different. Theſe Tables, therefore, are founded on a ſufficient number of Obſervations; and it appears, that there is an invariable law which governs the waſte of human life in theſe towns.—The ſame remark might be made concerning LONDON (*a*).

Secondly. An account was taken at SHREWSBURY, in 1750, of the *whole* number of inhabitants; diſtinguiſhing, particularly, the number at the age of 21 and upwards.—The former number was 8141; and the latter, 5187.—According to a Table

(*a*) Some have entertained a very wrong notion of the imperfections in the LONDON Bills. They do, indeed, give the *whole* number of births and deaths much too little; but the concluſions with reſpect to the probabilities of life in LONDON, and the proportion of inhabitants dying annually, depend only on the *proportions* of the numbers dying in the ſeveral diviſions of life; and theſe are given *right* in the LONDON Bills.—For firſt, There ſeems nothing in this caſe, that can be likely to cauſe the deficiencies in the Bills to fall in one diviſion of life more than in another: But what decides this point is, that theſe proportions, as given by the Bills for *any* ten, or even *five* years, come out nearly the ſame with one another; and always very different from the proportions given by other Bills.—There are no other variations, than ſuch as muſt ariſe from the fluctuations of LONDON as to increaſe and decreaſe; and alſo from ſome improvements in its ſtate, which have lately taken place, and particularly the law lately paſſed, ordering all pariſh infants to be nurſed in the country. See the note in page 251; and the Obſervations on Table XVI in the next volume.

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formed for NORTHAMPTON, in the ſame manner with Table XIV for LONDON, the whole number of the living is to the number of the living at 21 and upwards, as 26,411 to 16,586; that is, as 8141 to 5113.—According to a like Table for NORWICH, theſe numbers are to one another, as 24,500 to 15,680; that is, as 8141 to 5210.—Theſe Tables, therefore, give the proportion of the *whole* number of inhabitants, to the number of the living at 21 and upwards, almoſt exactly the ſame with the true proportion, as it is at SHREWSBURY (*a*): And this affords an additional proof of the rectitude of the principles on which theſe Tables have been formed.

But further.—The number of inhabitants, not reckoning children, in the pariſhes of *St. Giles* and *All-Saints*, NORTHAMPTON, was, in 1746, 2460; and the *whole* number of inhabitants in theſe two *pariſhes* was 3843.

(*a*) The annual medium of births at SHREWSBURY, for 7 years, from 1762 to 1768, was 301; of burials 329. It appears, therefore, that one in 24¾ of the inhabitants die annually. But it ſhould be remembered, that in 1766, the ſmall-pox and meaſles increaſed very much the mortality in this town; and I find alſo, that ſince 1750, a nurſery for *foundlings* from LONDON was eſtabliſhed here; and that in 1768 this nurſery contained 660 children and ſervants. It ſeems, therefore, probable, that the true *medium* of burials about the year 1750, muſt have been leſs than 329; and that the proportion of inhabitants dying annually, may not be much greater than it is at NORTHAMPTON; or 1 in 26.41.

See p. 350.—In the account I have received, the particular age at which the limit of childhood was fixed in taking this ſurvey, is not mentioned; but there ſeems reaſon to believe, that it was 21: And, taking this for granted, the number of inhabitants, not children, will come out (by ſuch a Table for NORTHAMPTON as Table XIV for LONDON) 2414; or, nearly the ſame with the number *really* found in theſe pariſhes.—Had this number been computed, from a Table formed for NORTHAMPTON, in the manner of Table XIII, in next volume, it would have come out only 2176. This remark is applicable to the Table for *Breſlaw*, formed by Dr. *Halley*, compared with the ſame Table, corrected for all the ages under 20 (*a*), by the rule, p. 338. The neceſſity, therefore, of that correction is verified by facts; and it appears, abundantly, that the Tables I

(*a*) I have given Dr. *Halley*'s Table in the next volume juſt as he framed it. A correction of it might be made from the proportion of births to burials, mentioned p. 332. And it would then appear, that a 25th part of the inhabitants at *Breſlaw* die annually; and that half the number born die there under ſix, as well as at *Norwich*. This Table, as we now have it, makes half live to 16; but the account mentioned in the note, page 332, ſhews this not to be the truth. It likewiſe makes the number of inhabitants at SHREWSBURY, above the age of 21, to be 4730; and in the pariſhes of *All-Saints* and *St. Giles*, in NORTHAMPTON, 2230. It gives, therefore, theſe numbers wrong; whereas, as obſerved above, a corrected Table would give them true.

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have given for NORTHAMPTON and NORWICH may be depended on.

But, thirdly. In comparing theſe two Tables, it may be obſerved, that there is a difference between them in favour of NORTHAMPTON, *fewer* dying there in childhood, and *more* in old age. The ſame would be found to be true, were the NORTHAMPTON Table to be compared with a corrected BRESLAW Table. It appears, therefore, agreeably to what might have been expected, that NORTHAMPTON, being a ſmall town compared with BRESLAW and NORWICH, is leſs unfavourable to health and longevity. The difference, however, is not conſiderable. After the age of 20, there is a ſtriking conformity between all the three Tables, which gives them great weight and authority.

Further. It ought to be noted, that theſe Tables prove the *decrements* of life between 25 and 75, in moderate towns, to be nearly *equal*. At NORTHAMPTON it appears that, of a given number of perſons alive at 20, the ſame number die every year 'till 78, without any conſiderable interruption, except between the ages of 30 and 40. —A like uniform decreaſe in the probabilities of the duration of life appears in the BRESLAW and NORWICH Tables; but not ſo remarkably. It was this circumſtance in the BRESLAW Table, that led Mr. *De Moivre* to the *hypotheſis*, deſcribed in p. 2, and ſo often

mentioned in this work.—It gives the values of lives in the middle ſtages nearly the ſame that they are by theſe three Tables; but it is far from being applicable with ſufficient correctneſs to the valuation of lives before 25 and after 75 years of age (*a*); nor does it at all correſpond to the law which governs the waſte of human life in great towns, and in country pariſhes and villages. This will appear immediately from inſpecting the Tables in the next volume. I will here only compare the *expectations* of life by it with the *expectations* at the ſame ages in *London*, and in a country pariſh, where the exacteſt obſervations have been made. I mean, in the pariſh of HOLY-CROSS near *Shrewſbury* (*b*), mentioned in the firſt Eſſay, page 261.

EXPEC-

(*a*) Having in the three firſt editions of this work given examples from this hypotheſis, and the Tables founded upon it and printed at the end of the next volume, I have been obliged to continue them in this edition; but the truth is, that it does not in *any* part of life give ſuch correct values, particularly of joint lives, as are neceſſary in ſome caſes. And it is this, together with the other reaſons mentioned here, and in the Poſtſcript to the ſecond Eſſay, that has induced me to employ a good deal of time in calculating the Tables of the values of lives from real obſervations, which will be found in the next volume.

(*b*) The regiſter of this pariſh, with a Table of the probabilities and expectations of lives deduced from it will be given among the Tables in the next volume.

The

EXPECTATIONS *of* LIFE *in* LONDON *at*

	By Hypotheſis.	*Holy-Croſs.*
Age 10 —— 34.8	—— 38	—— 46
20 —— 28.9	—— 33	—— 38.66
30 —— 23.6	—— 28	—— 32.66
40 —— 19.6	—— 23	—— 26.40
50 —— 16.0	—— 18	—— 20.40
60 —— 12.4	—— 13	—— 14.86
70 —— 8.9	—— 8	—— 10.00

There is one more fact which I ſhall here take notice of; and which deſerves more attention than has been hitherto beſtowed upon it.

The expectation of a child juſt born here is 33.9.—— At NORTHAMPTON it is $25\frac{1}{2}$. At NORWICH, $23\frac{3}{4}$. In LONDON, 18.—In this pariſh, 1 in 11 dies at 80, and upwards. In NORTHAMPTON; 1 in 22. In NORWICH; 1 in 27. In LONDON; 1 in 60. See Eſſay I. p. 273.

I will add, that the probabilities of life here appear to be much the ſame with the probabilities of life among the miniſters and profeſſors in SCOTLAND.—This is a fact of ſome conſequence; and, therefore, I will give a brief account of it.

The mean age at which the miniſters and profeſſors enter into benefices and profeſſorſhips in *Scotland* is reckoned to be 27. Their number is 974. The eſtabliſhment among them for providing for their widows begun on the 25th of *March* 1744; from which time to *November* 22, 1779, 1037 have died: That is, 29 annually; or 1 in $33\frac{3}{5}$. The *expectation*, therefore, of a life among them, at the age of 27, is 33.6; which is nearly the ſame with the *expectation* of a life of the ſame age in the

it. I mean; "the difference between the "probabilities of life among *males* and *fe*- "*males*, in favour of the latter."

From the account in p. 350, it appears, that at NORTHAMPTON, though more *males*

pariſh of *Holy-Croſs*; and $3\frac{1}{2}$ years more, than the *expectation* of the ſame age by Tables V, VI, and VII, in the next volume.—Now the *expectation* at a given age, being compoſed of all the probabilities of life from that age to the extremity of life; there ariſes from hence reaſon for concluding, that the probabilities of life among the miniſters in *Scotland*, cannot differ much in any part of life from thoſe in this pariſh.—But there is another fact that confirms this obſervation.

The annual average of weddings among the miniſters and profeſſors in SCOTLAND, for 35 years ending in 1779, has been 30. The average of married perſons among them, for 17 years ending in 1757, had been 667. This number, divided by 30, gives 22, the *expectation* of marriage among them; which is above 3 years more than the expectation of marriage would be, by Dr. *Halley*'s Table, on the ſuppoſition, that all marriages may be juſtly conſidered as commencing, one with another, ſo early as the age of 30.—The *expectation* of two equal *joint* lives is to the *expectation* of a *ſingle* life of the ſame age, as 2 to 3, by note (K) at the end of the next volume. It follows, therefore, that among the miniſters in *Scotland*, the *expectation* of a *ſingle* life at 30 cannot be leſs than 33. Moſt probably it is more; on account of the later commencement of marriage in the ſituation of the *Scotch* miniſters.—I reckon alſo that 27 muſt be leſs than the mean age at which they enter their benefices and profeſſorſhips; meaning by it, not the age on each ſide of which equal numbers enter, but the age at which the exceſs of the interval of time taken to enter on one ſide, is juſt ſuch as to compenſate the greater numbers who enter on the other ſide. See the concluſion of note (F) in the next volume.

are

are born than *females*, and nearly the ſame number die; yet the number of living *females* is greater than the number of *males*, in the proportion of 2301 to 1770, or 39 to 30. This cannot be accounted for without ſuppoſing, that *males* are more ſhort-lived than *females*.—One obvious reaſon of this fact is, that males are more ſubject to untimely deaths by accidents of various kinds; and alſo, in general, more addicted to the exceſſes and irregularities which ſhorten life. But this is by no means the *only* reaſon. For it ſhould be obſerved that at NORTHAMPTON, the number of *female* children was, in 1746, greater than the number of male children, in the proportion of 759 to 624.—The greater mortality of males, therefore, takes place among *children*.—But this, together with the greater mortality in general of males at all ages, will more particularly appear from the following recital of facts.

In the pariſh of *Holy-Croſs*, *Salop*, the ingenious Vicar, Mr. *Gorſuch*, in 1760, and again in 1770, took the number of male and female inhabitants turned of 80. In 1760, the number of females turned of this age, was 13; of males, 2. In 1770, theſe numbers were, females, 11; males, 6. And for 10 years to 1770, eleven out of 365 had died between the ages

ages of 85 and 102; and they were all females (*a*).

At BERLIN, it appeared, from the accurate account which was taken of the inhabitants in 1747, and which has been mentioned in p. 294, that the number of *female* citizens exceeded the number of *male* citizens, in the proportion of 459 to 391: And yet, out of this ſmaller number of males, more had died for 20 years preceding 1751, in the proportion of 19 to 17 (*b*).

At EDINBURGH, in 1743, the number of *females* was to the number of *males*, as 4 to 3; (See Eſſay I. p. 284) but the females that

(*a*) For 20 years, from 1760 to 1780, eleven out of 966 had died above 90 in this pariſh; and they were all females. See the Regiſter of this pariſh to 1780, among the collection of Tables in the next volume.

From an accurate ſurvey of the pariſh of *Skelton*, in *Yorkſhire*, taken 1777, under the direction of Dr *Biſſet*, it appeared that 39 (that is, an 18th part) were 75 and upwards, 25 of whom were females, and only 14 males.

According to an enumeration in 1762, a hundred and five pariſhes and villages in the generality of *Rouen* in *France*, conſiſted of 15,943 families, and 60,552 inhabitants, 6812 of whom were girls and 5670 boys, under the age of fourteen.

(*b*) Vid. *Suſmilch, Gottliche Ordnung*, &c. where a minute account is given of the number of males and females at BERLIN in 1747; and alſo, of the numbers of each ſex that had died from 1722 to 1750.

died

died annually, from 1749 to 1758, were to the males, in no higher proportion than $3\frac{1}{7}$ to 3. Before 1749, the Bills gave the totals of burials, without diſtinguiſhing them into the totals of males and females dying every year.

Mr. *Kerſeboom*, in his Eſſay on the numbers of people in HOLLAND, informs us, that from the Tables of aſſignable Annuities for lives in HOLLAND, which had been kept there for 125 years, wherein the ages of the perſons dying are truly entered; it appears, that females have, in all accidents of age, lived about 3 or 4 years longer than the ſame number of males. See *Philoſophical Tranſactions* abridged, Vol. IX. p. 326.

In Volume the 7th of the *Philoſophical Tranſactions* abridged, Part IV. p. 46, &c. there is an account of the numbers of *male* and *female* ſtill-born children and chryſoms, and of boys and girls under 10, of married men and married women, and of widows and widowers, who died for a courſe of years at *Vienna*, *Breſlaw*, *Dreſden*, *Leipſic*, *Ratiſbon*, and ſome other towns in GERMANY.

He that will take the pains to examine theſe accounts will find that, though in theſe towns the proportion of males and females born is no higher than 19 to 18, yet the pro-

proportion of boys and girls (*a*) that die is 8 to 7; and that, in particular, the *ſtill-born* and *chryſom males*, are to the ſtill-born and *chryſom females*, as 3 to 2.

In theſe accounts it appears alſo, that of 7270 *married* perſons who had died in theſe towns (*b*), 4336 were married men, and but 2934 married *women*; that is, *three* married *men* died to two married *women*.—In all POMERANIA, during 9 years, from 1748 to 1756, there died 13,556 married men, and 10,007 married women; that is, nearly 15 to 11. *Suſmilch, Gottliche Ordnung*, vol. i. Tables, p. 97. The ſcheme for making proviſion for the widows and orphans of the miniſters in SCOTLAND, has obliged them to keep an account of the number of *weddings* among them, and the number of *widows* left annually: and it appears from the *reports* of the *truſtees* for carrying this ſcheme into execution, that the annual medium of *weddings* is 30. And the annual medium of widows, who came upon the ſcheme for 35

(*a*) In the accounts from *Breſlaw* it is particularly mentioned, that by *boys* and *girls* are meant children to 10 years of age, of whom for 8 years from 1717 to 1725, *ſeven* males died to *ſix* females, excluſively of the *ſtill-born* and *chryſoms*.

(*b*) In *Breſlaw* alone, for the eight years mentioned in the laſt note, 1891 married men died, to 1196 married women; that is 5 to 3.

years, to 1779 was $19\frac{1}{10}$. Of 30 marriages then contracted annually, $19\frac{1}{10}$ became extinct by the deaths of *huſbands*; and not 11 by the deaths of *wives*. That is; among the miniſters and profeſſors in SCOTLAND, 19 married men die to 11 married women. It appears, therefore, that there is the chance of more than 7 to 4, that the *woman* ſhall be the ſurvivor of a marriage, and not the *man*. In order to account for this by the difference of age between men and their wives, this difference ought to be at leaſt 13 or 14 years. That is; ſuppoſing the mean age at which women marry to be 23, the mean age at which men marry ought to be 36 or 37. But this ſeems to exceed the bounds of credibility; and, therefore, very probably, the greater mortality of males muſt operate in this caſe.

It is further obſervable in the accounts from *Germany*, to which I have referred, that the number of *widows* dying annually, is four times the number of *widowers* (*a*); and,

(*a*) In *Dreſden* alone, the number of *widows* who died, in four years, was 584. The number of *widowers*, 149. That is: 4 to 1. At WITTENBERG, during 11 years, 98 *widowers* died, and 376 *widows*.—At GOTHA, during 20 years, 210 *widowers* and 760 *widows*. *Suſmilch's Gottliche Ordnung*, Vol. II. p. 273.—In the country, on account of a leſs difference between the ages of huſbands and wives and more early marriages, the deaths of *widowers* and *widows* come nearer to one another: for in POMERANIA,

and, as *widows* are certainly, one with another, ſeveral years younger than *widowers*; it may be concluded from hence, that the number of the former in life together could not be leſs than five times the latter.—This fact is likewiſe confirmed, by the obſervations which have been made among the miniſters in *Scotland*. The number of *widows* in life, derived from the whole body of miniſters and profeſſors, cannot be much ſhort of 400; but the number of *widowers* among them had, for many years before 1779, been ſcarcely 90; that is, not ſo much as a *quarter* of the number of *widows*.—It may be eaſily ſeen, and it would not be difficult to demonſtrate, that neither the greater number of perſons left widows, nor any probable ſuppoſition concerning the greater frequency of marriages among widowers, can completely account for this, without admitting the *greater mortality of males*.—This, therefore, appears on the whole to be a fact well eſtabliſhed: And it follows from it, that in order to calculate the values of *Life-An-*

MERANIA, during the 9 years mentioned in p. 364, the *widowers* that died were 411, the *widows* 1553; or 2 to 5. —At CHESTER, during 9 years, from 1772 to 1779, the number of *widowers* who died was 157; of *widows* 390. —The number of widowers in the town in 1774 was 258; of widows 736.—At *Warrington*, during 7 years, from 1773 to 1779, ſeventy-nine widowers died, and 155 widows. See the Introduction to the Tables in the next volume.

nuities

nuities and *Reverſions* with exactneſs, there ought to be diſtinct Tables of the Probabilities of life for *males* and *females*. All that is neceſſary to obtain the proper *data* for forming ſuch Tables is, that the *ſexes* as well as the *ages* of the dead ſhould be ſpecified in the Bills; and this improvement would be rendered more complete by diſtinguiſhing the males that die under the denominations of boys, married men, widowers and batchelors; and the females under the denominations of girls, married women, widows, and virgins (*a*).

It has been obſerved, that the Author of nature has provided, that more *males* ſhould

(*a*) Since the former editions of this work, Regiſters of mortality on the plan propoſed here and in the two following pages, have been eſtabliſhed at *Cheſter* under the direction of Dr. *Haygarth*; at *Warrington*, under the direction of Mr. *Aikin*; and at *Eccles* near *Mancheſter*, under the direction of Dr. *Percival*.——The two firſt of theſe Regiſters (abſtracts of which will be found in the next volume) have furniſhed already *data* nearly ſufficient for forming diſtinct Tables of the values of lives among males and females; and they confirm what has been here obſerved concerning the longer duration of human life among females. But the beſt information on this ſubject has been given by the Obſervations in SWEDEN, which came to my knowledge ſince the three firſt editions of this Treatiſe, and which have helped me not a little to improve it, as may be ſeen in the next volume.

For more facts relating to the longer duration of life among females, ſee page 126 and 127 of this volume, and the Supplement in the next volume.

be

be born than *females*, on account of the particular waſte of *males*, occaſioned by wars and other cauſes. Perhaps it might have been obſerved with more reaſon, that this proviſion had in view that particular weakneſs or delicacy in the conſtitution of males which makes them more ſubject to mortality; and which, conſequently, renders it neceſſary that more of them ſhould be produced, in order to preſerve in the world a due proportion between the two ſexes (*a*).

In the courſe of this Eſſay, it has often appeared, that I have been particularly indebted to an information which I have received from NORTHAMPTON.—I ſhould be inexcuſable, did I not mention, that I owe this information to Mr. *Lawton*, an ingenious gentleman in that town, who has preſerved the Bills of Mortality there with much care, and been very obliging in communicating them to me.—It is much to be deſired, that like accounts were kept in every town and pariſh. It would be extremely agreeable to learn from them the different rates of human mortality in different places, and the number of people and progreſs of population in the kingdom. The trouble of keeping them would be trifling; but the inſtruc-

(*a*) More will be ſaid on this ſubject in the *Supplement* in the next volume, and the true cauſe of that debility in the male conſtitution, which ſhortens its duration, will be there aſſigned.

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tion derived from them (*a*), would be very important.—I have already propoſed one improvement of ſuch accounts. I will add, that they would be ſtill more uſeful, did they give the ages of the dead after 10, within periods of *five* inſtead of *ten* years.—During every period ſo ſhort as *five* years, the decrements of life may, in conſtructing Tables, be ſafely taken to be *uniform*. But this cannot be equally depended on, in periods ſo long as ten years.

There is yet another improvement of theſe accounts, which I ſhall take this opportunity to mention. They ſhould contain not only a liſt of the diſtempers of which all die, like that in the *London* Bills; but they ſhould ſpecify particularly the numbers dying of theſe diſtempers, in the ſeveral diviſions of life. Accurate regiſters of mortality kept in this manner in all parts of the kingdom, and compared with records of the ſeaſons, and of the weather, and with the particular circumſtances which diſcriminate different ſituations, might contribute, more than can be eaſily imagined, to the increaſe of *phyſical* knowledge.—But to proceed no farther in theſe Obſervations; I ſhall now beg leave to ſhut up this Eſſay with the following general reflection.

(*a*) See Eſſay I. p. 280, 281.

I have repreſented particulary the great difference between the duration of human life in towns and in country pariſhes; and from the facts I have recited it appears, that the further we go from the artificial and irregular modes of living in great towns the fewer of mankind die in the *firſt* ſtages of life, and the more in its *laſt* ſtages. The lower animals (except ſuch (*a*) as have been taken under human management) ſeem in general to enjoy the full period of exiſtence allotted them, and to die chiefly of old age: And were any obſervations to be made among *ſavages*, perhaps the ſame would be found to be true of them.—DEATH is an evil to which the order of Providence has ſubjected every inhabitant of this earth; but to man it has been rendered unſpeakably more an evil than

(*a*) Calves are the only animals taken under our peculiar care immediately after birth; and in conſequence of then adminiſtring to them the ſame ſort of phyſic that is given to infants, and treating them in other reſpects in the ſame manner; it is probable, that more of them die ſoon after being born, than of *all* the other ſpecies of animals, which we ſee in the ſame circumſtances. See the *Comparative View of the State and Faculties of Man with thoſe of the Animal World*, p. 23.—It is indeed, melancholy to think of the havock made among the human ſpecies by the unnatural *cuſtoms* as well as the *vices* which prevail in poliſhed ſocieties. I have no doubt, but that the cuſtom, in particular, of committing infants, as ſoon as born, to the care of *foſter-mothers*, deſtroys more lives than the ſword, famine, and peſtilence put together.

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it was deſigned to be. The greateſt part of that black catalogue of diſeaſes which ravage human life, is the off-ſpring of the tenderneſs, the luxury, and the corruptions introduced by the vices and falſe refinements of civil ſociety (*a*). That delicacy which is injured by every breath of air, and that rottenneſs of conſtitution which is the effect of indolence, intemperance, and debauchery, were never intended by the Author of Nature; and it is impoſſible, that they ſhould not lay the foundation of numberleſs ſufferings, and terminate in premature and miſerable deaths.—Let us then value more the ſimplicity and innocence of a life agreeable to nature; and learn to conſider nothing as ſavageneſs but malevolence, ignorance, and wickedneſs. The order of nature is wiſe and kind. In a conformity to it conſiſt health and long life; grace, honour, virtue and joy. But nature turned out of its way will always puniſh. *The wicked ſhall not*

(*a*) The ingenious and excellent writer quoted in the laſt note, obſerves, that the whole claſs of diſeaſes which ariſe from catching cold, are found only among the civilized part of mankind, p. 51.—And, concerning that loſs of all our higher powers which ſo often attends the decline of life, and which is ſo humiliating to human pride, he obſerves, that it exhibits a ſcene ſingular in nature, and that there is the greateſt reaſon to believe, that it proceeds from adventitious cauſes, and would not take place among us if we led natural lives, p. 62.

live

live out half their days. Criminal exceſſes embitter and cut ſhort our *preſent* lives; and the higheſt authority has taught us to expect, that they will not only kill the *body*, but the *ſoul*; and deprive of ETERNAL LIFE.

END OF VOL. I.

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